Strain

Simple Strain

Strain: Is a measure of the deformation of the material which is subjected to an external load, and its non-dimensional. The strain may divide into: 1) Normal strain. 2) Shear strain. If a bar is subjected to a direct tension, the bar will change in length. If the bar has an original length "L" and change in length by an amount "ΔL" the strain produces is defined as follows. Strain represents a change in length divided by the original length, strain is dimensionless quantity. Strain assumed to be constant over the length under certain condition: -

1. The specimen must be constant cross section.

2. The material must be homogenous.

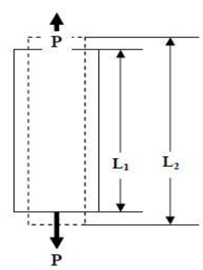
3. The load must be axial, that is produces uniform stress.

1) Normal Strain: It is occurred due to normal stresses (tensile causes +ve strain and compressive stress causes –ve strain).

$$\varepsilon = \frac{\Delta L}{L} = \frac{L_2 - L_1}{L_1}$$

In tension :

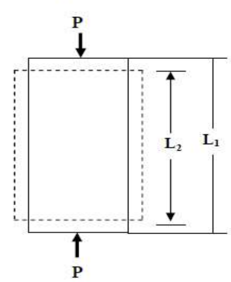
$$\varepsilon_t = \frac{\Delta L}{L_1} = \frac{L_2 - L_1}{L_1} \quad \textbf{(+ve strain)}$$



In compression :

$$\varepsilon_c = \frac{\Delta L}{L_1} = \frac{L_2 - L_1}{L_1} \quad \textbf{(-ve strain)}$$

as L_1 **larger than** L_2



Where, **ε** is the normal strain (Epsilon)

L: Original length

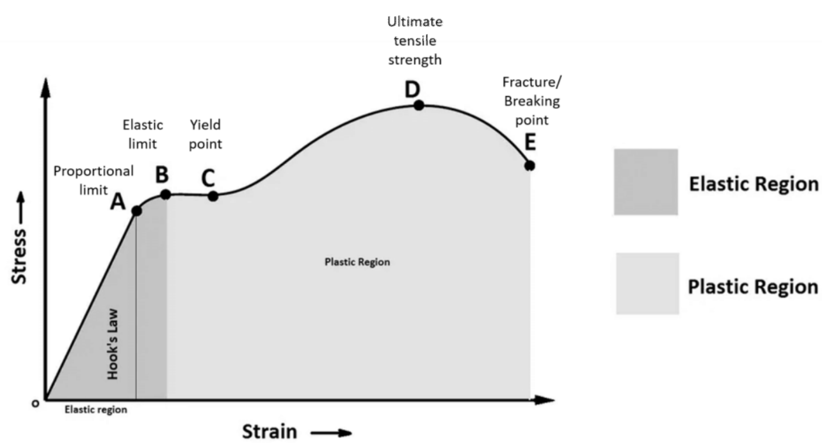
ΔL: Change in length

Stress-Strain Diagram

A stress-strain curve is a graphical depiction of a material's behavior when subjected to increasing loads. Stress-strain curves can be generated to investigate a material's behavior when any type of load (tensile, compression, shear, bending, torsion) is applied. Stress-strain curves generated for tensile loads are important because they enable engineers to quickly determine several mechanical properties of a material including: modulus of elasticity (Young's modulus), yield strength, ultimate strength, and ductility. A stress-strain curve is obtained by conducting a tensile test (a type of test where a load is continuously applied to a test specimen until it fractures). A single tensile test can produce a stress-strain graph, which then allows the following properties of a material to be obtained:

1. Young's modulus
2. Yield strength
3. Ultimate tensile strength
4. Ductility
5. Poisson's ratio

Stress-strain curves are generated automatically by modern tensile testing machines. These machines continuously monitor and record the force applied to a test specimen and the amount of deformation it experiences as a result of that load. The most commonly used test methods for tensile testing and creating standardized stress-strain curves are those issued by ASTM International. ASTM E8 standardizes tensile tests for metallic materials while ASTM D638 standardizes tensile tests for plastic materials. Such Stress Strain diagrams are used to study the behavior of a material from the point it is loaded until it breaks. Each material produces a different stress-strain diagram.

