Class: 2nd Class
Subject: Mechanics of Materials
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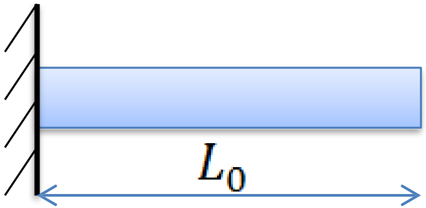
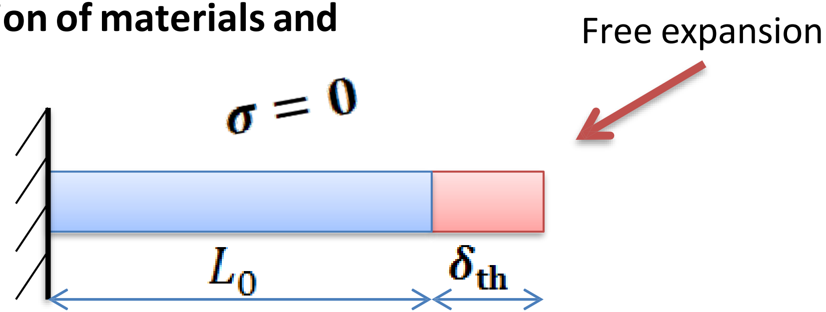
Lec4/Thermal stress

Thermal stress

Changes in temperature produce expansion or contraction of materials and result in thermal strains and thermal stress.



At room temperature



After temperature increased a bar is expanded

Heat

$$\delta_{th} = L\,\alpha\,\Delta T$$

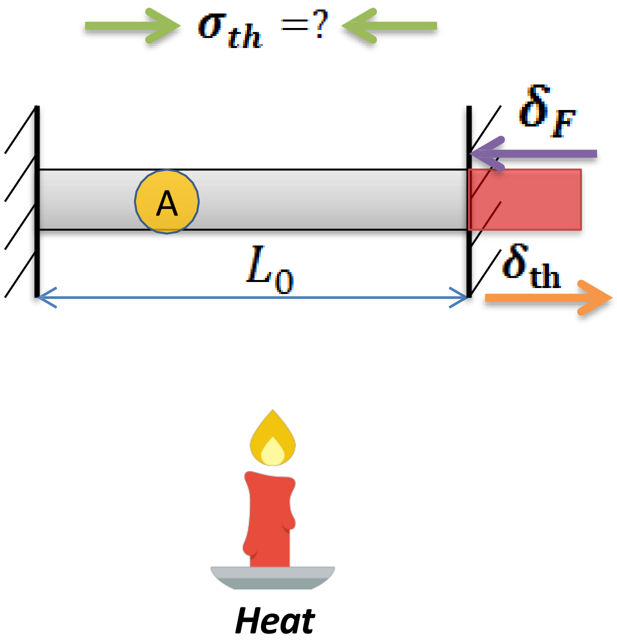
δ_{th} = *thermal strain*

L = *length of bar* (m)

α = *coefficient of thermal expansion or contraction* $(\frac{m}{m}\,°C)$

ΔT = *change in the temperature* (°C)

In case rigid support:



$$\delta_F = \delta_{th}$$

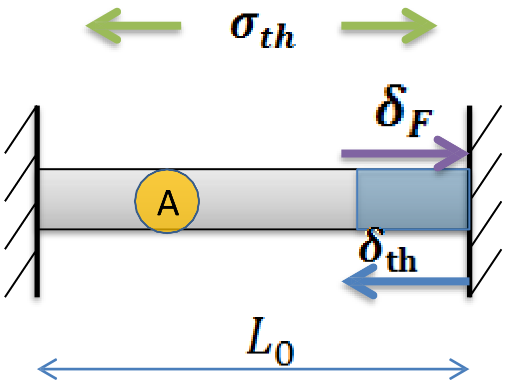
$$\frac{PL}{AE} = L\,\alpha\,\Delta T$$

$$\frac{P}{A} = E\,\alpha\,\Delta T$$

$$\sigma_{th} = E\,\alpha\,\Delta T$$

Compression

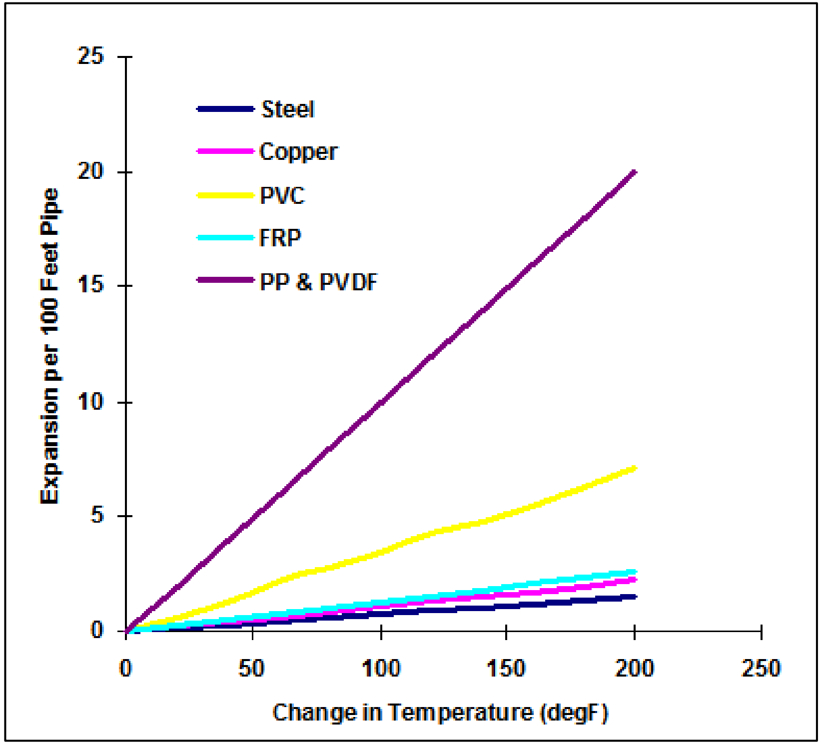
In case rigid support:



