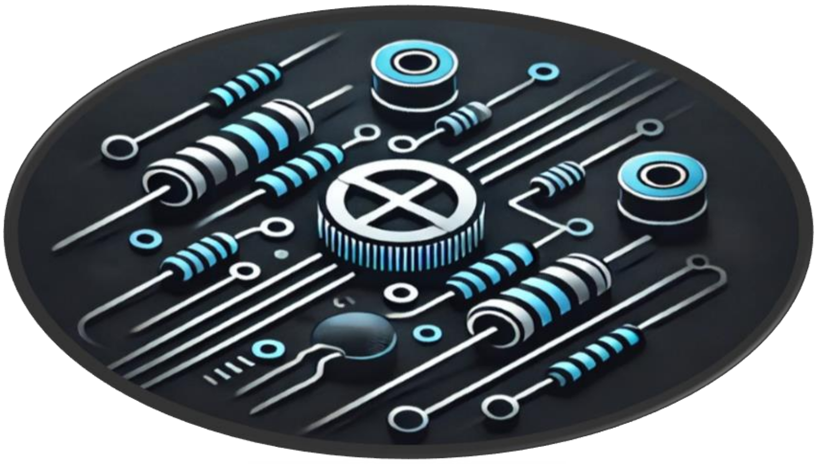
Ministry of Higher Education and Scientific Research

Al-Mustaqbal University

Electrical Engineering Techniques Department



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Second-Order Circuits

Lecture (3)

Transient
Second-Order Circuits

1) Introduction

In the previous lecture we considered circuits with a single storage element (a capacitor or an inductor). **Such circuits are first-order because the differential equations describing them are first-order.** In this lecture we will consider circuits containing two storage elements. **These are known as second-order circuits because their responses are described by differential equations that contain second derivatives.**

Typical examples of second-order circuits are RLC circuits, in which the three kinds of passive elements are present. Examples of such circuits are shown in Fig. 1.1(a) and (b). Other examples are RL and RC circuits, as shown in Fig. 1.1(c) and (d). It is apparent from Fig. 1.1 that **a second-order circuit may have two storage elements of different type or the same type(provided elements of the same type cannot be represented by an equivalent single element).**

A second-order circuit is characterized by a second-order differential equation. It consists of resistors and the equivalent of two energy storage elements.

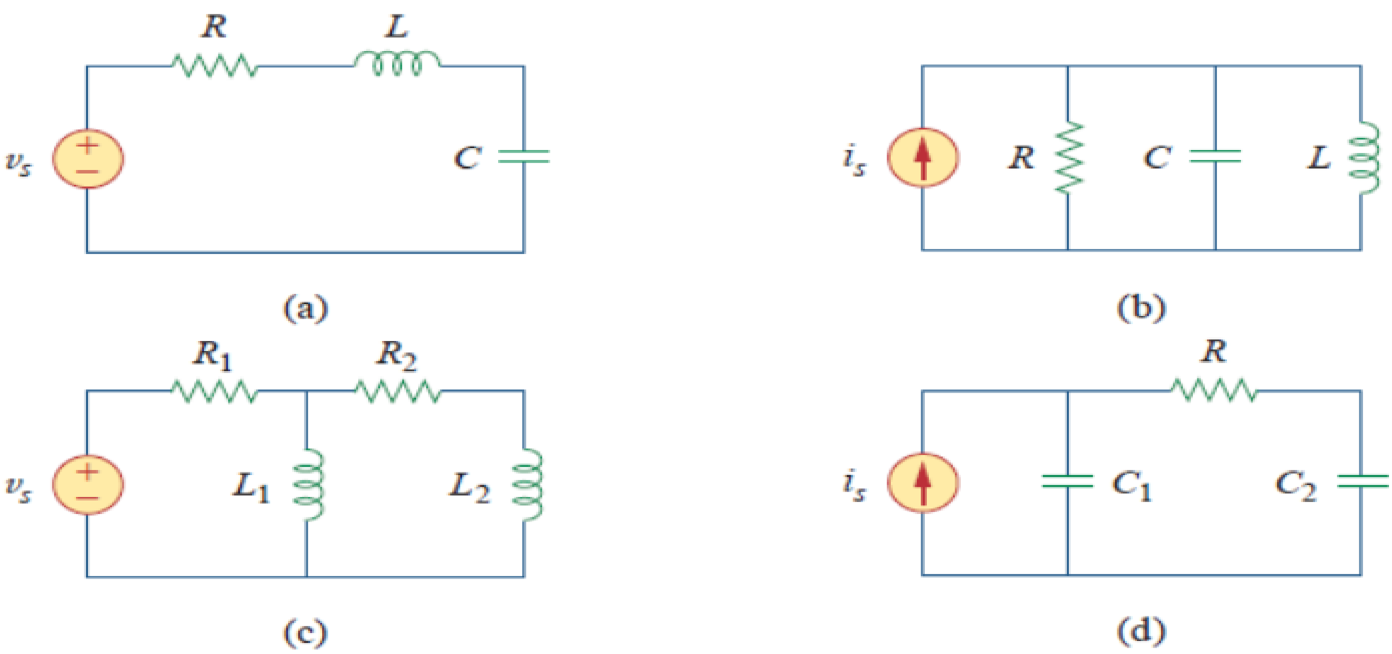


Fig. 1.1 Typical examples of second-order circuits: (a) series *RLC* circuit, (b) parallel *RLC* circuit, (c) *RL* circuit, (d) *RC* circuit.

