CHPTER THREE

Fluid Static and Its Applications

3.1 Introduction

Static fluids means that the fluids are at rest.

The pressure in a static fluid is familiar as a surface force exerted by the fluid ageist a unit area of the wall of its container. Pressure also exists at every point within a volume of fluid. It is a scalar quantity; at any given point its magnitude is the same in all directions.

3.2 Pressure in a Fluid

In Figure (1) a stationary column of fluid of height (h_2) and cross-sectional area A, where $A=A_o=A_1=A_2$, is shown. The pressure above the fluid is P_o, it could be the pressure of atmosphere above the fluid. The fluid at any point, say h_1, must support all the fluid above it. It can be shown that the forces at any point in a nonmoving or static fluid must be the same in all directions. Also, for a fluid at rest, the pressure or (force / unit area) in the same at all points with the same elevation. For example, at h_1 from the top, the pressure is the same at all points on the cross-sectional area A_1.

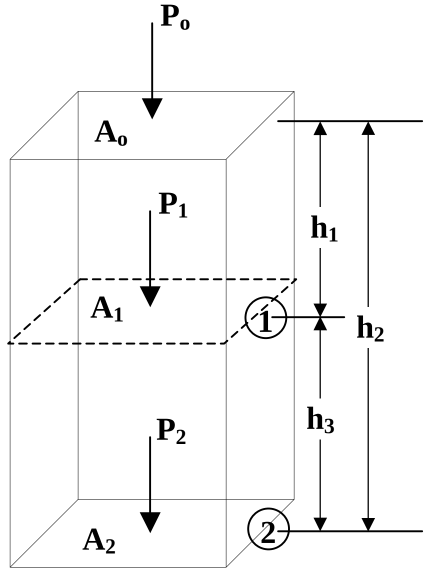


Figure (1): Pressure in a static fluid.

The total mass of fluid for h_2, height and ρ density is: - (h_2 A ρ) (kg)

But from Newton's 2nd law in motion the total force of the fluid on area (A) due to the fluid only is: - (h_2 A ρ g) (N) i.e. $F = h_2\ A\ \rho\ g$ (N)

The pressure is defined as ($P = F/A = h_2\ \rho\ g$) (N/m^2 or Pa)

This is the pressure on A_2 due to the weight of the fluid column above it. However to get the total pressure P_2 on A_2, the pressure P_o on the top of the fluid must be added,

i.e. $P_2 = h_2\ \rho\ g + P_o$ (N/m^2 or Pa)

Thus to calculate P_1, $P_1 = h_1\ \rho\ g + P_o$ (N/m^2 or Pa)

The pressure difference between points ① and ② is: -

$P_2 - P_1 = (h_2\ \rho\ g + P_o) - (h_1\ \rho\ g + P_o)$

⇒ $P_2 - P_1 = (h_2 - h_1)\ \rho\ g$ SI units

$P_2 - P_1 = (h_2 - h_1)\ \rho\ g / g_c$ English units

Since it is vertical height of a fluid that determines the pressure in a fluid, the shape of the vessel does not affect the pressure. For example in Figure (2) the pressure P_1 at the bottom of all three vessels is the same and equal to ($h_1\ \rho\ g + P_o$).

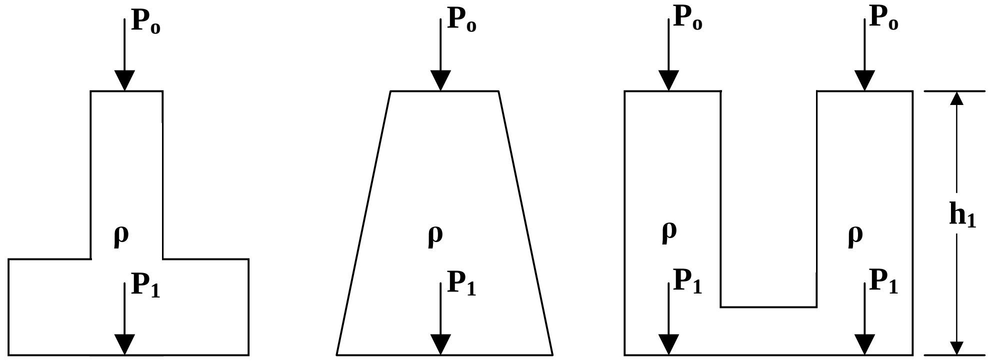


Figure (2): Pressure in vessel of various shapes.

3.3 Absolute and Relative Pressure

The term pressure is sometimes associated with different terms such as *atmospheric*, *gauge*, *absolute*, and *vacuum*. The meanings of these terms have to be understood well before solving problems in hydraulic and fluid mechanics.

