Parallel vector and Cross product

Cross product

Example: calculator cross product

$$\vec{A} = 3i + 8j + 2k \qquad \vec{B} = 4i + 5j + 7k$$

$$A \times B = \begin{vmatrix} i & j & k \\ 3 & 8 & 2 \\ 4 & 5 & 7 \end{vmatrix} \begin{matrix} i & j \\ 3 & 8 \\ 4 & 5 \end{matrix}$$

= 56i + 8j +15k - (21j + 10i +32k)

= 56i+8j+15k -21j -10i – 32k

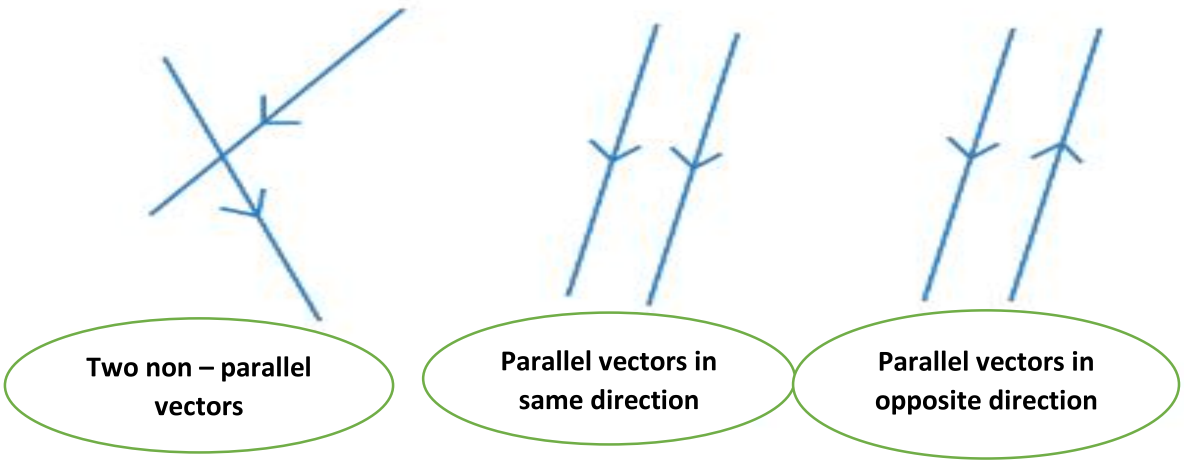
= 46 I -13 j – 17 k

Parallel vector

The parallel vectors are vectors that have the same direction or exactly the opposite direction. **i.e**., for any vector a , the vector itself and its opposite vector -a are vectors that are always parallel to a. Extending this further, any scalar multiple of a is parallel to a. **i.e**., a vector a and (ka) are always parallel vectors where 'k' is a scalar (real number).

What are Parallel Vectors?

Two vectors are said to be parallel if and only if the angle between them is 0 degrees. Parallel vectors are also known as collinear vectors. i.e., two parallel vectors will be always parallel to the same line but they can be either in the same direction or in the exact opposite direction. In the following image, the vectors shown in the left-most figure are NOT parallel as they have different directions (i.e., neither the same nor opposite directions).



The parallel vectors that are in opposite directions are sometimes referred to as anti-parallel vectors too. In the above image, the last figure shows the anti-parallel vectors.

How to Find Parallel Vectors?

Two vectors **a** and **b** are said to be parallel vectors if one is a scalar multiple of the other. i.e., **a** = k **b**, where 'k' is a scalar (real number). Here, 'k' can be positive, negative, or 0. In this case,

- **a** and **b** have the same directions if k is positive.
- **a** and **b** have opposite directions if k is negative.

Here are some examples of parallel vectors:

- **a** and 3**a** are parallel and they are in the same directions as 3 > 0.
- **v** and (-1/2) **v** are parallel and they are in the same directions as (-1/2) < 0.
- **a** = <1, -3> and **b** = <3, -9> are parallel as **b** = <3, -9> = 3 <1, -3> = 3**a**.

In the above examples, example 2 refers to the anti-parallel vectors.

Parallel Vectors Formula

The parallel vectors can be determined by using the scalar multiple, or cross product. Here is the parallel vectors formula according to its meaning explained in the previous sections.

Parallel Vectors Formula

$\vec{a}$ and $\vec{b}$ are parallel if

(i) $\vec{a} = k\vec{b}$, where 'k' is a scalar

(or)

(ii) $\vec{a} \times \vec{b} = 0$

(or)

(iii) $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|$

Example 1: Determine whether the vectors **a** = <10, -6> and **b** = <15, -9> are parallel.

Solution:

a = k**b**

< 10 , - 6 > = k <15 , -9 >

< 10 , - 6 > = < 15 k , -9 k>

10 = 15 k ; - 6 = -9 k

$$k = \frac{10}{15} = \frac{2}{3}, \quad k = \frac{-6}{-9} = \frac{2}{3}$$

Since the value of 'k' is the same in both cases,

a = (2/3) **b**

a and **b** are parallel vectors.

Example 2 : Determine whether the vectors **a** = <-2, 3> and **b** = < 4, -6 > are parallel.

Solution:

b=ka

$$\langle 4\ ,-6\rangle = \ k\,\langle -2\ ,\ 3\rangle$$

$$\langle 4\ ,-6\rangle = \langle -2k\ ,3k\rangle$$

$$-2k = 4 \quad \rightarrow k = \frac{4}{-2} = -2$$

$$3k = -6 \ \rightarrow k = \frac{-6}{3} = -2$$

K= -2

a and b are parallel

Example 3 : Determine whether the vectors **a** = <2, 3> and **b** = < 4, 7 > are parallel.

Solution: **b = ka**

$$\langle 4\ ,7\ \rangle = \ k\,\langle 2,3\rangle$$

$$\langle 4\ ,7\rangle = \langle 2k\ ,3k\rangle$$

$$2k = 4 \quad \rightarrow k = \frac{4}{2} = 2$$

$$3k = 7 \rightarrow k = \frac{7}{3}$$

K= -2

a and b are not parallel

Example 4 : $a = \langle -1\,,2,3\rangle \quad b = \langle 2\ ,\ -4\,,-6\rangle$ $two\ vector\ are\ a\ \ and\ \ b\ \ is\ parallel$

$$A \times B = \begin{vmatrix} i & j & k \\ -1 & 2 & 3 \\ 2 & -4 & -6 \end{vmatrix} \begin{matrix} i & j \\ -1 & 2 \\ 2 & -4 \end{matrix}$$

(-12i+ 6j +4k) - (6j – 12i +4k) = -12i +6j +4k – 6j + 12i -4k = 0

A and b are parallel

Example 4 : $\boldsymbol{a} = \langle 1, 2, -1 \rangle$ $\boldsymbol{b} = \langle \mathbf{2}, 4, 2 \rangle$ ***two vector are a and b is parallel***

$$A \times B = \begin{vmatrix} i & j & k \\ 1 & 2 & -1 \\ 2 & 4 & 2 \end{vmatrix} \begin{matrix} i & j \\ 1 & 2 \\ 2 & 4 \end{matrix}$$

A x B = (4i -2j +4k) - (2j -4i +4k)

= 4i -2j +4k -2j +4i – 4k = 8i -4j +8k

a and b are not parallel