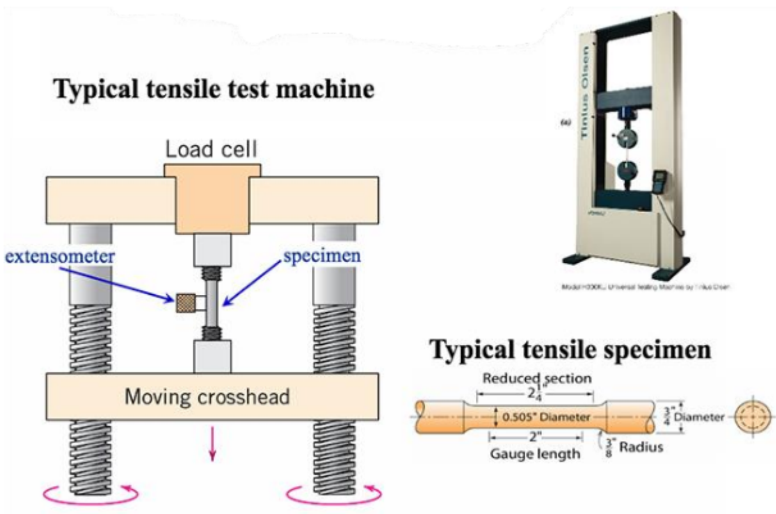


Point O on the diagram represents the original undeformed, unloaded condition of the material. As the material is loaded, both stress and strain increase, and the plot proceeds from Point O to Point A. If the material is unloaded before Point A is reached, then the plot would proceed back down the same line to Point O. If the material is unloaded anywhere between Points O and A, then it will return to its original shape, like a rubber band. This type of behavior is termed Elastic and the region between Points O and A is the Elastic Region. The Stress-Strain curve also appears linear between Points O and A. In this region stress and strain are proportional. The constant of proportionality is called the Elastic Modulus or Young's Modulus (E).

$$E = \frac{\sigma}{\varepsilon} \qquad or \qquad \sigma = E\varepsilon \quad \textbf{(Hook's law)}$$

where: σ is the stress (psi)
E is the Elastic Modulus (psi)
ε is the strain (in/in)

Point B is called the Yield Strength (σy). If it is passed, the material will no longer return to its original length. It will have some permanent deformation. This area beyond Point B is the Plastic Region. Consider, for example, what happens if we continue along the curve from Point B to Point C, the stress required to continue deformation increases with increasing strain. If the material is unloaded the curve will proceed from Point C to Point D. The slope (Elastic Modulus) will be the same as the slope between Points 1 and 2. The difference between Points O and D represents the permanent strain of the material. If the material is loaded again, the curve will proceed from Point D to Point C with the same Elastic Modulus (slope). The Elastic Modulus will be unchanged, but the Yield Strength will be increased. Permanently straining the material in order to increase the Yield Strength is called Strain Hardening. If the material is strained beyond Point C stress decreases as non-uniform deformation and necking occur. The sample will eventually reach Point E at which it fractures. The largest value of stress on the diagram is called the Tensile Strength (TS) or Ultimate Tensile Strength (UTS). This is the most stress the material can support without breaking.



- **Hook's law:**

  Can be defined as the linear relationship between stress and strain for a bar under uniaxial tension or compression and can be expressed by

$$E = \frac{\sigma}{\varepsilon} \qquad or \qquad \sigma = E\varepsilon$$

where: σ is the stress (psi)

E is the Elastic Modulus (psi)

ε is the strain (in/in)

The Units of E Is: Pa, Kpa, Mpa, Gpa

Its also called young modulus. The value of E is high for the stiff materials such as:

Steel Es= 200 Gpa

Aluminum Ea= 70 Gpa

Wood Ew = 11 Gpa

Concrete Ec= $4.7\sqrt{fc'}$

Example: A Steel rod (E=200 GPa) has a circular cross section and is 10m long. Determine the minimum diameter if the rod must hold a 30 kN tensile force without deforming more than 5mm. Assume the steel stays in the elastic region. Note, 1 GPa = 10^9 Pa.