1- Atmospheric Pressure

It is the pressure exerted by atmospheric air on the earth due to its weight. This pressure is change as the density of air varies according to the altitudes. Greater the height lesser the density. Also it may vary because of the temperature and humidity of air. Hence for all purposes of calculations the pressure exerted by air at sea level is taken as standard and that is equal to: -

1 atm = 1.01325 bar = 101.325 kPa = 10.328 m H_2o = 760 torr (mm Hg) = 14.7 psi

2- Gauge Pressure or Positive Pressure

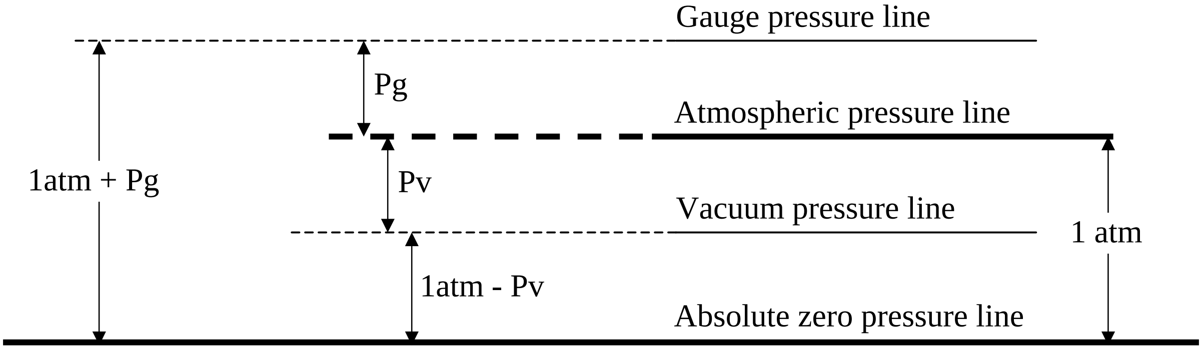
It is the pressure recorded by an instrument. This is always above atmospheric. The zero mark of the dial will have been adjusted to atmospheric pressure.

3- Vacuum Pressure or Negative Pressure

This pressure is caused either artificially or by flow conditions. The pressure intensity will be less than the atmospheric pressure whenever vacuum is formed.

4- Absolute Pressure

Absolute pressure is the algebraic sum of atmospheric pressure and gauge pressure. Atmospheric pressure is usually considered as the datum line and all other pressures are recorded either above or below it.



Absolute Pressure = Atmospheric Pressure + Gauge Pressure

Absolute Pressure = Atmospheric Pressure – Vacuum Pressure

For example if the vacuum pressure is 0.3 atm ⇒ absolute pressure = 1.0 – 0.3 = 0.7 atm

Note: -

Barometric pressure is the pressure that recorded from the barometer (apparatus used to measure atmospheric pressure).

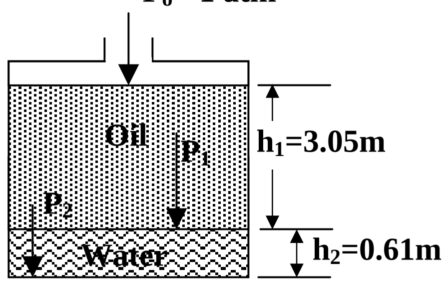
3.4 Head of Fluid

Pressures are given in many different sets of units, such as N/m^2, or Pa, $dyne/cm^2$, psi, lb_f/ft^2. However a common method of expressing pressures is in terms of head (m, cm, mm, in, or ft) of a particular fluid. This height or head of the given fluid will exert the same pressure as the pressures it represents. P = h ρ g.

Example -3.1-

A large storage tank contains oil having a density of 917 kg/m^3. The tank is 3.66 m tall and vented (open) to the atmosphere of 1 atm at the top. The tank is filled with oil to a depth of 3.05 m (10 ft) and also contains 0.61 m (2 ft) of water in the bottom of the tank. Calculate the pressure in Pa and psia at 3.05 m from the top of the tank and at the bottom. And calculate the gauge pressure at the bottom of the tank.

