Al-Mustaqbal University
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Digital Communication

Lecture 2

Sampling Theorem

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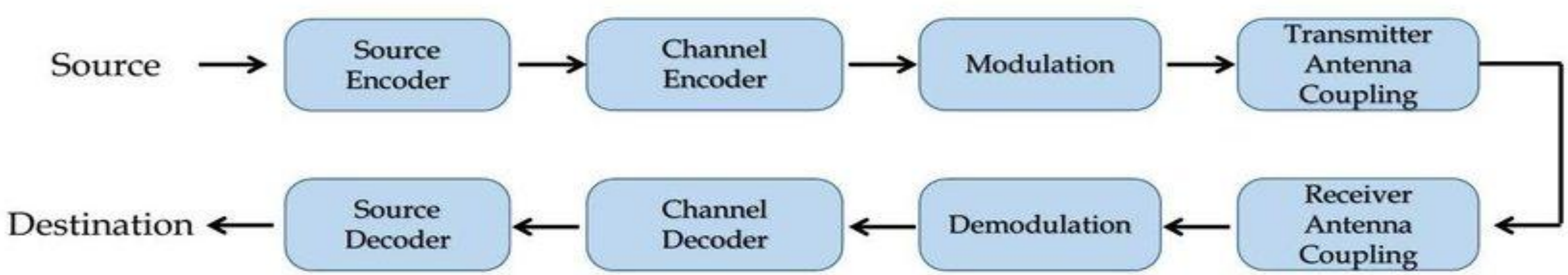
Aims of this Lecture

By the end of this lecture, students will **be able** to:

- **Identify** the key concepts of sampling, Nyquist rate, and signal classification
- **Identify** different types of signals.
- **Explain** the role of the Nyquist rate in preventing aliasing in digital communication
- **Calculate** the Nyquist rate and Nyquist interval for different signals using examples
- **Compare** analog and discrete signals and their role in communication systems.

Information Source

- It is a device producing the data to be communicated.
- Can be **analog** or **discrete**.
- Analog sources are **transformed** into digital through **sampling** and **quantization**.



Binary Digit (Bit) and Bit Stream

- **Bit**: The fundamental unit of digital information (0 or 1).
- **Bit Stream**: A sequence of binary digits, forming a baseband signal for digital transmission.

Data Rate

- **Data Rate (R):** The number of bits per second (bits/s) transmitted.

- **Formula**: $\mathrm{R} = \frac{K}{T}$

where k bits identify a symbol, and T is the symbol duration.

Classification of Signals

- **Deterministic Signals**: Modeled by explicit mathematical expressions.
- **Random Signals**: Described by probabilities and averages.
- **Periodic Signals:** Repeat over time, defined by period T 0 T 0 .
- **Nonperiodic Signals:** Do not repeat over time, meaning no T 0 T 0 satisfies the periodic condition.
- **Analog Signals:** Continuous functions of time.
- Discrete Signals: Exist at specific, discrete times.

Sampling Theorem

- A continuous-time signal can be converted to discrete-time using **sampling**.
- **Nyquist Rate**: A signal with no frequency components higher than W Hz can be recovered if sampled at a rate of **2W** samples per second.

Sampling Theorem

Concept:

- The **Sampling Theorem** states that a **continuous-time** signal can be converted into a **discrete-time** signal by taking samples at regular intervals. This process is called **sampling**.
- To ensure that no information is lost during sampling, the sampling rate must be at least **twice the highest frequency** of the original signal. This minimum rate is known as the **Nyquist Rate**.

Sampling Theorem

Key Formula (Nyquist Rate):

- **Nyquist Rate**:
  $f_s \geq 2f_m$
  where f_s is the sampling rate and f_m is the highest frequency of the signal.

Explanation:

1. **A band-limited signal**:
   - A signal with no frequency components higher than W Hz (also called a **band-limited signal**) can be fully recovered from its samples if the sampling rate is at least **2W samples per second**. This is the **Nyquist rate**.

Sampling Theorem

2. **Sampling Interval**:
   - The time between two consecutive samples is called the **sampling interval**, denoted as T_s.
     The relationship between the sampling rate f_s and the sampling interval T_s is:

$$T_s = \frac{1}{f_s}$$

For example, if the Nyquist rate is 500 Hz, the sampling interval T_s is $\frac{1}{500} = 2$ milliseconds.