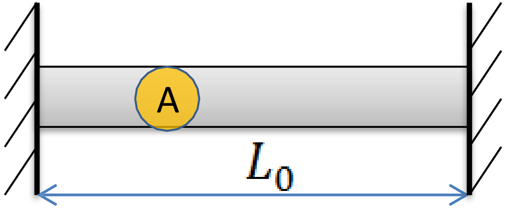
Cold

$$\sigma_{th} = E\,\alpha\,\Delta T$$

Tension

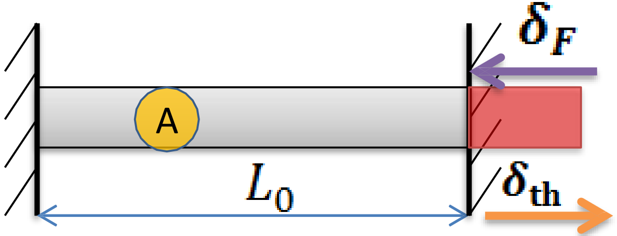


Q/ *Fixed-Fixed bar was 20°C at room temperature, Determine the stress when the bar is heated to 100°C ?*



$$E_{st} = 200\ Gpa$$
$$\alpha_{st} = 11.7\ \mu m/m°C$$
$$A_{st} = 200\ mm^2$$

Sol:



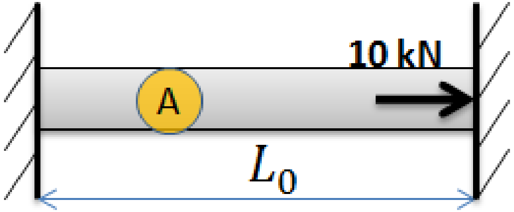
$$\sigma_{th} = E\ \alpha\ \Delta T$$

$$\sigma_{th} = E\ \alpha\ (T_2 - T_1)$$

$$\sigma_{th} = \left(200x10^9\right)\left(11.7x10^{-6}\right)(100 - 20)$$

$$\sigma_{th} = 187.2\ Mpa\ \ (-Compression)$$

Q/ *Fixed-Fixed bar was 20°C at room temperature, Determine the stress when the bar is initially stretched by a force at* ***10 KN*** *,then heated to* ***100****°C* ***?***



$E_{st} = 200\ Gpa$

$\alpha_{st} = 11.7\ \mu m/m°C$

$A_{st} = 200\ mm^2$

Sol:

$$\sigma_{th} = E\ \alpha\ \Delta T$$

$$\sigma_{th} = E\ \alpha\ (T_2 - T_1)$$

$$\sigma_{th} = (200x10^9)(11.7x10^{-6})(100 - 20)$$

$$\sigma_{th} = 187.2\ Mpa\ \ (-Compression)$$

$$\sigma_{initial} = \frac{P}{A} = \frac{10x10^3}{200x10^{-6}}$$

$$\sigma_{initial} = 50\ Mpa\ (+Tension)$$

$$\sigma_T = -187.2 + 50$$

$$\sigma_T = -137.2\ Mpa\ (-Compression)$$

Q/The composite bar shown in figure below is firmly attached to unyielding supports. An axial load P= 200 kN is applied at 20°C, Find the stress in each materials at 60°C ?