2) Finding Initial and Final Values

There are two key points to keep in mind in determining the initial conditions. **First**—as always in circuit analysis—we must carefully handle the polarity of voltage $v(t)$ across thecapacitor and the direction of the current $i(t)$ through the inductor. Keep in mind that v and i are defined strictly according to the passive sign convention. **Second**, keep in mind that the capacitor voltage is always continuous so that

$$v(0^+) = v(0^-) \tag{2.1a}$$

and the inductor current is always continuous so that

$$i(0^+) = i(0^-) \tag{2.1b}$$

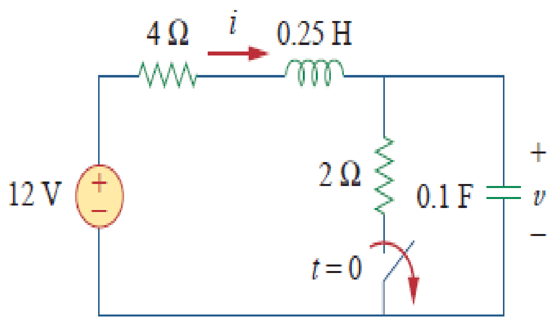
where $t = 0^-$ denotes the time just before a switching event and $t = 0^+$ is the time just after the switching event, assuming that the switching event takes place at $t = 0$.

Thus, in finding initial conditions, we first focus on those variables that cannot changeabruptly, capacitor voltage and inductor current, by applying Eq. (2.1). The following examples illustrate these ideas.

Example 1: The switch in Fig.1 has been closed for a long time. It is open at $t = 0$. Find: (a) $i(0^+)$, $v(0^+)$, $(b) di(0^+)/dt$, $dv(0^+)/dt$, $(c) i(\infty)$, $v(\infty)$.

Solution:

(a) If the switch is closed a long time before $t = 0$, itmeans that the circuit has reached dc steady state at $t = 0$. At dc steady state, the inductor acts like a shortcircuit, while the capacitor acts like an open circuit, sowe have the circuit in Fig.2 (a) at $t = 0^-$. Thus, Fig.1



$$i(0^-) = \frac{12}{4+2} = 2A, \qquad v(0^-) = 2i(0^-) = 4V$$

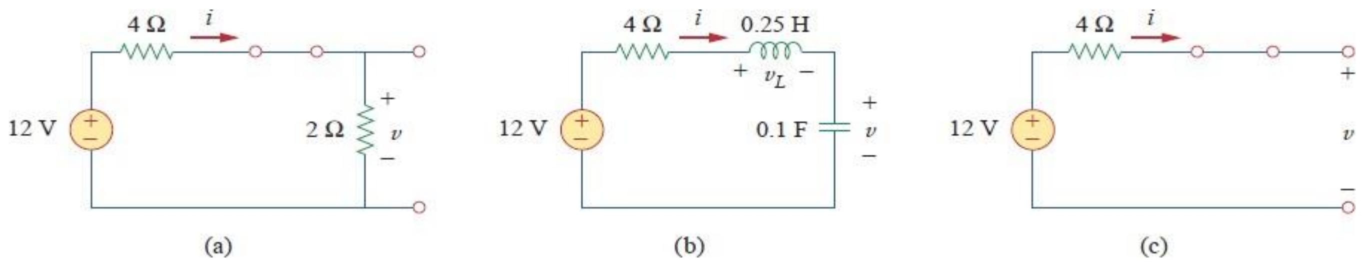


Fig.2 Equivalent circuit of that in Fig.1 for: (a) $t = 0^-$, $(b) t = 0^+$, $(c) t \to \infty$.

As the inductor current and the capacitor voltage cannot change abruptly,

$i(0^+) = i(0^-) = 2A, v(0^+) = v(0^-) = 4V$

(b) At $t = 0^+$, the switch is open; the equivalent circuit is as shown in Fig. 2(b). The same current flows through both the inductor and capacitor. Hence,