Solution: Knowing the initial length and the change in length permits the calculation of strain.

$$\varepsilon = \frac{\Delta l}{l_o} = \frac{5mm(\frac{1m}{1000mm})}{20m} = 0.0005$$

In the elastic region, the stress σ is directly proportional to the strain ε, by the Modulus of Elasticity, E

$$\frac{F}{A_o} = \sigma = E\varepsilon$$

Rearranging, substituting values and converting units,

$$\sigma = E\varepsilon = (200\ GPa)0.0005 = 0.1\ GPa = 0.1 \times 10^9 Pa = 0.1 \times 10^9\ N/m^2$$

The definition of stress $\sigma = \frac{F}{A_o}$ *can be used to find the required cross section area.*

$$A_O = \frac{F}{\sigma} = \frac{30kN\left(\frac{1000N}{kN}\right)}{0.1 \times 10^9\ N/m^2} = 0.0003m^2(\frac{1000mm}{m})(\frac{1000mm}{m}) = 300mm^2$$

The diameter, d_o *is solved from the area of a circle*

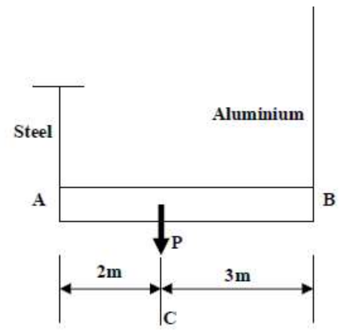
$$A_o = \frac{\pi d_o^2}{4}$$

$$d_o^2 = \frac{A_o 4}{\pi}$$

$$d_o = \sqrt{\frac{A_o 4}{\pi}} = \sqrt{\frac{300mm^2 * 4}{3.14}} = 19.5mm$$

Ex: The rigid bar (AB) attached to two vertical rods as shown in figure, horizontally before the load (P) applied. If the load (P=50kN), determine its vertical movement.

	St.	**Al.**
L (m)	**3**	**4**
Area (mm^2)	**300**	**500**
E (GPa)	**200**	**70**

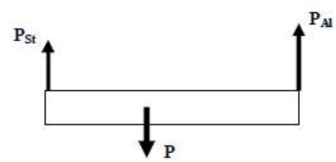


Sol :

From F.B.D

$$\therefore \sum F_y = 0 \Rightarrow P_{St} + P_{Al} = P = 50kN \quad \text{------(1)}$$

$$\therefore \sum M_C = 0 \Rightarrow 2 * P_{St} = 3 * P_{Al} \Rightarrow P_{St} = 1.5 P_{Al} \quad \text{-----(2)}$$



F.B.D

Sub. (1) into (2)

$$\therefore 1.5 P_{Al} + P_{Al} = 50kN \Rightarrow P_{Al} = 20kN$$

$$\therefore P_{St} = 30kN$$

$$\therefore \delta_{St} = \frac{P_{St} * L_{St}}{A_{St} * E_{St}} = \frac{30 * 10^3 * 3}{300 * 10^{-6} * 200 * 10^9} = 1.5 * 10^{-3} m = 1.5mm$$

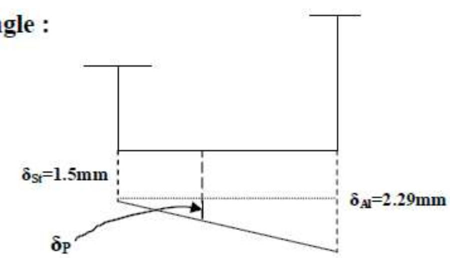
$$\therefore \delta_{Al} = \frac{P_{Al} * L_{Al}}{A_{Al} * E_{Al}} = \frac{20 * 10^3 * 4}{500 * 10^{-6} * 70 * 10^9} = 2.29 * 10^{-3} m = 2.29mm$$

For triangle shown by similarity of triangle :

$$\therefore \frac{\delta_P - 1.5}{2} = \frac{2.29 - 1.5}{5}$$

$$\delta_P = \frac{2(0.79)}{5} + 1.5$$

$$\delta_P = 1.816 \ mm$$



Poisson's Ratio: Biaxial and Triaxial Deformations

The ratio of strain in the lateral direction to the linear strain in the axial direction and it's denoted by the Greek letter υ (nu) and can be expressed as:

$$\upsilon = \frac{Lateral\ Strain}{Longitudinal\ Strain} = -\frac{\varepsilon y}{\varepsilon x}$$

υ : *Poison's Ratio*

$$\upsilon = -\frac{\varepsilon y}{\varepsilon x}$$

$$\varepsilon_y = -\upsilon \times \varepsilon_x = -\upsilon \frac{\sigma x}{E}$$

for biaxial stress state:

$\sigma_x \neq 0 \quad \sigma_y \neq 0 \ \sigma_z = 0$

In the x- direction resulting from
σ_x ,$\epsilon_x = \sigma_x / E$
In the y-direction resulting from
σ_y ,$\epsilon_y = \sigma_y / E$
In the x-direction resulting from
σ_y ,$\epsilon_x = -\upsilon(\sigma x E)$
In the y-direction resulting from the
σ_x , $\epsilon_y = -\upsilon(\sigma x E)$
The total strain in the x-direction will be:

$\epsilon_x = \sigma_x / E - \upsilon\ \sigma_y / E$

$\epsilon_y = \sigma_y / E - \upsilon\ \sigma_x / E$

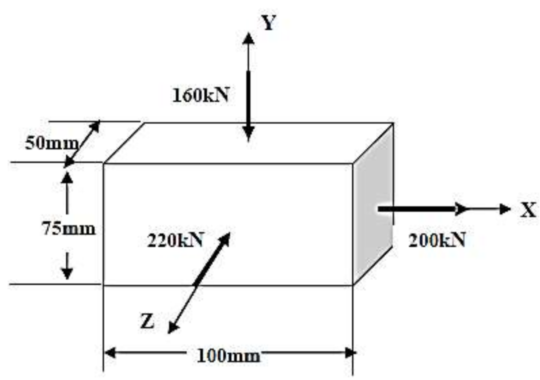
$\epsilon_z = -\upsilon\ \sigma_x / E - \upsilon\ \sigma_y / E$

Triaxial tensile stresses:

$\sigma_x \neq 0 \quad \sigma_y \neq 0 \; \sigma_z \neq 0$

$\epsilon_x = \sigma_x/E - \upsilon\, \sigma_y/E - \upsilon\, \sigma_z/E$
$\epsilon_y = \sigma_y/E - \upsilon\, \sigma_x/E - \upsilon\, \sigma_z/E$
$\epsilon_z = \sigma_z/E - \upsilon\, \sigma_x/E - \upsilon\, \sigma_y/E$

Ex: -9- A rectangular Aluminum block is (100mm) long in X-direction , (75mm) wide in Y-direction and (50mm) thick in Z-direction . It is subjected to try axial loading consisting of uniformly distributed tensile force of (200kN) in the X-direction and uniformly distributed compressive forces of 160kN in Y-direction and (220kN) in Z-direction. If the Poisson's ratio (υ =0.333) and (E=70GPa). Determine a single distributed load that must applied in Xdirection that would produce the same deformation in Z-direction as original loading.



$$\varepsilon_z = \frac{\sigma_z}{E} - \upsilon\frac{\sigma_y}{E} - \upsilon\frac{\sigma_x}{E}$$

$$\sigma_x = \frac{P_x}{A} = \frac{200*10^3}{0.05*0.075} = 53.3MPa\ (tension)$$

$$\sigma_y = \frac{160*10^3}{0.05*0.1} = 32MPa\ (compression)$$

$$\sigma_z = \frac{220*10^3}{0.075*0.1} = 24.34MPa\ (compression)$$

$$\therefore \varepsilon_z = \frac{1}{70*10^9}[-24.34 - 0.333(-32+53.3)]*10^6$$

$$\varepsilon_z = -0.52*10^{-3}$$

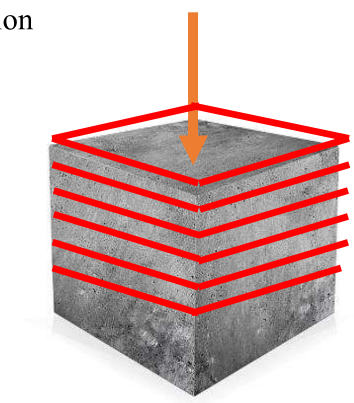
$$\varepsilon_z = -\upsilon\frac{\sigma_x}{E} = -\upsilon\frac{P_x}{A*E} \Rightarrow -0.52*10^{-3} = -0.333\frac{P_x}{0.075*0.05*70*10^9}$$

$$\therefore P_x = 410\ kN\ (Tension)$$

Example: a concrete cube of dimensions (150x150x150) mm is supported by rigid base and walls as shown in figure. Find the transverse stress and longitudinal deformation. use Ec=20 Gpa and υ =0.15.