Solution:



P_o = 1 atm = 14.696 psia = 1.01325 x 10^5 Pa

$P_1 = h_1 \rho_{oil} g + P_o$

= 3.05 m (917 kg/m^3) 9.81 m/s^2 + 1.01325 x 10^5 Pa

= 1.28762 x 10^5 Pa

P_1 = 1.28762 x 10^5 Pa (14.696 psia/1.01325 x 10^5 Pa)

= 18.675 psia

or

$P_1 = h_1 \rho_{oil} g + P_o$

= 10 ft m [917 kg/m^3 (62.43 lb/ft^3/1000 kg/m^3)] (32.174 ft/s^2/32.174 $lb.ft/lb_f.s^2$) 1/144 ft^2/in^2 +14.696 = 18.675 psia

$P_2 = P_1 + h_2 \rho_{water} g$

= 1.28762 x 10^5 Pa + 0.61 m (1000 kg/m^3) 9.81 m/s^2

= 1.347461 x 10^5 Pa

P_2 = 1.347461 x 10^5 Pa (14.696 psia/1.01325 x 10^5 Pa)

= 19.5433 psia

The gauge pressure = abs – atm

= 33421.1 Pa = 4.9472 psig

Example -3.2-

Convert the pressure of [1 atm =101.325 kPa] to

a- head of water in (m) at 4°C

b- head of Hg in (m) at 0°C

Solution:

a- The density of water at 4°C is approximatly1000 kg/m^3

$h = P / \rho_{water}\, g = 1.01325 \times 10^5$ Pa/(1000 kg/m^3 x 9.81m/s^2) =10.33 m H_2o

b- The density of mercury at 0°C is approximatly13595.5 kg/m^3

$h = P / \rho_{mercury}\, g = 1.01325 \times 10^5$ Pa/(13595.5 kg/m^3 x 9.81m/s^2) =0.76 m Hg

or

$P = (h\,\rho\,g)_{water} = (h\,\rho\,g)_{mercury} \quad \Rightarrow \; h_{Hg} = h_{water}\,(\rho_{water} / \rho_{Hg})$

$h_{Hg} = 10.33\ (1000 / 13595.5) = 0.76$ m Hg

Example -3.3-

Find the static head of a liquid of sp.gr. 0.8 and pressure equivalent to 5 x 10^4 Pa.

Solution:

$\rho = 0.8\ (1000) = 800$ kg/m^3

$h = P / \rho\, g = 5\times10^4/(800 \times 9.81) = 6.37$ m H_2o

3.5 Measurement of Fluid Pressure

In chemical and other industrial processing plants it is often to measure and control the pressure in vessel or process and/or the liquid level vessel.

The pressure measuring devices are: -

1- Piezometer tube

The piezometer consists a tube open at one end to atmosphere, the other end is capable of being inserted into vessel or pipe of which pressure is to be measured. The height to which liquid rises up in the vertical tube gives the pressure head directly.

i.e. $\boxed{P = h\,\rho\,g}$

Piezometer is used for measuring moderate pressures. It is meant for measuring *gauge pressure* **only** as the end is open to atmosphere. It cannot be used for *vacuum pressures*.

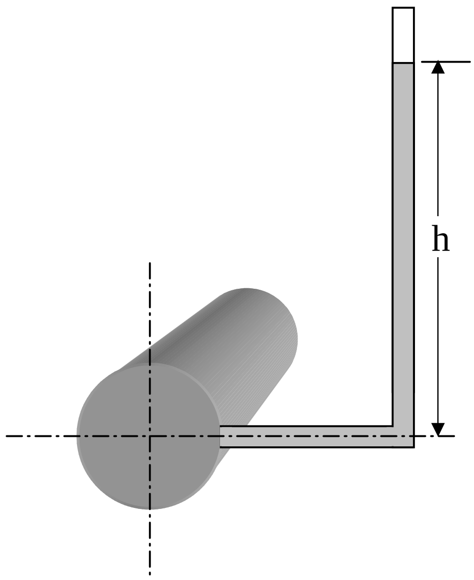


Figure (3): The Piezometer

2- Manometers

The manometer is an improved (modified) form of a piezometer. It can be used for measurement of comparatively *high pressures* and of both *gauge and vacuum pressures*.

Following are the various types of manometers: -

a- Simple manometer
b- The well type manometer
c- Inclined manometer
d- The inverted manometer
e- The two-liquid manometer

a- Simple manometer

It consists of a transparent U-tube containing the fluid A of density (ρ_A) whose pressure is to be measured and an immiscible fluid (B) of higher density (ρ_B). The limbs are connected to the two points between which the pressure difference (P_2 - P_1) is required; the connecting leads should be completely full of fluid A. If P_2 is greater than P_1, the interface between the two liquids in limb ② will be depressed a distance (h_m) (say) below that in limb ①.