$i_C(0^+) = i(0^+) = 2A$

Since $Cdv/dt = i_C,\ dv/dt = i_C/C$, and

$$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = \frac{2}{0.1} = 20V/s$$

Similarly, since $Ldi/dt = v_L,\ di/dt = v_L/L$. We now obtain v_L by applying KVL to the loop in Fig. 2(b). The result is

$$-12 + 4i(0^+) + v_L(0^+) + v(0^+) = 0$$

or

$$v_L(0^+) = 12 - 8 - 4 = 0$$

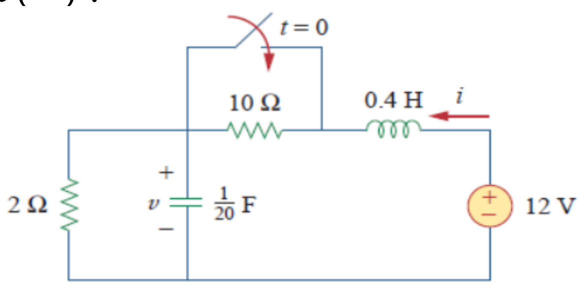
Thus,

$$\frac{di(0^+)}{dt} = \frac{v_L(0^+)}{L} = \frac{0}{0.25} = 0A/s$$

b) For $t > 0$, the circuit undergoes transience. But as $t \to \infty$, the circuit reaches steady stateagain. The inductor acts like a short circuit and the capacitor like an open circuit, so that thecircuit in Fig. 2(b) becomes that shown in Fig. 2(c), from which we have

$i(\infty) = 0A, v(\infty) = 12V$

H.W.1: The switch in Fig. 1 was open for a long time but closed at $t = 0$. Determine: (a) $i(0^+),\ v(0^+),\ (b) di(0^+)/dt,\ dv(0^+)/dt,\ (c) i(\infty),\ v(\infty)$.



Answer: (a) $1\,A,\ 2V,\ (b) 25A/s,\ 0V/s,\ (c) 6A,\ 12v$.

Example 2: In the circuit of Fig. 1 , calculate: (a) $i_L(0^+)$, $v_C(0^+)$, $v_R(0^+)$, (b)$di_L(0^+)/dt$, $dv_c(0^+)/dt$, $dv_R(0^+)/dt$, $(c) i_L(\infty)$, $v_c(\infty)$, $v_R(\infty)$.

Solution:

(a) For $t < 0$, $3u(t) = 0$. At $t = 0^-$ since the circuit has reached steady state, the inductor can be replaced by a short circuit, while the capacitor is replaced by an open circuit as shown in Fig. 2(a). From this figure we obtain

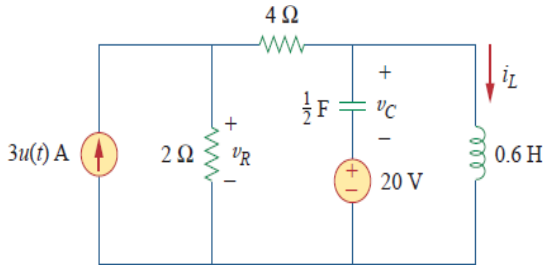


Fig. 1

$$i_L(0^-) = 0, v_R(0^-) = 0, v_C(0^-) = -20V \tag{1.1}$$

Although the derivatives of these quantities at $t = 0^-$ are not required, it is evident that they are all zero, since the circuit has reached steady state and nothing changes.

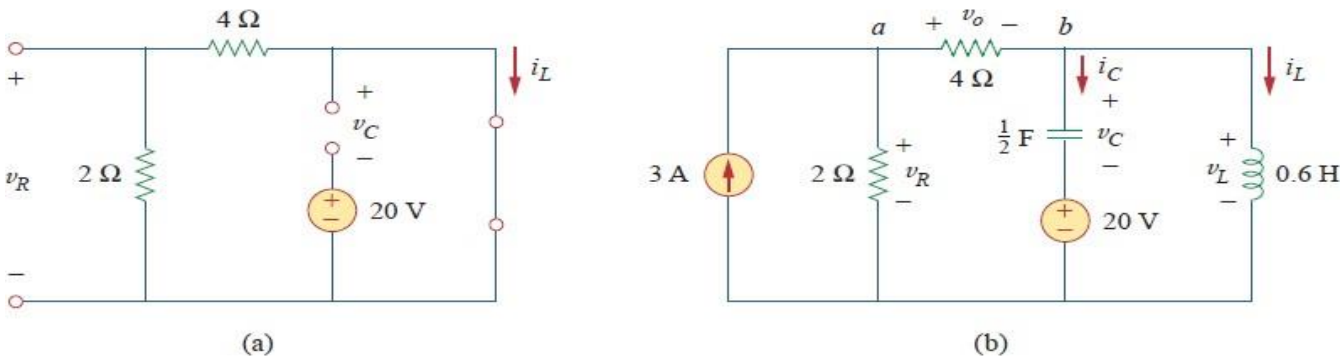


Fig. 2 The circuit in Fig. 1 for: (a) $t = 0^-$, $(b) t = 0^+$

For $t > 0$, $3u(t) = 3$, so that the circuit is now equivalent to that in Fig. 2(b). Since the inductor current and capacitor voltage cannot change abruptly,

$$i_L(0^+) = i_L(0^-) = 0, \quad v_C(0^+) = v_C(0^-) = -20V \tag{1.2}$$

Although the voltage across the $4-\Omega$ resistor is not required, we will use it to apply KVL and KCL; let it be called v_o. Applying KCL at node a in Fig. 2(b) gives