Steel

$E = 200\ Gpa$

$A = 1200\ mm^2$

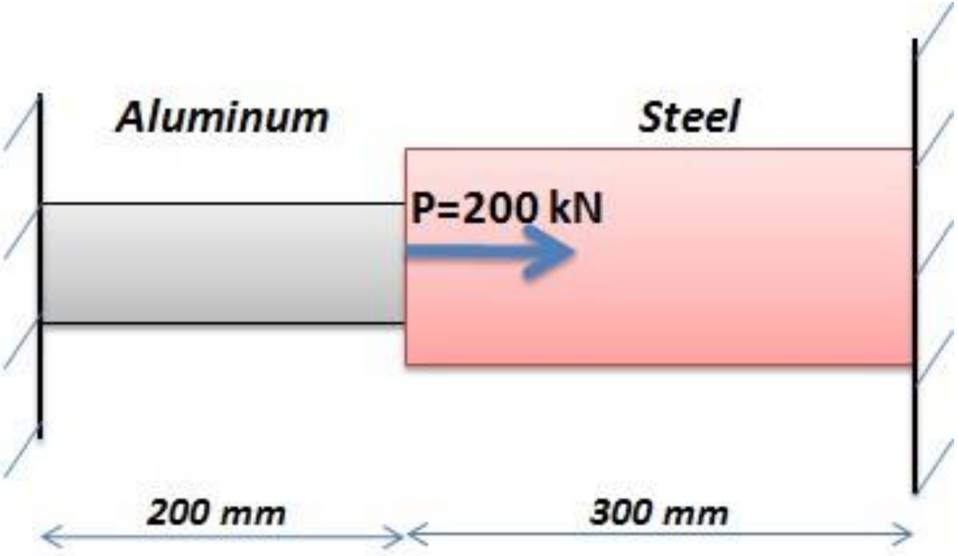
$\alpha_{st} = 11.7\ \mu m/m°C$

Aluminum

$E = 70\ Gpa$

$A = 900\ mm^2$

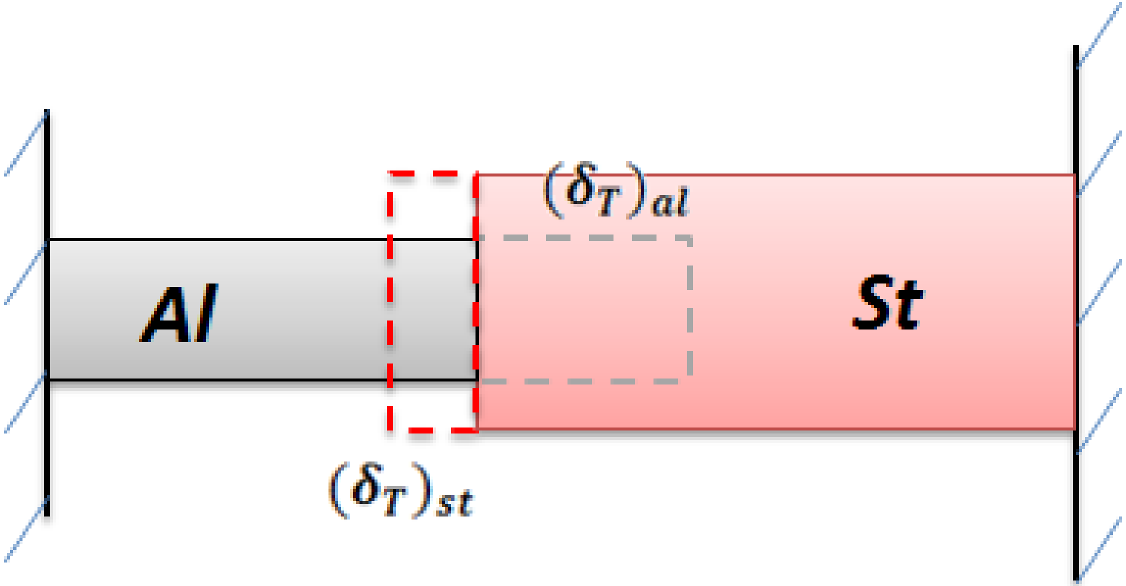
$\alpha_{al} = 23\ \mu m/m°C$



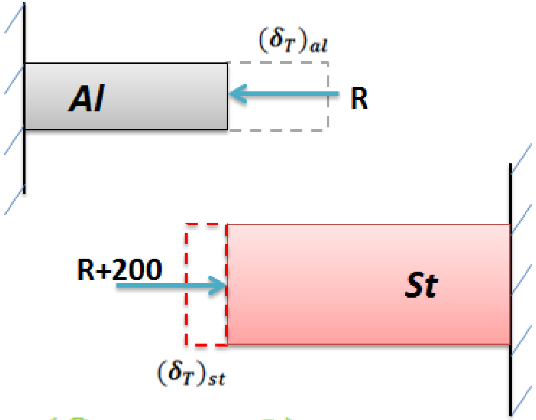
$\Delta T = 60 - 20 = 40°C$

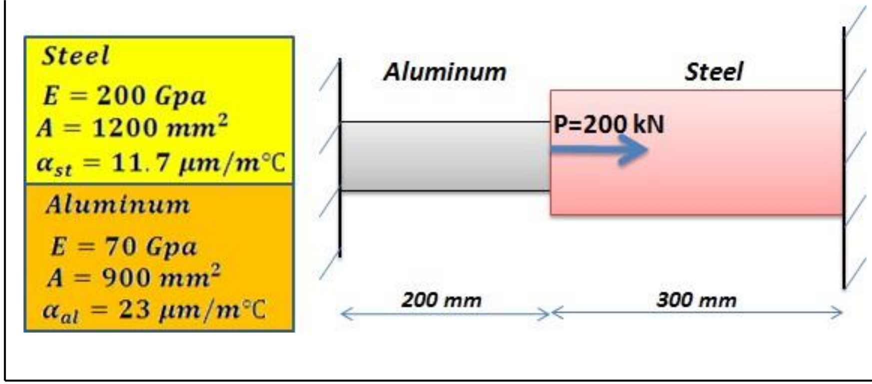
Steel
$E = 200\ Gpa$
$A = 1200\ mm^2$
$\alpha_{st} = 11.7\ \mu m/m°C$

Aluminum
$E = 70\ Gpa$
$A = 900\ mm^2$
$\alpha_{al} = 23\ \mu m/m°C$



$$\alpha_{al} > \alpha_{st}$$





$$\delta_{total} = \delta_{st} + \delta_{al} \quad , \quad (\delta_{total} = 0)$$

$$\Delta T = 60 - 20 = 40°C$$

$$0 = \delta_{st} + \delta_{al}$$

$$0 = (\delta_T)_{st} - (\delta_F)_{st} + (\delta_T)_{al} - (\delta_F)_{al}$$

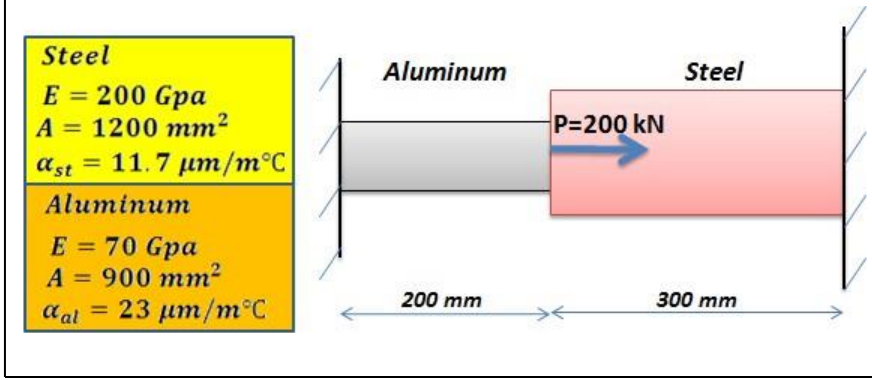
$$(\delta_T)_{st} + (\delta_T)_{al} = (\delta_F)_{al} + (\delta_F)_{st}$$

$$(\alpha\, L\, \Delta T)_{st} + (\alpha\, L\, \Delta T)_{al} = \left(\frac{PL}{AE}\right)_{al} + \left(\frac{PL}{AE}\right)_{st}$$

$$(10^{-6})(40)[(11.7x0.3) + (23x0.2)] = \frac{10^{-3}}{(10^{-6})(10^{9})}\left[\frac{R(200)}{900(70)} + \frac{(R + 200x10^3)(300)}{1200(200)}\right]$$