The pressure at the level a — a must be the same in each of the limbs and, therefore:

$$P_2 + Z_m \rho_A g = P_1 + (Z_m - h_m) \rho_A g + h_m \rho_B g$$

$$\Rightarrow \quad \Delta p = P_2 - P_1 = h_m (\rho_B - \rho_A) g$$

If fluid A is a gas, the density ρ_A will normally be small compared with the density of the manometer fluid pm so that:

$$\Delta p = P_2 - P_1 = h_m \rho_B g$$

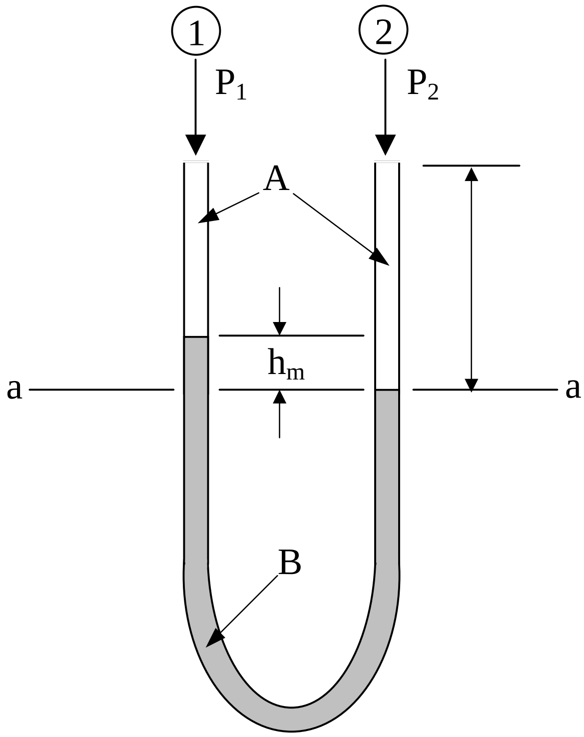


Figure (4): The simple manometer

b- The well-type manometer

In order to avoid the inconvenience of having to read two limbs, and in order to measure low pressures, where accuracy id of much importance, the well-type manometer shown in Figure (5) can be used. If A_w and A_c are the cross-sectional areas of the well and the column and h_m is the increase in the level of the column and h_w the decrease in the level of the well, then:

$$P_2 = P1 + (h_m + h_w) \rho g$$

or: $$\Delta p = P_2 - P_1 = (h_m + h_w) \rho g$$

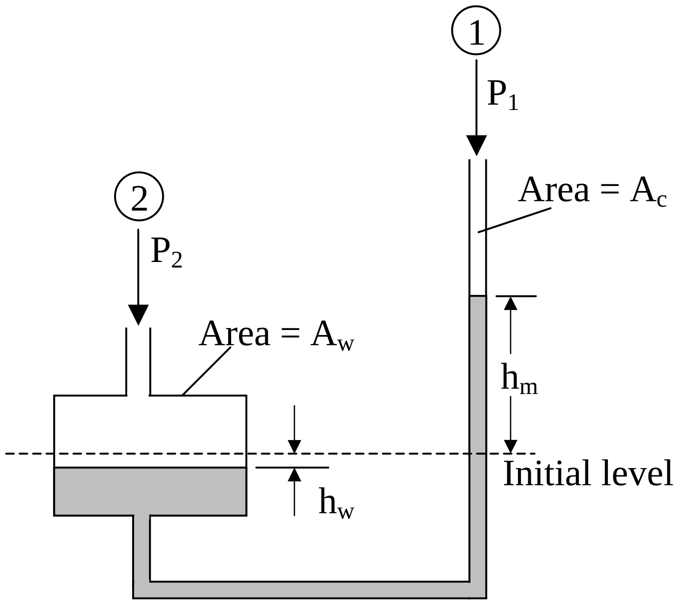


Figure (5): The well-type manometer

The quantity of liquid expelled from the well is equal to the quantity pushed into the column so that:

$$A_w h_w = A_c h_m \Rightarrow h_w = (A_c/A_w) h_m$$

$$\Rightarrow \Delta p = P_2 - P_1 = \rho g h_m (1 + A_c/A_w)$$

If the well is large in comparison to the column then:

i.e. $(A_c/A_w) \rightarrow \approx 0 \Rightarrow \quad \Delta p = P_2 - P_1 = \rho g h_m$

c- The inclined manometer

Shown in Figure (6) enables the sensitivity of the manometers described previously to be increased by measuring the length of the column of liquid. If θ is the angle of inclination of the manometer (typically about 10-20°) and L is the movement of the column of liquid along the limb, then:

$$h_m = L \sin\theta$$

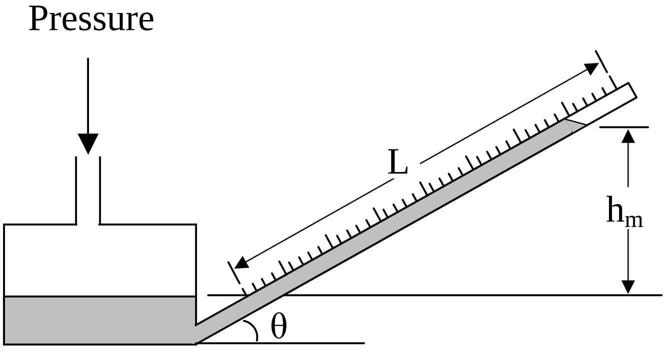


Figure (6): The inclined manometer

If θ = 10°, the manometer reading L is increased by about 5.7 times compared with the reading h_m which would have been obtained from a simple manometer.

d- The inverted manometer

Figure (7) is used for measuring <u>*pressure differences in liquids*</u>. The space above the liquid in the manometer is filled with air, which can be admitted or expelled through the tap A in order to adjust the level of the liquid in the manometer.