$$3 = \frac{v_R(0^+)}{2} + \frac{v_o(0^+)}{4} \tag{1.3}$$

Applying KVL to the middle mesh in Fig. 2(b) yields

$$-v_R(0^+) + v_o(0^+) + v_C(0^+) + 20 = 0 \tag{1.4}$$

Since $v_C(0^+) = -20V$ from Eq. (1.2), Eq. (1.4) implies that

$$v_R(0^+) = v_o(0^+) \tag{1.5}$$

From Eqs. (1.3) and (1.5), we obtain

$$v_R(0^+) = v_o(0^+) = 4V \tag{1.6}$$

(b) Since $Ldi_L/dt = v_L$,

$$\frac{di_L(0^+)}{dt} = \frac{v_L(0^+)}{L}$$

But applying KVL to the right mesh in Fig. 2(b) gives

$$v_L(0^+) = v_c(0^+) + 20 = 0$$

Hence,

$$\frac{di_L(0^+)}{dt} = 0 \tag{1.7}$$

Similarly, since $Cdv_C/dt = i_C$, then $dv_C/dt = i_C/C$. We apply KCL at node b in Fig. 2(b) to get i_C:

$$\frac{v_o(0^+)}{4} = i_C(0^+) + i_L(0^+) \tag{1.8}$$

Since $v_o(0^+) = 4$ and $i_L(0^+) = 0$, $i_C(0^+) = 4/4 = 1$ A. Then

$$\frac{dv_C(0^+)}{dt} = \frac{i_C(0^+)}{C} = \frac{1}{0.5} = 2V/s \tag{1.9}$$

To get $dv_R(0^+)/dt$, we apply KCL to node a and obtain

$$3 = \frac{v_R}{2} + \frac{v_o}{4}$$

Taking the derivative of each term and setting $t = 0^+$ gives

$$0 = 2\frac{dv_R(0^+)}{dt} + \frac{dv_o(0^+)}{dt} \tag{1.10}$$

We also apply KVL to the middle mesh in Fig. 2(b) and obtain

$$-v_R + v_c + 20 + v_o = 0$$

Again, taking the derivative of each term and setting $t = 0^+$ yields

$$-\frac{dv_R(0^+)}{dt}+\frac{dv_c(0^+)}{dt}+\frac{dv_o(0^+)}{dt}=0$$

Substituting for $dv_C(0^+)/dt = 2$ gives

$$\frac{dv_R(0^+)}{dt}=2+\frac{dv_o(0^+)}{dt} \quad (1.11)$$

From Eqs. (1.10) and (1.11), we get

$$\frac{dv_R(0^+)}{dt}=\frac{2}{3}V/s$$

We can find $di_R(0^+)/dt$ although it is not required. Since $v_R = 5i_R$,

$$\frac{di_R(0^+)}{dt}=\frac{1}{5}\frac{dv_R(0^+)}{dt}=\frac{1}{5}\frac{2}{3}=\frac{2}{15}A/s$$

(c) As $t \to \infty$, the circuit reaches steady state. We have the equivalent circuit in Fig. 2(a) except that the 3-A current source is now operative. By current division principle,

$$i_L(\infty)=\frac{2}{2+4}3A=1A \quad (1.12)$$

$$v_R(\infty)=\frac{4}{2+4}3A\times 2=4V, \quad v_C(\infty)=-20V$$

H.W.2: For the circuit in Fig. 1, find: (a) $i_L(0^+)$, $v_C(0^+)$, $v_R(0^+)$, (b) $di_L(0^+)/dt$, $dv_C(0^+)/dt$, $dv_R(0^+)/dt$, $(c) i_L(\infty)$, $v_C(\infty)$, $v_R(\infty)$.

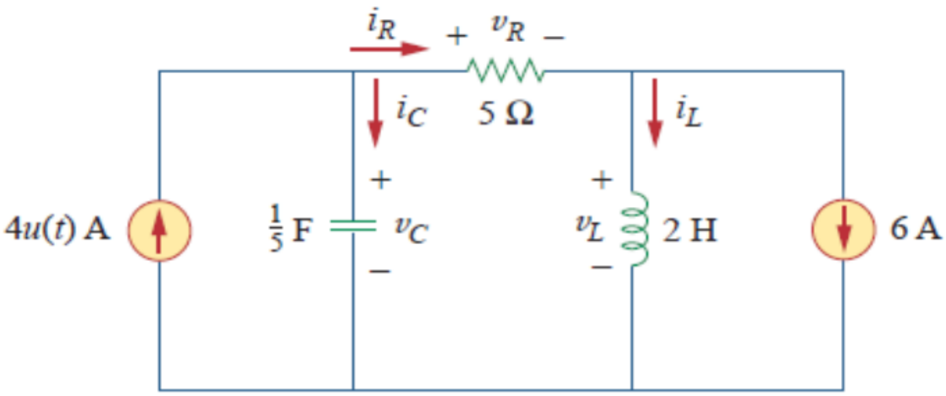


Fig. 1

Answer: (a) $-6A, 0, 0, (b) 0, 20V/s, 0, (c) -2A, 20V, 20V$.