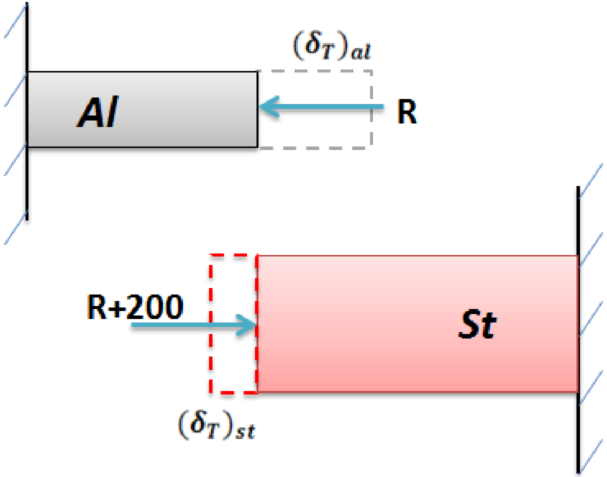
$$R = 16.8\ kN$$

$$R = 16.8\ kN$$



<u>*Calculate the stress by:*</u>

$$\sigma = \frac{P}{A}$$



$$\sigma_{al} = \frac{16.8x10^3}{900}\left(\frac{N}{mm^2}\right) = 18.7\ Mpa$$

$$\sigma_{st} = \frac{(16.8 + 200)x10^3}{1200}\left(\frac{N}{mm^2}\right) = 181\ Mpa$$