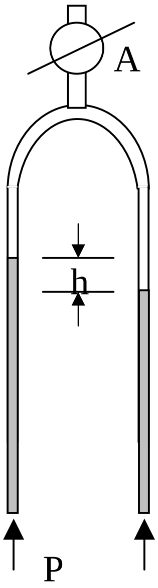


Figure (6): The inverted manometer

e- The two-liquid manometer

Small differences in pressure in gases are often measured with a manometer of the form shown in Figure 6.5. The reservoir at the top of each limb is of a sufficiently large cross-section for the liquid level to remain approximately the on each side of the manometer.

The difference in pressure is then given by:

$$\Delta p = P_2 - P_1 = h_m (\rho_{m1} - \rho_{m2})\, g$$

where ρ_{m1} and ρm2 are the densities of the two manometer liquids. The sensitivity of the instrument is very high if the densities of the two liquids are nearly the same. To obtain accurate readings it is necessary to choose liquids, which give sharp interfaces: paraffin oil and industrial alcohol are commonly used.

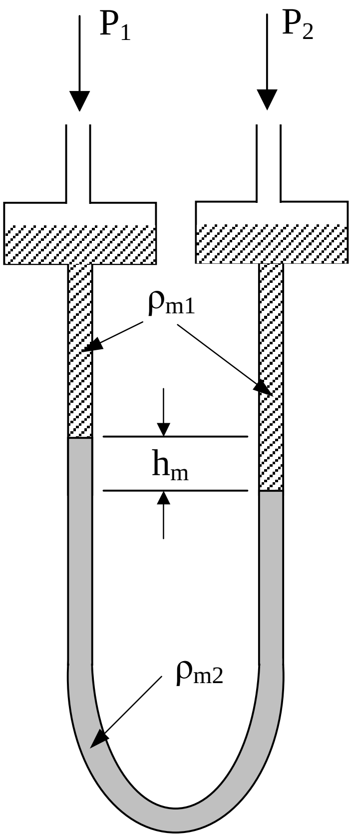


Figure (7): The two-liquid manometer

3- Mechanical Gauges

Whenever a *very high fluid pressure* is to be measured, and a *very great sensitivity* a mechanical gauge is best suited for these purposes. They are also designed to read vacuum pressure. A mechanical gauge is also used for measurement of pressure in boilers or other pipes, where tube manometer cannot be conveniently used.

There are many types of gauge available in the market. But the principle on which all these gauge work is almost the same. The followings are some of the important types of mechanical gauges: -

1- The Bourdon gauge
2- Diaphragm pressure gauge
3- Dead weight pressure gauge

The Bourdon gauge

The pressure to be measured is applied to a curved tube, oval in cross-section, and the deflection of the end of the tube is communicated through a system of levers to a recording needle. This gauge is widely used for *steam* and *compressed gases*, and frequently *forms the indicating element on flow controllers*. The simple form of the gauge is illustrated in Figures (7a) and (7b). Figure (7c) shows a Bourdon type gauge with the sensing element in the form of a helix; this instrument has a very much greater sensitivity and is suitable for very high pressures.