3) The Source-Free Series RLC Circuit

Consider the series *RLC* circuit shown in Fig. 3.1. The circuit is being excited by the energy initially stored in the capacitor and inductor. The energy is represented by the initial capacitor voltage V_0 and initial inductor current I_0. Thus, at $t = 0$,

$$v(O) = \frac{1}{C}\int_{-\infty}^{0} i\,dt = V_0 \tag{3.1a}$$

$$i(0) = I_0 \tag{3.1b}$$

Applying KVL around the loop in Fig. 3.1,

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{t} i\,dt = 0 \tag{3.2}$$

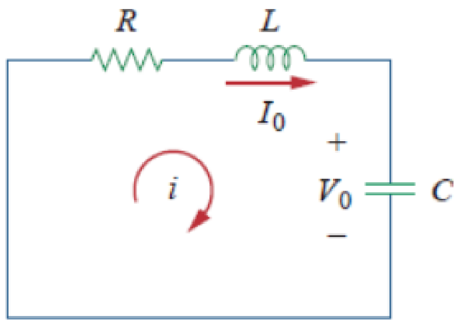


Fig. 3.1. A source-free series *RLC* circuit.

To eliminate the integral, we differentiate with respect to t and rearrange terms. We get

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0 \tag{3.3}$$

This is a ***second-order differential equation*** and is the reason for calling the *RLC* circuits in this lecture ***second-order circuits***. To solve such a second-order differential equation requires that we have two initial conditions, such as the initial value of i and its first derivative or initial values of some i and v. The initial value of i is given in Eq. (8.2b). We get the initial value of the derivative of i from Eqs. (3.1a) and (3.2); that is,

$$Ri(O) + L\frac{di(0)}{dt} + V_0 = 0$$

or

$$\frac{di(0)}{dt} = -\frac{1}{L}(RI_0 + V_0) \tag{3.4}$$

With the two initial conditions in Eqs. (3.1b) and (3.4), we can now solve Eq. (3.4). Our experience in the preceding lecture on first-order circuits suggests that the solution is of exponential form. So we let

$$i = Ae^{st} \tag{3.5}$$

where A and s are constants to be determined. Substituting Eq. (3.5) into Eq. (3.3) and carrying out the necessary differentiations, we obtain

$$As^2e^{st} + \frac{AR}{L}se^{st} + \frac{A}{LC}e^{st} = 0$$

or

$$Ae^{st}(s^2 + \frac{R}{L}s + \frac{1}{LC}) = 0 \tag{3.6}$$

Since $i = Ae^{st}$ is the assumed solution we are trying to find, only the expression in parentheses can be zero:

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \tag{3.7}$$

This quadratic equation is known as the ***characteristic equation*** of the formula to differential Eq. (3.3), since the roots of the equation dictate the character of i. The two roots of Eq. (3.7)are

$$s_1 = -\frac{R}{2L} + \sqrt{(\frac{R}{2L})^2 - \frac{1}{LC}} \tag{3.8a}$$

$$s_2 = -\frac{R}{2L} - \sqrt{(\frac{R}{2L})^2 - \frac{1}{LC}} \tag{3.8b}$$

A more compact way of expressing the roots is

$$\boxed{s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}, \quad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}} \tag{3.9}$$

where

$$\alpha = \frac{R}{2L}, \omega_0 = \frac{1}{\sqrt{LC}} \tag{3.10}$$

The roots s_1 and s_2 are called **natural frequencies**, measured in **nepers per second** ($\boldsymbol{Np/s}$) , because they are associated with the natural response of the circuit; ω_0 is known as the **resonant frequency** or strictly as the **undamped natural frequency**, expressed in **radians per second** ($\boldsymbol{rad/s}$) ;and α is the **neper frequency** or the **damping factor**, expressed in nepers per second. In terms of α and ω_0, Eq. (3.7) can be written

$$s^2 + 2\alpha s + \omega_0^2 = 0 \qquad (3.7a)$$

Notes:

1) The neper (Np) is a dimensionless unit named after John Napier (1550–1617), a Scottish mathematician.
2) The ratio α/ω_0 is known as the damping ratio ζ.

The variables s and ω_0 are important quantities we will be discussing throughout the rest of the lecture.

The two values of s in Eq. (3.9) indicate that there are two possible solutions for i, each of which is of the form of the assumed solution in Eq. (3.5); that is,

$$i_1 = A_1 e^{s_1 t}, i_2 = A_2 e^{s_2 t} \quad (3.11)$$

Since Eq. (3.3) is a linear equation, any linear combination of the two distinct solutions i_1 and i_2 is also a solution of Eq. (3.3). A complete or total solution of Eq. (3.3) would therefore require a linear combination of i_1 and i_2. Thus, the natural response ofthe series RLC circuit is

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad (3.12)$$

where the constants A_1 and A_2 are determined ffom the initial values $i(O)$ and $di(O)/dt$ in Eqs. (3.1b) and (3.4).

From Eq. (3.9), we can infer that there are three types of solutions

1. If $\alpha > \omega_0$, we have the ***overdamped*** case.

2. If $\alpha = \omega_0$, we have the ***critically damped*** case.

3. If $\alpha < \omega_0$, we have the ***underdamped*** case.

We will consider each of these cases separately.