Steel

$E = 200\ Gpa$

$A = 1200\ mm^2$

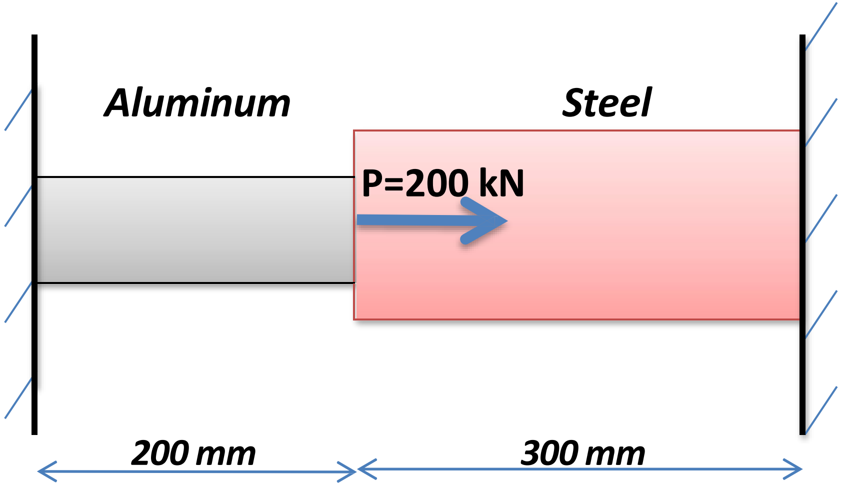
$\alpha_{st} = 11.7\ \mu m/m°C$

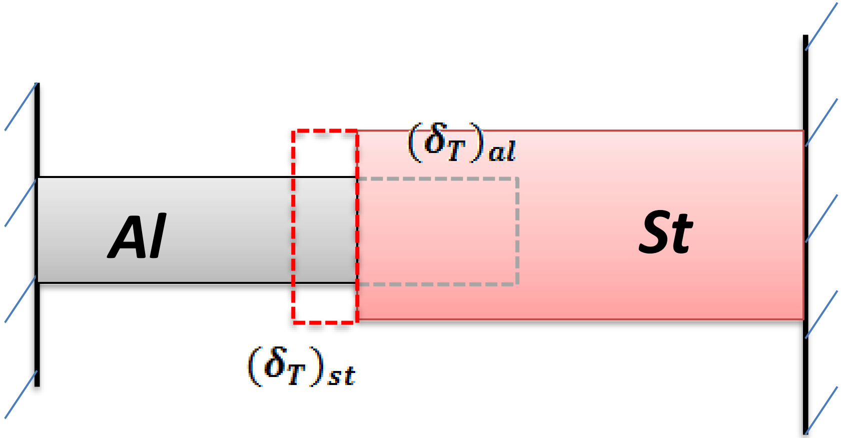
Aluminum

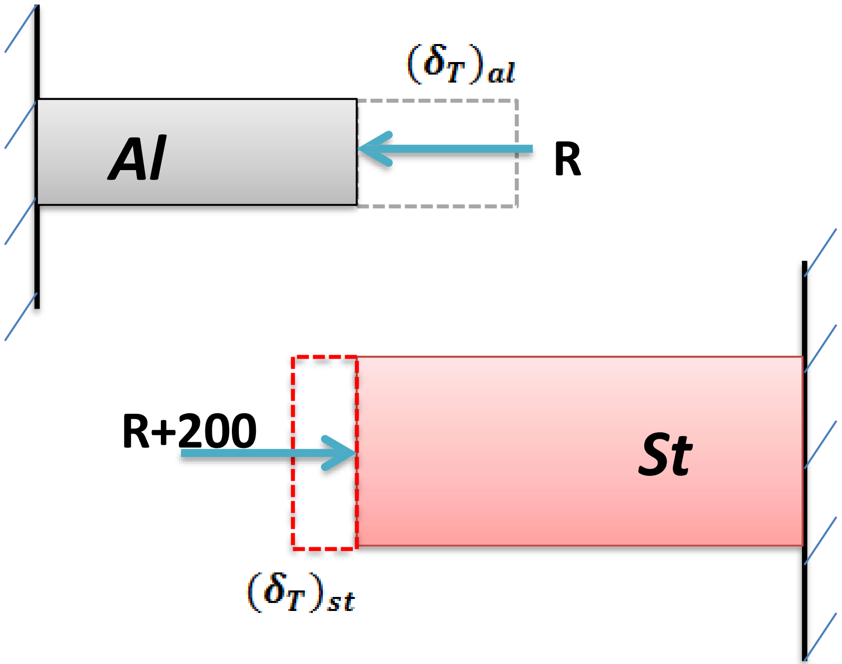
$E = 70\ Gpa$

$A = 900\ mm^2$

$\alpha_{al} = 23\ \mu m/m°C$





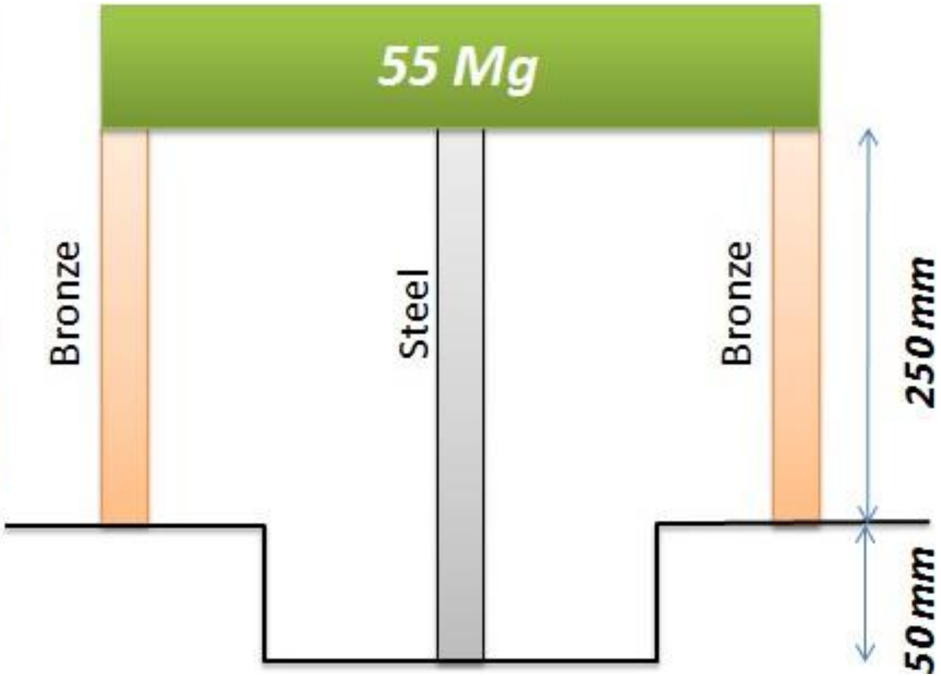

$(\delta_T)_{al}$
Al
R
R+200
St
$(\delta_T)_{st}$

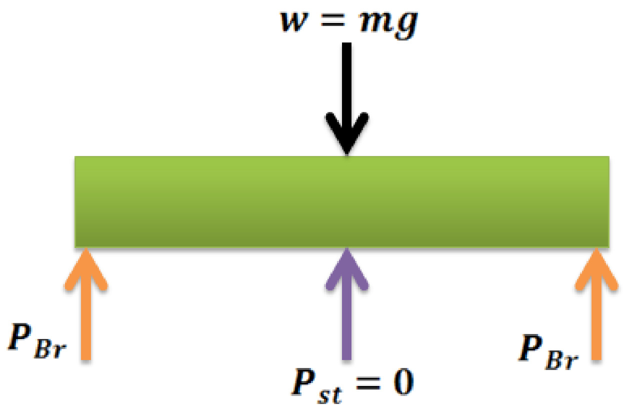
Q/ At 20℃, a rigid slab having a mass of 55 Mg is placed upon two bronze rods and one steel rod as shown in figure below. At what temperature will the stress in the steel rod be Zero ?