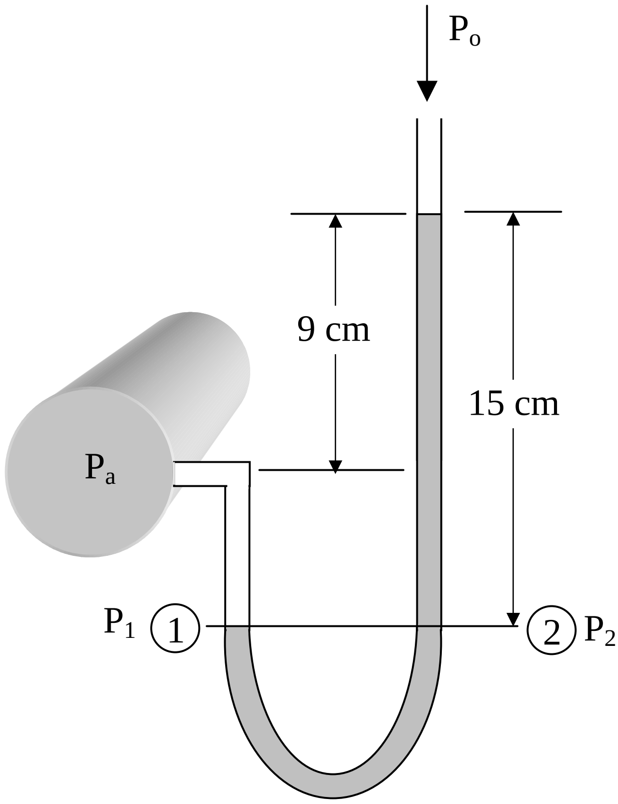
It may be noted that the pressure measuring devices of category (2) all measure a pressure difference ($\Delta p = P_2 - P_1$). In the case of the Bourdon gauge (1) of category (3), the pressure indicated is the difference between that communicated by the system to the tube and the external (ambient) pressure, and this is usually referred to as *the gauge pressure*. It is then necessary to add on the ambient pressure in order to obtain the *(absolute) pressure*.

Figure (7) Bourdon gauge

Gauge pressures are not, however, used in the SI System of units.

Example -3.4-

A simple manometer is used to measure the pressure of oil sp.gr. 0.8 flowing in a pipeline. Its right limb is open to atmosphere and the left limb is connected to the pipe. The center of the pipe is 9.0 cm below the level of the mercury in the right limb. If the difference of the mercury level in the two limbs is 15 cm, determine the absolute and the gauge pressures of the oil in the pipe.



Solution:

ρ = 0.8 (1000) = 800 kg/m^3

$P_1 = P_2$

P_1 = (0.15 –0.0 9)m(800 kg/m^3)9.81m/s^2+ P_a

P_2 = (0.15) m (13600 kg/m^3) 9.81 m/s^2 + P_o

P_a = 15 (13600) 9.81 + P_o + [(15 – 9)cm (800 kg/m^3) 9.81 m/s^2]

= 1.20866 x 10^5 Pa (Absolute pressure)

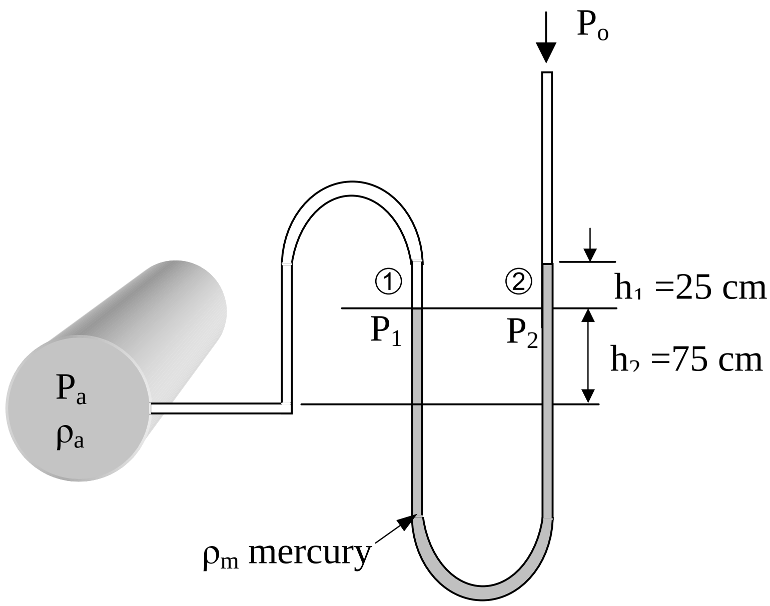
The gauge press. = Abs. press. – Atm. Press.

= 1.20866 x 10^5 - 1.0325 x 10^5

= 1.9541 x 10^4 Pa

Example -3.5-

The following Figure shows a manometer connected to the pipeline containing oil of sp.gr. 0.8. Determine the absolute pressure of the oil in the pipe, and the gauge pressure.



Solution:

ρ_a = 0.8 (1000) = 800 kg/m^3

$P_1 = P_2$

$P_1 = P_a - h_2 \rho_a g$

$P_2 = P_o + h_1 \rho_m g$

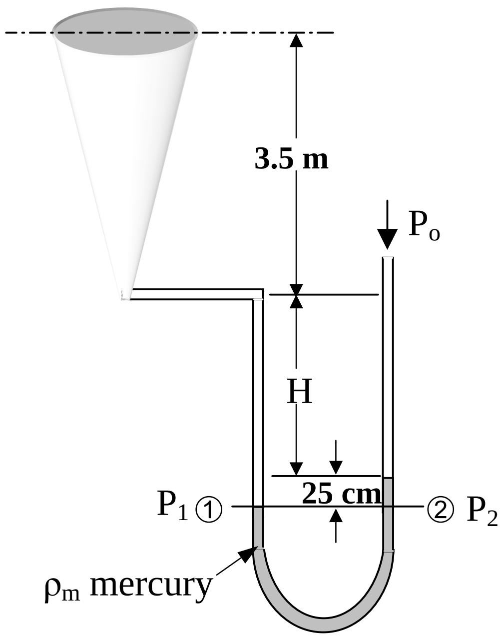
$\Rightarrow P_a = P_o + h_1 \rho_m g + h_2 \rho_a g$