Note: The response is overdamped when the roots of the circuit's characteristic equation are unequal and real, critically damped when the roots are equal and real, and underdamped when the roots are complex.

Overdamped Case $(\alpha > \omega_0)$

From Eqs. (3.8) and (3.9), $\alpha > \omega_0$ implies $C > 4L/R^2$. When this happens, both roots s_1 and s_2 are negative and real. The response is

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \tag{3.13}$$

which decays and approaches zero as t increases. Fig. 3.2(a) illustrates a typical overdamped response.

Critically Damped Case $(\alpha = \omega_0)$

When $\alpha = \omega_0$, $C = 4L/R^2$ and

$$s_1 = s_2 = -\alpha = -\frac{R}{2L} \tag{3.14}$$

For this case, Eq. (3.12) yields

$$i(t) = A_1 e^{-\alpha t} + A_2 e^{-\alpha t} = A_3 e^{-\alpha t}$$

where $A_3 = A_1 + A_2$. This cannot be the solution, because the two initial conditions cannot

$$\frac{d^2 i}{dt^2} + 2\alpha \frac{di}{dt} + \alpha^2 i = 0$$

or

$$\frac{d}{dt}\left(\frac{di}{dt} + \alpha i\right) + \alpha\left(\frac{di}{dt} + \alpha i\right) = 0 \tag{3.15}$$

If we let

$$f = \frac{di}{dt} + \alpha i \tag{3.16}$$

then Eq. (8. 15) becomes

$$\frac{df}{dt} + \alpha f = 0$$

which is a first-order differential equation with solution $f = A_1 e^{-\alpha t}$, where A_1 is a constant. Equation (3.16) then becomes

$$\frac{di}{dt} + \alpha i = A_1 e^{-\alpha t}$$

be satisfied with the single constant A_3. What then could be wrong? Our assumption of an exponential solution is incorrect for the special case of critical damping. Let us go back to Eq. (3.3). When $\alpha = \omega_0 = R/2L$, Eq. (3.3) becomes or

$$e^{\alpha t}\frac{di}{dt} + e^{\alpha t}\alpha i = A_1 \tag{3.17}$$

This can be written as

$$\frac{d}{dt}(e^{\alpha t} i) = A_1 \tag{3.18}$$

Integrating both sides yields

$$e^{\alpha t} i = A_1 t + A_2$$

or

$$i = (A_1 t + A_2)e^{-\alpha t} \tag{3.19}$$

where A_2 is another constant. Hence, the natural response of the critically damped circuit is a sum of two terms: a negative exponential and a negative exponential multiplied by a linear term, or

$$i(t) = (A_2 + A_1 t)e^{-\alpha t} \quad (3.20)$$

A typical critically damped response is shown in Fig. 3.2 (b). In fact, Fig. 3.2 (b) is a sketch of $i(t) = te^{-\alpha t}$, which reaches a maximum value of e^{-1}/α at $t = 1/\alpha$, one time constant, and then decays all the way to zero.

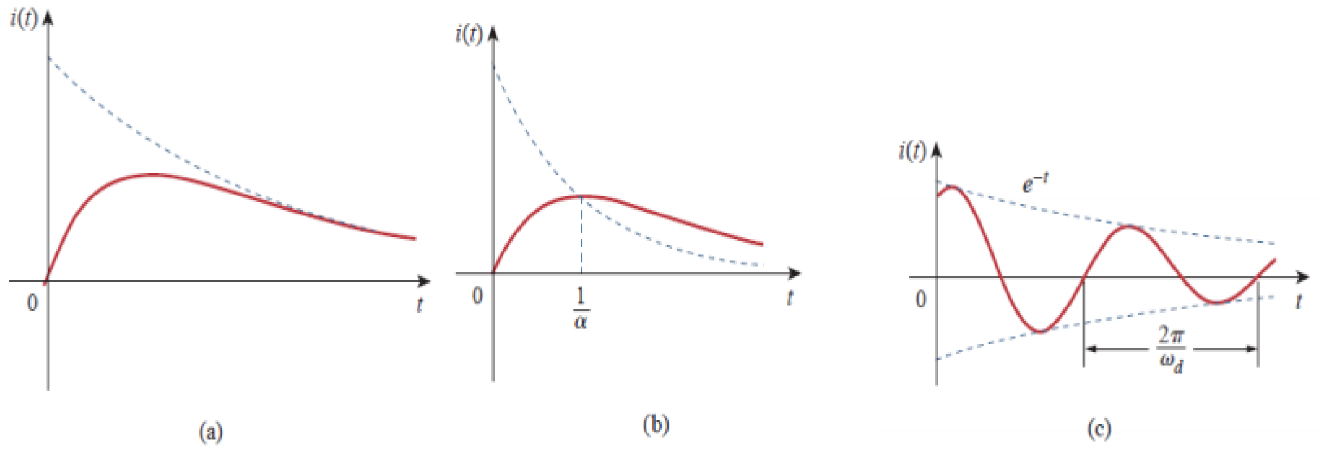


Fig. 3.2 (a) Overdamped response, (b) critically damped response, (c) underdamped response.