Sol/

Since the cube is supported by walls in x and z direction



So

$\epsilon_x = \epsilon_z$

$\epsilon_x = \sigma_x/E - \upsilon\ \sigma_y/E - \upsilon\ \sigma_z/E$

$0 = \sigma_x/E - 0.15\ \sigma_y/E - 0.15\ \sigma_z/E$

$\sigma_{y= P/A} = \frac{-100\ X\ 10^3}{150\ X 150} = 0.677$

SO $\sigma_x = 0.15\ \sigma_Z - 0.677$ ……………..1

$\epsilon_z = \sigma_z/E - \upsilon\ \sigma_x/E - \upsilon\ \sigma_y/E$

$0 = \sigma_z/E - 0.15\ \sigma_x/E - 0.15\ \sigma_y/E$

SO $\sigma_Z = 0.15\ \sigma_X - 0.677$ ……………..2

By solving eq1 and 2 we get

$\sigma_x = -0.785$ MPA

$\sigma_z = -0.785$ MPA

SO:

$\epsilon_y = \sigma_y/E - \upsilon\ \sigma_x/E - \upsilon\ \sigma_z/E$

$\epsilon_y = -2.0144\ X\ 10^{-4}$

$\epsilon_y = \frac{\Delta L}{L} =$

$-2.0144\ X\ 10^{-4}\ X\ 150 = -0.0316$ MM (compression) OK

Deformation Of Axially Loaded Members

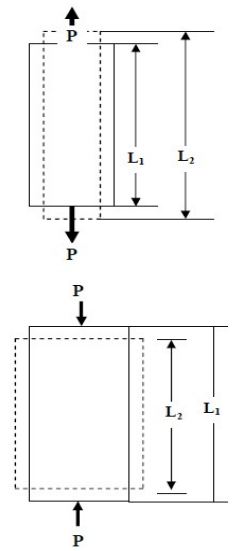
1- **prismatic bodies**
prismatic bar of homogenous materials loaded by constant force :

$\sigma = \frac{P}{A}$ ………………..1

$\epsilon = \frac{\Delta L}{L}$………………..2

$E = \frac{\sigma}{\varepsilon}$ ……………….3

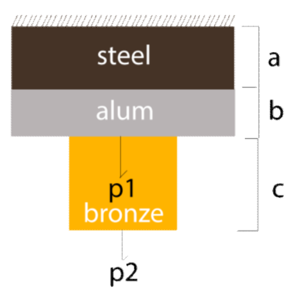
$\Delta L = \frac{PL}{AE}$ …………… 4



2- **non prismatic bodies**
non prismatic bar (different in dimensions) of non-homogenous materials loaded by multiple forces:

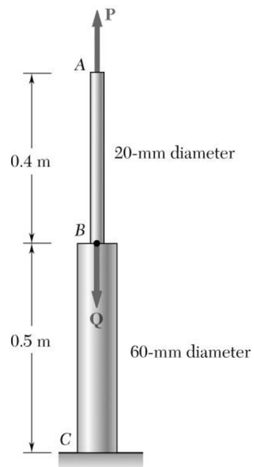
$\Delta L = \sum \frac{PL}{AE} = \Delta L1 + \Delta L2 + \Delta L3 + \Delta L4 + \Delta L5 + \ldots\ldots$

$= \frac{(P1+P2)a}{As\ Es} + \frac{(P1+p2)b}{Aa\ Ea} + \frac{P2\ c}{Ab\ Eb}$



Example:

The rod *ABC* is made of an aluminum for which *E* = 70 GPa. Knowing that P = 6 kN and Q = 42 kN, determine the deflection of (*a*) point *A*, (*b*) point *B*.



$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(0.020)^2 = 314.16\times10^{-6}\,\text{m}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(0.060)^2 = 2.8274\times10^{-3}\,\text{m}^2$$

$$P_{AB} = P = 6\times10^3\,\text{N}$$

$$P_{BC} = P - Q = 6\times10^3 - 42\times10^3 = -36\times10^3\,\text{N}$$

$$L_{AB} = 0.4\text{ m} \quad L_{BC} = 0.5\text{ m}$$

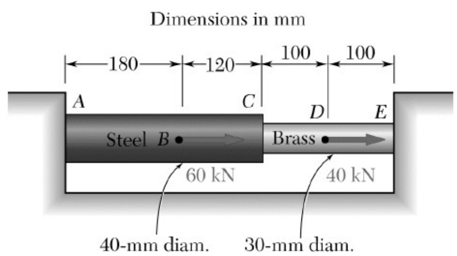
$$\delta_{AB} = \frac{P_{AB}L_{AB}}{A_{AB}E_A} = \frac{(6\times10^3)(0.4)}{(314.16\times10^{-6})(70\times10^9)} = 109.135\times10^{-6}\,\text{m}$$

$$\delta_{BC} = \frac{P_{BC}L_{BC}}{A_{BC}E} = \frac{(-36\times10^3)(0.5)}{(2.8274\times10^{-3})(70\times10^9)} = -90.947\times10^{-6}\,\text{m}$$