= 1.0325 x 10^5 + (0.25) m (13600 kg/m^3) 9.81 m/s^2 + (0.75) m (800 kg/m^3) 9.81 m/s^2

= 1.40565 x 10^5 Pa

Example -3.6-

A conical vessel is connected to a U-tube having mercury and water as shown in the Figure. When the vessel is empty the manometer reads 0.25 m. find the reading in manometer, when the vessel is full of water.



Solution:

$P_1 = P_2$

$P_1 = (0.25 + H)\,\rho_w\, g + P_o$

$P_2 = 0.25\,\rho_m\, g + P_o$

$\Rightarrow (0.25 + H)\,\rho_w\, g + P_o = 0.25\,\rho_m\, g + P_o$

$\Rightarrow H = 0.25\,(\rho_m - \rho_w)/\,\rho_w$

$= 0.25\,(12600\,/1000) = 3.15$ m

When the vessel is full of water, let the mercury level in the left limp go down by (x) meter and the mercury level in the right limp go to up by the same amount (x) meter.

i.e. the reading manometer = (0.25 + 2x)

$P_1 = P_2$

$P_1 = (0.25 + x + H + 3.5)\,\rho_w\, g + P_o$

$P_2 = (0.25 + 2x)\,\rho_m\, g + P_o$

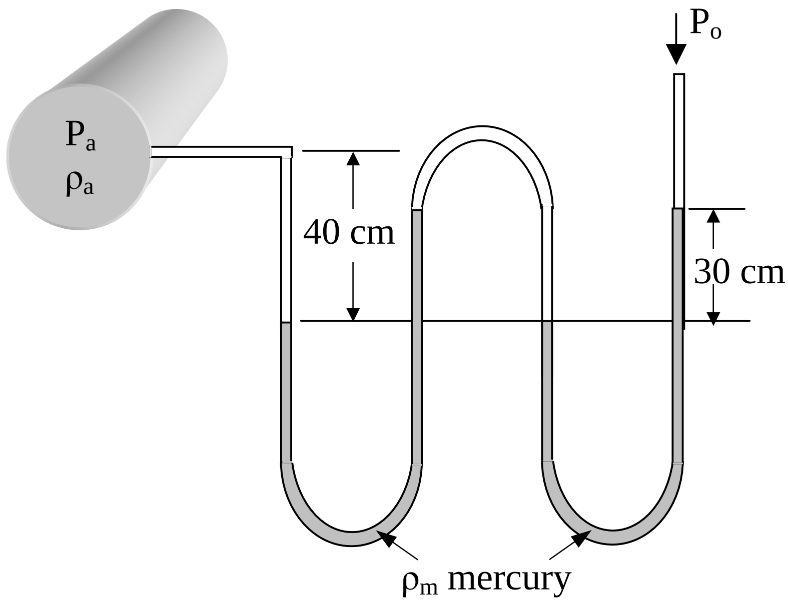
$\Rightarrow (0.25 + x + H + 3.5)\,\rho_w\, g + P_o = (0.25 + 2x)\,\rho_m\, g + P_o$

$\Rightarrow 6.9 + x = (0.25 + 2x)\,(\rho_m/\,\rho_w) \Rightarrow x = 0.1431$ m

The manometer reading = 0.25 + 2 (0.1431) = 0.536 m

Example -3.7-

The following Figure shows a compound manometer connected to the pipeline containing oil of sp.gr. 0.8. Calculate P_a.