Underdamped Case $(a < m_0)$

For $\alpha < \omega_0$, $C < 4L/R^2$. The roots may be written as

$$s_1 = -\alpha + \sqrt{-(\omega_0^2 - \alpha^2)} = -\alpha + j\omega_d \quad (3.21a)$$

$$s_2 = -\alpha - \sqrt{-(\omega_0^2 - \alpha^2)} = -\alpha - j\omega_d \quad (3.21b)$$

where $j = \sqrt{-1}$ and $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$, which is called the ***damping frequency***. Both ω_0 and ω_d are natural frequencies because they help determine the natural response; while ω_0 is often called the ***undamped natural frequency***, ω_d is called the ***damped natural frequency***. The natural response is

$$i(t) = A_1 e^{-(\alpha - j\omega_d)t} + A_2 e^{-(\alpha + j\omega_d)t} = e^{-\alpha t}(A_1 e^{j\omega_d t} + A_2 e^{-j\omega_d t}) \quad (3.22)$$

Using Euler's identities,

$$e^{j\theta} = \cos\theta + j\sin\theta, \; e^{-j\theta} = \cos\theta - j\sin\theta \quad (3.23)$$

we get

$$\begin{aligned} i(t) &= e^{-\alpha t}[A_1(\cos\omega_d t + j\sin\omega_d t) + A_2(\cos\omega_d t - j\sin\omega_d t)] \\ &= e^{-\alpha t}[(A_1 + A_2)\cos\omega_d t + j(A_1 - A_2)\sin\omega_d t] \end{aligned} \quad (3.24)$$

Replacing constants $(A_1 + A_2)$ and $j(A_1 - A_2)$ with constants B_1 and B_2, we write

$$i(t) = e^{-\alpha t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t) \quad (3.25)$$

With the presence of sine and cosine functions, it is clear that the natural response for this case is exponentially damped and oscillatory in nature. The response has a time constant of $1/\alpha$ and a period of $T = 2\pi/\omega_d$. Fig. 3.2(c) depicts a typical underdamped response. [Fig.3.2 assumes for each case that $i(O) = 0$.]

Once the inductor current $i(t)$ is found for the RLC series circuit as shown above, other circuit quantities such as individual element voltages can easily be found. For example, theresistor voltage is $v_R = Ri$, and the inductor voltage is $v_L = Ldi/dt$. The inductor current $i(t)$ is selected as the key variable to be determined first in order to take advantage of Eq. (2.lb).

We conclude this section by noting the following interesting, peculiar properties of an *RLC* network:

1. The behavior of such a network is captured by the idea of ***damping***, which is the gradualloss of the initial stored energy, as evidenced by because of the inherent losses in them. thecontinuous decrease in the amplitude of the response. The damping effect is due to the

2. presence of resistance R. The damping factor α determines the rate at which the response isdamped. If $R = 0$, then $\alpha = 0$, and we have an LC circuit with $1/\sqrt{LC}$ as the undamped natural frequency. Since $\alpha < \omega_0$ in this case, the response is not only undamped but also oscillatory. The circuit is said to be ***loss-less***, because the dissipating or damping element (*R*) is absent. By adjusting the value of R, the response may be made ***undamped, overdamped, critically damped*** or ***undamped***.

3. Oscillatory response is possible due to the presence of the two types of storage elements. Having both L and C allows the flow of energy back and forth between the two. The damped oscillation exhibited by the underdamped response is known as ***ringing***. It stems from the ability of the storage elements L and C to transfer energy back and forth between them.

4. Observe from Fig. 3.2 that the waveforms of the responses differ. In general, it is difficult to tell from the waveforms the difference between the overdamped and critically damped responses. The critically damped case is the borderline between the underdamped and overdamped cases and it decays the fastest. With the same initial conditions, the overdamped case has the longest settling time, because it takes the longest time to dissipate the initial stored energy. If we desire the response that approaches the final value most rapidly without oscillation or ringing, the critically damped critically damped circuit. circuit is the right choice.

Example 3: In Fig.3.1, $R = 40\Omega, L = 4H$, and $C = 1/4$ F. Calculate the characteristic roots of the circuit. Is the natural response overdamped, under- damped, or critically damped?
Solution: We first calculate

$$\alpha = \frac{R}{2L} = \frac{40}{2(4)} = 5, \quad \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4\times\frac{1}{4}}} = 1$$

The roots are

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -5 \pm \sqrt{25 - 1}$$

or

$$s_1 = -0.101, \quad s_2 = -9.899$$

Since $\alpha > \omega_0$, we conclude that the response is overdamped. This is also evident from the fact that the roots are real and negative.

Since $\alpha > \omega_0$, we conclude that the response is overdamped. This is also evident from the fact that the roots are real and negative.

H.W. 3: If $R = 10\Omega$, $L = 5H$, and $C = 2mF$ in Fig.3.1, find α, ω_0, s_1, and s_2. What typeof natural response will the circuit have?

Answer: 1, 10, $-1 \pm j9.95$, underdamped.