(*a*) $\delta_A = \delta_{AB} + \delta_{BC} = 109.135\times10^{-6} - 90.947\times10^{-6}\,\text{m} = 18.19\times10^{-6}\,\text{m}$ $\qquad \delta_A = 0.01819\text{ mm}\uparrow$ ◄

(*b*) $\delta_B = \delta_{BC} = -90.9\times10^{-6}\,\text{m} = -0.0909\text{ mm}$ or $\qquad \delta_B = 0.0919\text{ mm}\downarrow$ ◄

Example:
Two cylindrical rods, one of steel and the other of brass, are joined at C and restrained by rigid supports at A and E. For the loading shown and knowing that Es = 200 GPa and Eb =105 GPa, determine (a) the reactions at A and E, (b) the deflection of point C.



SOLUTION

$\underline{A \text{ to } C}$: $E = 200\times10^{9}\,\text{Pa}$

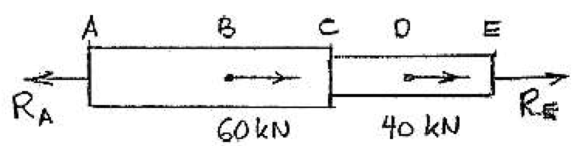
$$A = \frac{\pi}{4}(40)^2 = 1.25664\times10^{3}\,\text{mm}^2 = 1.25664\times10^{-3}\,\text{m}^2$$

$$EA = 251.327\times10^{6}\,\text{N}$$

$\underline{C \text{ to } E}$: $E = 105\times10^{9}\,\text{Pa}$

$$A = \frac{\pi}{4}(30)^2 = 706.86\ \text{mm}^2 = 706.86\times10^{-6}\,\text{m}^2$$

$$EA = 74.220\times10^{6}\,\text{N}$$



$\underline{A \text{ to } B}$: $P = R_A$

$$L = 180\ \text{mm} = 0.180\ \text{m}$$

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.180)}{251.327\times10^{6}} = 716.20\times10^{-12}R_A$$

$\underline{B \text{ to } C}$: $P = R_A - 60\times10^{3}$

$$L = 120\ \text{mm} = 0.120\ \text{m}$$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60\times10^{3})(0.120)}{251.327\times10^{6}} = 447.47\times10^{-12}R_A - 26.848\times10^{-6}$$

<u>*C* to *D*</u>: $P = R_A - 60\times10^3$

$L = 100\text{ mm} = 0.100\text{ m}$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60\times10^3)(0.100)}{74.220\times10^6}$$

$$= 1.34735\times10^{-9}R_A - 80.841\times10^{-6}$$

<u>*D* to *E*</u>: $P = R_A - 100\times10^3$

$L = 100\text{ mm} = 0.100\text{ m}$

$$\delta_{DE} = \frac{PL}{EA} = \frac{(R_A - 100\times10^3)(0.100)}{74.220\times10^6}$$

$$= 1.34735\times10^{-9}R_A - 134.735\times10^{-6}$$

<u>*A* to *E*</u>: $\delta_{AE} = \delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE}$

$$= 3.85837\times10^{-9}R_A - 242.424\times10^{-6}$$

Since point *E* cannot move relative to *A*, $\delta_{AE} = 0$

(*a*) $3.85837\times10^{-9}R_A - 242.424\times10^{-6} = 0 \quad R_A = 62.831\times10^3\text{ N}$

$R_E = R_A - 100\times10^3 = 62.8\times10^3 - 100\times10^3 = -37.2\times10^3\text{ N}$

(*b*) $\delta_C = \delta_{AB} + \delta_{BC} = 1.16367\times10^{-9}R_A - 26.848\times10^{-6}$

$$= (1.16369\times10^{-9})(62.831\times10^3) - 26.848\times10^{-6}$$

$$= 46.3\times10^{-6}\text{m}$$

Thermal Stresses:

The change in temperature causes bodies to expand or contract, the amount of linear deformation (Δth) is expressed as follows:

$\Delta th = \alpha \times L \times \Delta T$

where:

α -------The coefficient of linear deformation in unit of (m/m.C°)

L-------Length of the body (m)

ΔT------Temperature change (C°).