3. **Sampling Theorem Statement**:
   - If a signal is band-limited to W Hz, the signal can be **completely described** by its values at instants separated by $\frac{1}{2W}$ seconds.
   - It can also be **completely recovered** if sampled at a rate of at least **2W samples per second**.

Proof of Sampling Theorem

Proof of Sampling Theorem:

- Let $x(t)$ be a continuous-time signal that is band-limited with no frequency components higher than W Hz.
- The signal is sampled at regular intervals with a sampling rate f_s, and the sampled signal is represented as:

$$x_s(t) = x(t) \cdot \delta(t - nT_s)$$

  where $T_s = \frac{1}{f_s}$ is the sampling interval and $\delta(t)$ is the Dirac delta function (impulse function).

- The Fourier transform of the sampled signal becomes an impulse train in the frequency domain:

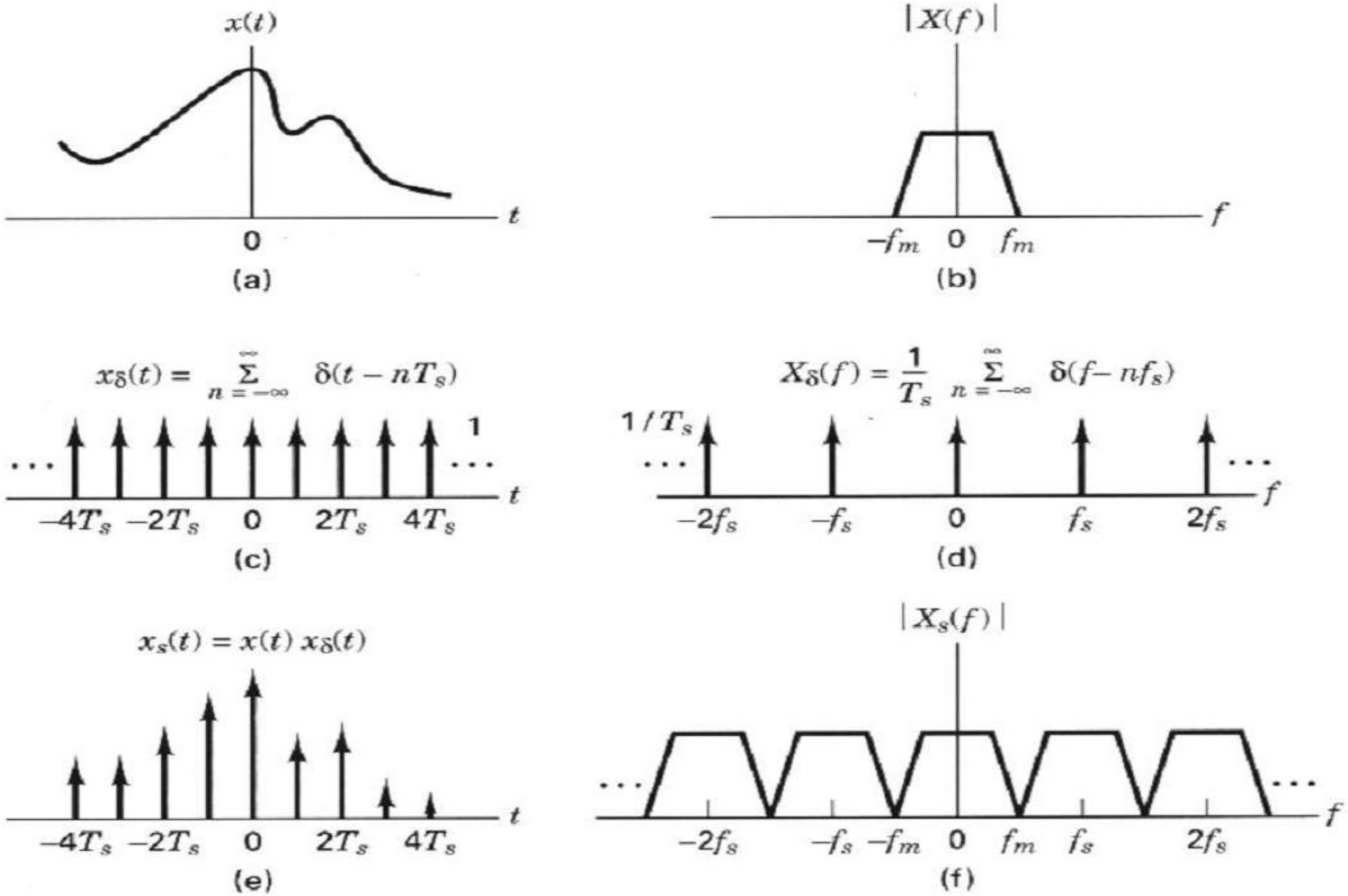
$$X_s(f) = \sum X(f - nf_s)$$

  - If the sampling rate $f_s \geq 2f_m$, each spectral replicate is separated by a frequency band of f_s, allowing the original signal to be recovered using a **low-pass filter**.

Aliasing:

- **Aliasing** occurs when the sampling rate is less than $2f_m$, causing spectral overlaps that result in loss of information. The overlapping frequencies make it impossible to distinguish the original signal.
- To avoid **aliasing**, we must ensure the sampling rate $f_s \geq 2f_m$.

Proof of Sampling Theorem



Example 1:

Find the **Nyquist rate** and **Nyquist interval** for the following signal:

$$m(t) = \frac{\sin(500\pi t)}{\pi t}$$

Solution:

1. **Identify the highest frequency:**
   - The signal $m(t) = \frac{\sin(500\pi t)}{\pi t}$ is a sinc function, which has a frequency component given by 500π.
   - Using the general relation for angular frequency $\omega = 2\pi f$, we can solve for the frequency f:

   $$2\pi f = 500\pi \quad \Rightarrow \quad f = \frac{500\pi}{2\pi} = 250\,\text{Hz}$$

   - Therefore, the highest frequency component f_m is **250 Hz**.
2. **Calculate the Nyquist rate:**
   - The Nyquist rate is twice the highest frequency component:

   $$f_s = 2f_m = 2 \times 250 = 500\,\text{Hz}$$

3. **Calculate the Nyquist interval:**
   - The Nyquist interval is the time between consecutive samples:

   $$T_s = \frac{1}{f_s} = \frac{1}{500} = 0.002\,\text{seconds} = 2\,\text{milliseconds}$$

Summary:

- **Nyquist Rate:** $500\,\text{Hz}$
- **Nyquist Interval:** $2\,\text{milliseconds}$

Find the **Nyquist rate** and **Nyquist interval** for the following signal:

$$m(t) = \frac{1}{2\pi}\cos(4000\pi t)\cdot\cos(1000\pi t)$$

Solution:

1. **Simplify the product of cosines**:

   - The signal involves a product of two cosine terms. Using the trigonometric identity for the product of cosines:

   $$\cos A\cdot\cos B = \frac{1}{2}[\cos(A-B)+\cos(A+B)]$$

   - Apply this identity to the given signal:

   $$m(t) = \frac{1}{2\pi}\left[\frac{1}{2}\left(\cos(4000\pi t - 1000\pi t) + \cos(4000\pi t + 1000\pi t)\right)\right]$$

   $$= \frac{1}{4\pi}\left[\cos(3000\pi t) + \cos(5000\pi t)\right]$$

   - The frequencies involved are 3000π and 5000π. Dividing by 2π, the corresponding frequencies are:

   $$f_1 = 1500\,\mathrm{Hz},\quad f_2 = 2500\,\mathrm{Hz}$$

   - The highest frequency component is $2500\,\mathrm{Hz}$.

2. **Calculate the Nyquist rate**:

    - The Nyquist rate is twice the highest frequency component:

$$f_s = 2f_m = 2 \times 2500 = 5000\,\text{Hz}$$

3. **Calculate the Nyquist interval**:

    - The Nyquist interval is the time between consecutive samples:

$$T_s = \frac{1}{f_s} = \frac{1}{5000} = 0.0002\,\text{seconds} = 0.2\,\text{milliseconds}$$

Summary:

- **Nyquist Rate**: $5000\,\text{Hz}$
- **Nyquist Interval**: $0.2\,\text{milliseconds}$