Example 4: Find $i(t)$in the circuit of Fig.1. Assume that the circuit has reached steady state at $t = 0^-$.

Solution:

$$i(O) = \frac{10}{4+6} = 1A, \qquad v(O) = 6i(O) = 6V$$

where $i(O)$ is the initial current through the inductor and $v(O)$ is the initial voltage across the capacitor.

For $t < 0$, the switch is closed. The capacitor acts like an open circuit while the inductor acts like a shunted circuit. The equivalent circuit is shown in Fig.2(a). Thus, at $t = 0$,

For $t > 0$, the switch is opened and the voltage source is disconnected. The equivalent circuit is shown in Fig.2(b), which is a source- free series RLC circuit. Notice that the 3Ω and 6Ω resistors, which are in series in Fig.1 when the switch is opened, have been combined to give $R = 9\Omega$ in Fig.2(b). The roots are calculated as follows:

$$\alpha = \frac{R}{2L} = \frac{9}{2(\frac{1}{2})} = 9, \quad \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{2} \times \frac{1}{50}}} = 10$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -9 \pm \sqrt{81 - 100} = -9 \pm j4.359$$

Hence, the response is underdamped $(\alpha < \omega)$; that is,

$$i(t) = e^{-9t}(A_1 \cos 4.359t + A_2 \sin 4.359t) \quad (1)$$

We now obtain A_1 and A_2 using the initial conditions. At $t = 0$,

$$i(O) = 1 = A_1 \quad (2)$$

From Eq. (3.4),

$$\left.\frac{di}{dt}\right|_{t=0} = -\frac{1}{L}[Ri(O) + v(O)] = -2[9(1) - 6] = -6A/s \quad (3)$$

Note that $v(O) = V_0 = -6V$ is used, because the polarity of v in Fig.2 (b) is opposite that in Fig. 3.1. Taking the derivative of $i(t)$ in Eq. (1),

$$\frac{di}{dt} = -9e^{-9t}(A_1 \cos 4.359t + A_2 \sin 4.359t) + e^{-9t}(4.359)(-A_1 \sin 4.359t + A_2 \cos 4.359t)$$

Imposing the condition in Eq. (3) at $t = 0$ gives

$$-6 = -9(A_1 + 0) + 4.359(-0 + A_2)$$

But $A_1 = 1$ from Eq. (2). Then

$$-6 = -9 + 4.359A_2 \;\Rightarrow\; A_2 = 0.6882$$

Substituting the values of A_1 and A_2 in Eq. (1) yields the complete solution as

$$i(t) = e^{-9t}(\cos 4.359t + 0.6882 \sin 4.359t)A$$

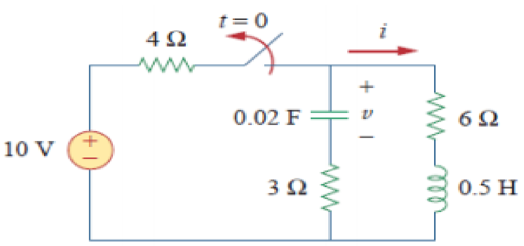


Fig.1

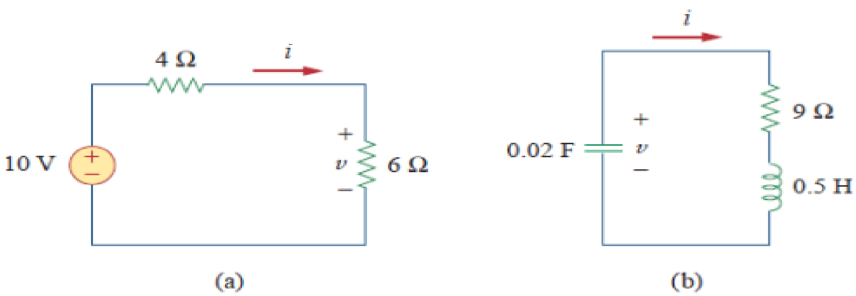


Fig.2 The circuit in Fig.1: (a) for $t < 0$, (b) for $t > 0$.

H.W. 4: The circuit in Fig.1 has reached steady state at $t = 0^-$. If the make before-break switch moves to position b at $t = 0$, calculate $i(t)$ for t > 0

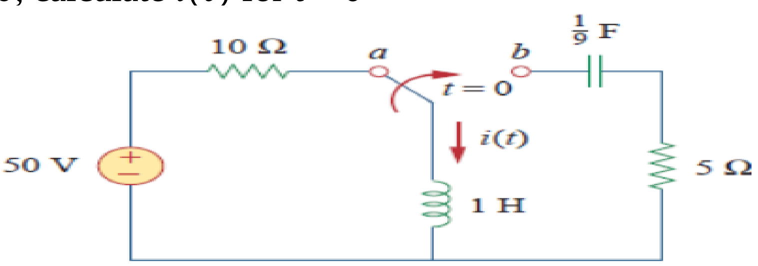


Fig. 1

Answer: $e^{-2.5t}(5\cos 1.6583t - 7.5378 \sin 1.6583t)A$

Thank you very much

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