A general procedure for computing the loads and stresses caused when thermal deformation happened as result for temperature changing is outlines in steps:

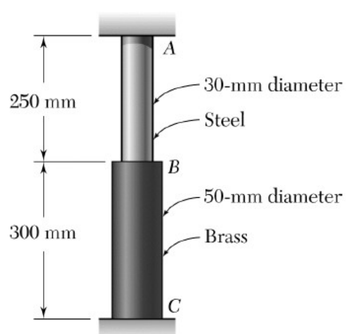
1-Assume that the body is free from all applied loads and constraints so that thermal deformations can occur freely.

2- Apply sufficient load to the body to restore it to the original condition

3- Solve to find unknowns, using equations of equilibrium and equations which are obtained from geometric relations between the temperature and load deformation.

Example:

A rod consisting of two cylindrical portions *AB* and *BC* is restrained at both ends. Portion *AB* is made of steel ($E_s = 200$ GPa, $\alpha_s = 11.7 \times 10^{-6}/°C$) and portion *BC* is made of brass ($E_b = 105$ GPa, $\alpha_b = 20.9 \times 10^{-6}/°C$). Knowing that the rod is initially unstressed, determine the compressive force induced in *ABC* when there is a temperature rise of 50 °C.



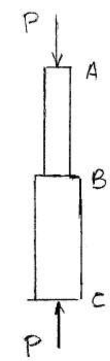
SOLUTION

$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86 \times 10^{-6} \text{m}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(50)^2 = 1.9635 \times 10^3 \text{mm}^2 = 1.9635 \times 10^{-3} \text{m}^2$$

Free thermal expansion:

$$\delta_T = L_{AB}\alpha_s(\Delta T) + L_{BC}\alpha_b(\Delta T)$$
$$= (0.250)(11.7 \times 10^{-6})(50) + (0.300)(20.9 \times 10^{-6})(50)$$
$$= 459.75 \times 10^{-6} \text{m}$$



Shortening due to induced compressive force P:

$$\delta_P = \frac{PL}{E_s A_{AB}} + \frac{PL}{E_b A_{BC}}$$
$$= \frac{0.250P}{(200 \times 10^9)(706.86 \times 10^{-6})} + \frac{0.300P}{(105 \times 10^9)(1.9635 \times 10^{-3})}$$
$$= 3.2235 \times 10^{-9} P$$

For zero net deflection, $\delta_P = \delta_T$

$$3.2235 \times 10^{-9} P = 459.75 \times 10^{-6}$$
$$P = 142.62 \times 10^3 \text{N}$$

$P = 142.6$ kN ◀

Example:

A steel railroad track $(E_s = 200 \text{ GPa}, \alpha_s = 11.7 \times 10^{-6} /°\text{C})$ was laid out at a temperature of 6°C. Determine the normal stress in the rails when the temperature reaches 48°C, assuming that the rails (*a*) are welded to form a continuous track, (*b*) are 10 m long with 3-mm gaps between them.

SOLUTION

(*a*) $\delta_T = \alpha(\Delta T)L = (11.7 \times 10^{-6})(48 - 6)(10) = 4.914 \times 10^{-3}$ m

$$\delta_P = \frac{PL}{AE} = \frac{L\sigma}{E} = \frac{(10)\sigma}{200 \times 10^9} = 50 \times 10^{-12}\sigma$$

$$\delta = \delta_T + \delta_P = 4.914 \times 10^{-3} + 50 \times 10^{-12}\sigma = 0$$

$$\sigma = -98.3 \times 10^6 \text{ Pa}$$

(*b*) $\delta = \delta_T + \delta_P = 4.914 \times 10^{-3} + 50 \times 10^{-12}\sigma = 3 \times 10^{-3}$

$$\sigma = \frac{3 \times 10^{-3} - 4.914 \times 10^{-3}}{50 \times 10^{-12}}$$
$$= -38.3 \times 10^6 \text{ Pa}$$