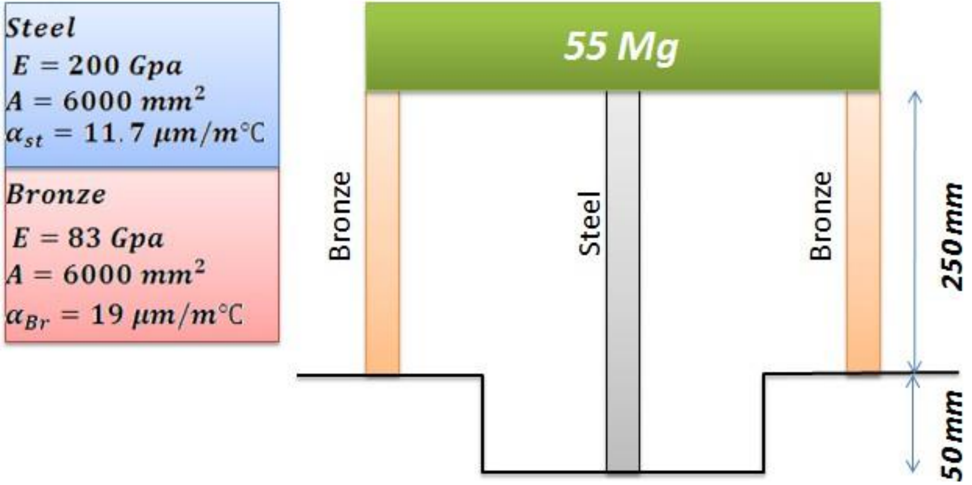
$$\sigma_{st} = 0$$

Steel
$E = 200\ Gpa$
$A = 6000\ mm^2$
$\alpha_{st} = 11.7\ \mu m/m℃$

Bronze
$E = 83\ Gpa$
$A = 6000\ mm^2$
$\alpha_{Br} = 19\ \mu m/m℃$







$$\sigma_{st} = 0$$

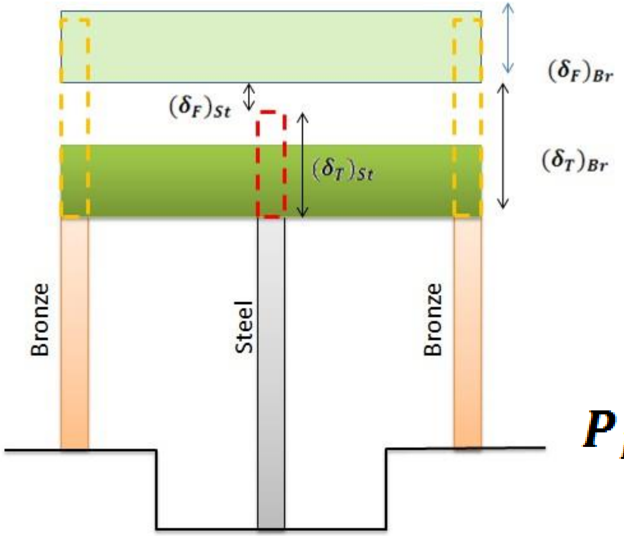
$$P = \sigma_{st} A = 0$$

$$\sum F_y = 0$$

$$2P_{Br} - mg = 0$$

$$P_{Br} = \frac{(55x10^3)(9.81)}{2} = 270kN$$

$$\alpha_{Br} > \alpha_{St}$$



$$\sigma_{st} = 0$$

$$P = \sigma_{st}\, A = 0$$

$$(\delta_F)_{St} = \frac{(0)L}{AE} = 0$$

$$\delta_T = \delta_F$$

Steel

$E = 200\ Gpa$

$A = 6000\ mm^2$

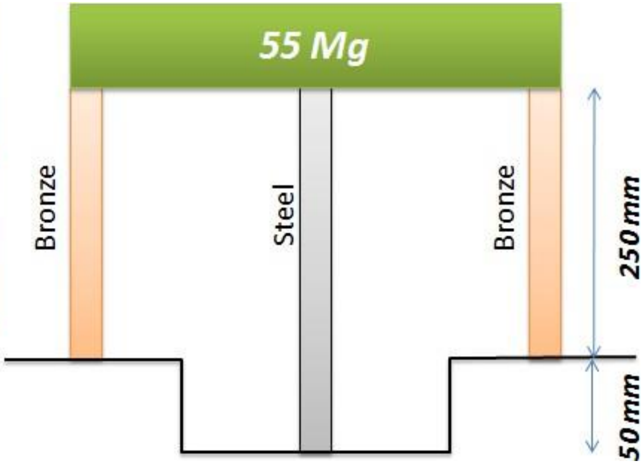
$\alpha_{st} = 11.7\ \mu m/m°C$

Bronze

$E = 83\ Gpa$

$A = 6000\ mm^2$

$\alpha_{Br} = 19\ \mu m/m°C$



$$P_{Br} = 270kN$$

$$(\delta_T)_{Br} - (\delta_F)_{Br} = 0$$

$$(\delta_T)_{St} - (\delta_F)_{St} = 0$$

$$\delta_{total} = 0$$

$$(\delta_T)_{Br} - (\delta_F)_{Br} = (\delta_T)_{St} - (\delta_F)_{St}$$

$$(\alpha\, L\, \Delta T)_{Br} - \left(\frac{PL}{AE}\right)_{Br} = (\alpha\, L\, \Delta T)_{St}$$

$$(19x10^{-6})(0.25)\Delta T - \left(\frac{270x10^3(0.25)}{6000x10^{-6}(83x10^9)}\right) = (11.7x10^{-6})(0.3)\Delta T$$

$$\Delta T = 109\ °C$$

$$T_2 = T_1 + \Delta T = 20 + 109$$

$$T_2 = 129\ °C$$

Steel

$E = 200\ Gpa$

$A = 6000\ mm^2$

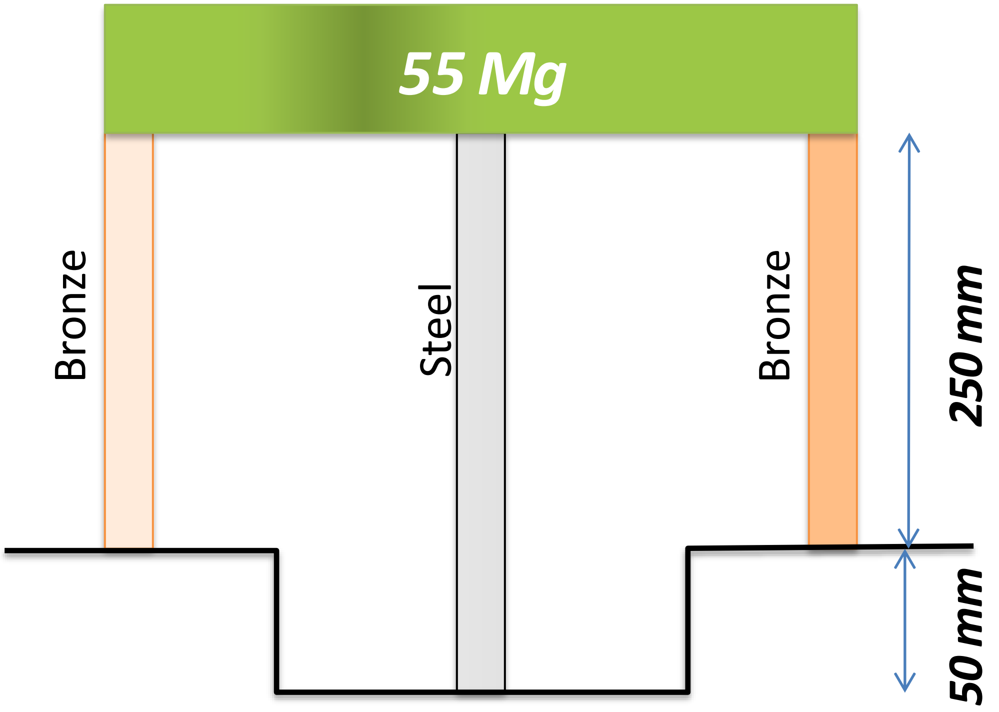
$\alpha_{st} = 11.7\ \mu m/m°C$

Bronze

$E = 83\ Gpa$

$A = 6000\ mm^2$

$\alpha_{Br} = 19\ \mu m/m°C$

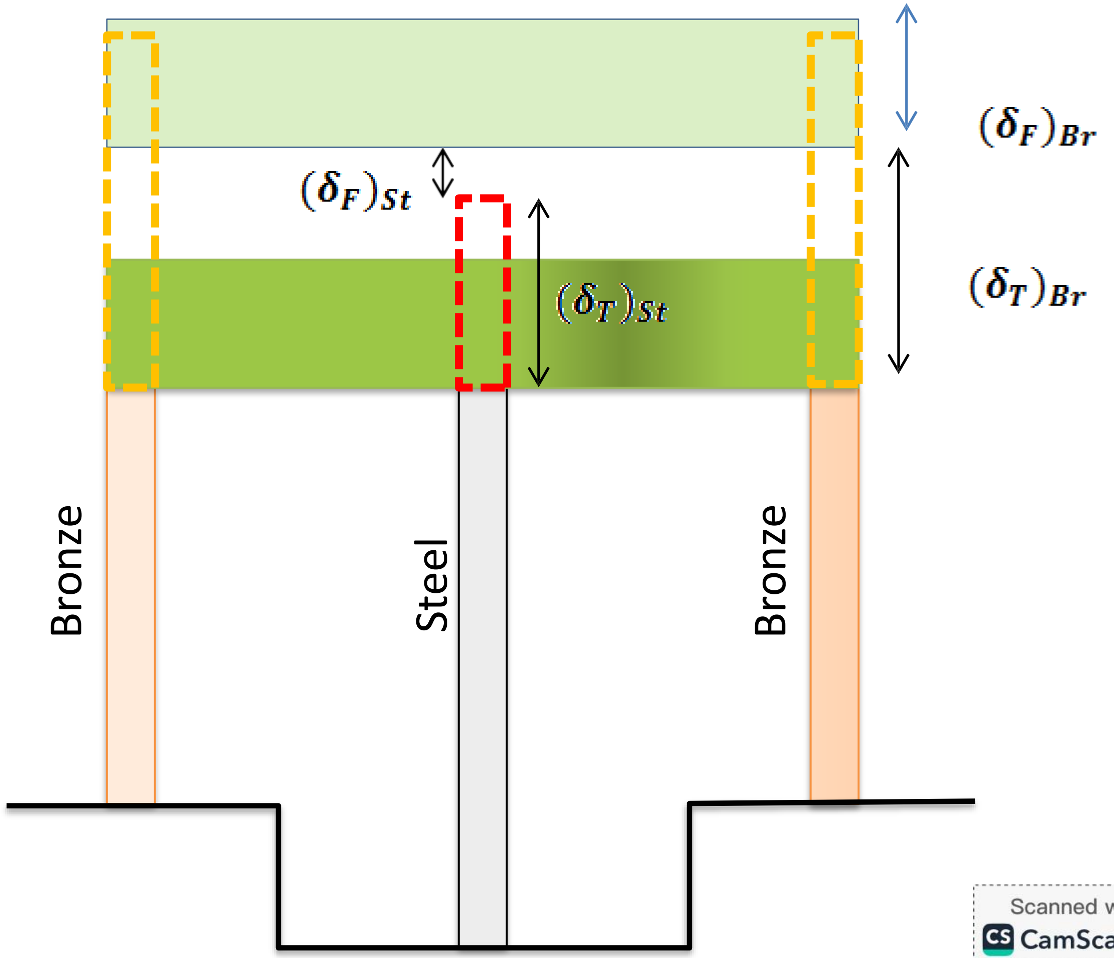


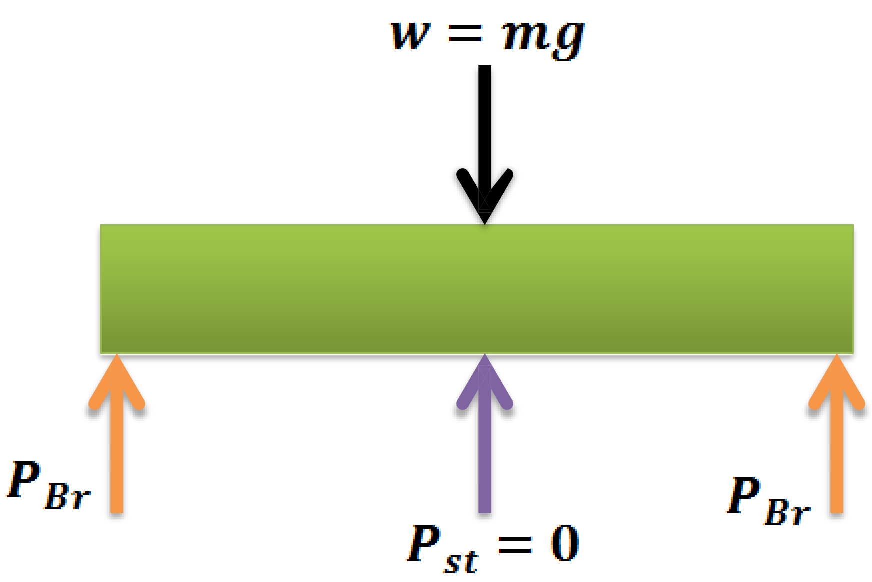
$$\alpha_{Br} > \alpha_{St}$$

$$\sigma_{st} = 0$$

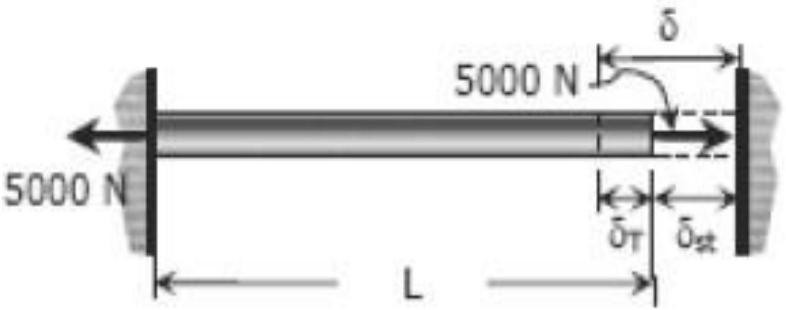
$$P = \sigma_{st} A = 0$$

$$(\delta_F)_{St} = \frac{(0)L}{AE} = 0$$




$w = mg$
P_{Br}
$P_{st} = 0$
P_{Br}

Q/ A steel rod is stretched between two rigid walls and carries a tensile load of 5000 N at 20°C. If the allowable stress is not to exceed 130 MPa at -20°C, what is the minimum diameter of the rod? Assume α = 11.7 µm/(m·°C) and E = 200 GPa.



$$\delta = \delta_T + \delta_{st}$$

$$\frac{\sigma \cancel{L}}{E} = \alpha \cancel{L}(\Delta T) + \frac{P\cancel{L}}{AE}$$

$$\sigma = \alpha E(\Delta T) + \frac{P}{A}$$

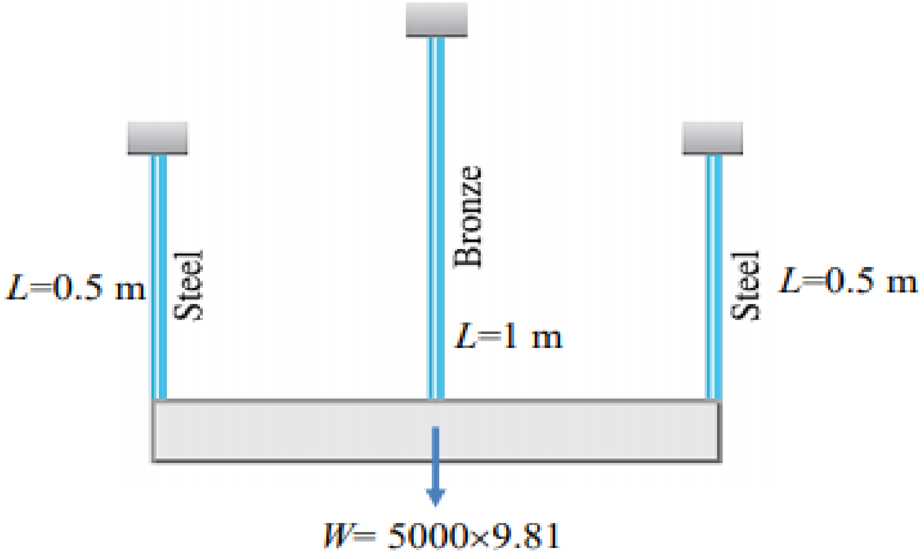
$$130 = (11.7 \times 10^{-6})(200\,000)(40) + \frac{5000}{A}$$

$$A = \frac{5000}{36.4} = 137.36 \text{ mm}^2$$

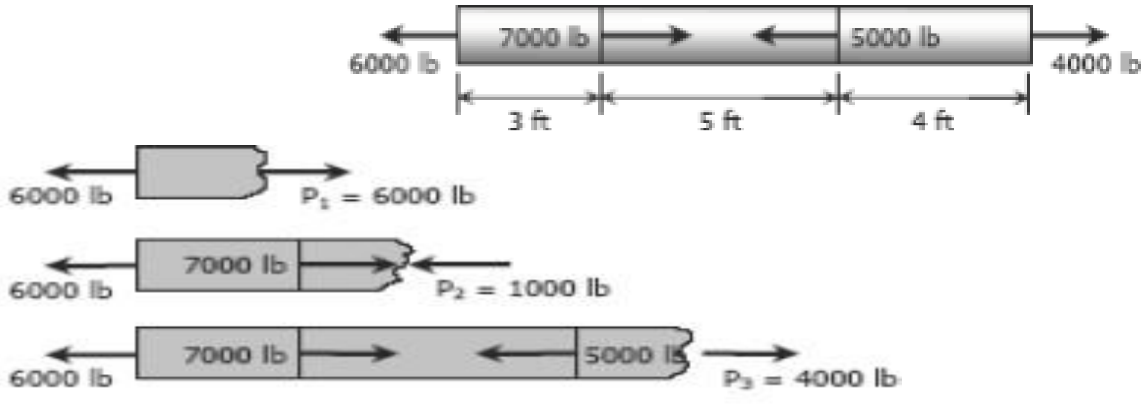
$$\frac{1}{4}\pi d^2 = 137.36; \quad d = 13.22 \text{ mm}$$

H.W/

A rigid block having a mass 5 Mg is supported by three rods symmetrically placed, as shown in the figure. Determine the stress in each rod after a temperature rise of 40 °C. Use E_s=200 GPa, α_s=11.7 μm/m·°C, A_s=500 mm^2, E_b=83 GPa, α_b=18. 9 μm/m·°C, and A_b=900 mm^2.



Q/ An aluminum bar having a cross-sectional area of 0.5 in^2 carries the axial loads applied at the positions shown in Fig. P-209. Compute the total change in length of the bar if E = 10 × 10^6 psi. Assume the bar is suitably braced to prevent lateral buckling.



P_1 = 6000 lb tension
P_2 = 1000 lb compression
P_3 = 4000 lb tension

$$\delta = \frac{PL}{AE}$$

$$\delta = \delta_1 - \delta_2 + \delta_3$$

$$\delta = \frac{6000(3\times12)}{0.5(10\times10^6)} - \frac{1000(5\times12)}{0.5(10\times10^6)} + \frac{4000(4\times12)}{0.5(10\times10^6)}$$

$\delta = 0.0696$ in (lengthening)