Solution:

$\rho_a = 0.8\,(1000) = 800\ \text{kg/m}^3$

$P_a + 0.4\,\rho_a\, g - 0.3\,\rho_m\, g + 0.3\,\rho_a\, g - 0.3\,\rho_m\, g - P_o = 0$

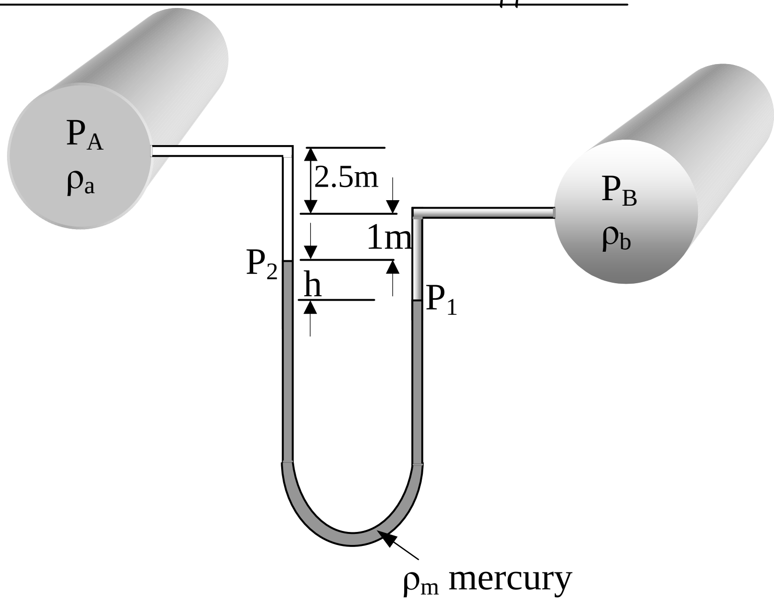
$\Rightarrow P_a = P_o + 0.7\,\rho_a\, g - 0.6\,\rho_m\, g$

$= 1.01325 \times 10^5 - 0.7\,(800)\,9.81 + 0.6\,(13600)\,9.81$

$= 1.75881 \times 10^5$ Pa

Example -3.8-

A differential manometer is connected to two pipes as shown in Figure. The pipe A is containing carbon tetrachloride sp.gr. = 1.594 and the pipe B is contain an oil of sp.gr. = 0.8. Find the difference of mercury level if the pressure difference in the two pipes be 0.8 kg/cm^2.



Solution:

$P_1 = P_2$

$P_1 = P_B + (1 + h)\, \rho_b\, g$

$P_2 = P_A + 3.5\, \rho_a\, g + h\, \rho_m\, g$

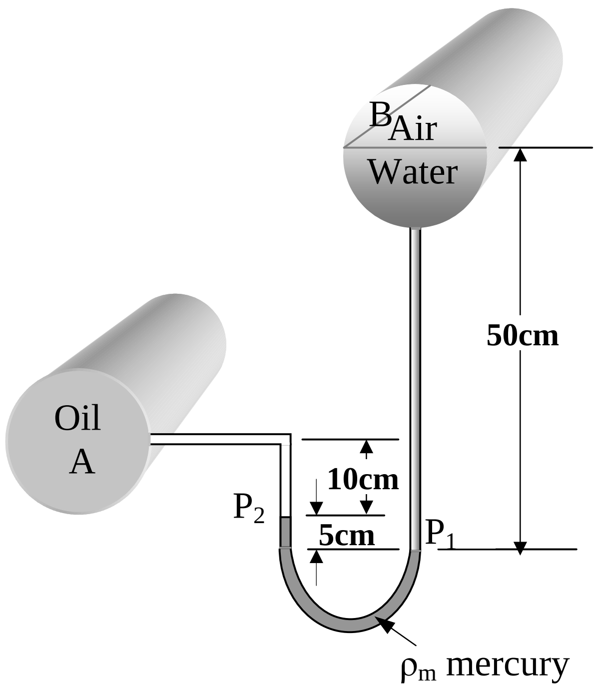
$\Rightarrow\ P_A - P_B = 3.5\, \rho_a\, g + h\, \rho_m\, g - (1 + h)\, \rho_b\, g = (0.8 \text{ kg/cm}^2)\,(9.81 \text{ m/s}^2)\,(10^4 \text{ cm}^2/\text{m}^2)$

$\Rightarrow\ 7.848 \times 10^4 = 3.5\,(1594)\,9.81 + h\,(13600)\,9.81 - (1+h)\,800\,(9.81)$

$\Rightarrow h = 25.16$ cm.

Example -3.9-

A differential manometer is connected to two pipes as shown in Figure. At B the air pressure is 1.0 kg/cm^2 (abs), find the absolute pressure at A.



Solution:

$P_1 = P_2$

$P_1 = P_{air} + 0.5\, \rho_w\, g$

$P_2 = P_A + 0.1\, \rho_a\, g + 0.05\, \rho_m\, g$

$\Rightarrow P_A = P_{air} + 0.5\, \rho_w\, g - 0.1\, \rho_a\, g - 0.05\, \rho_m\, g$

$\Rightarrow\ P_{air} = (1.0 \text{ kg/cm}^2\ P_B)\,(9.81 \text{m/s}^2)\,(10^4 \text{ cm}^2/\text{ m}^2)$

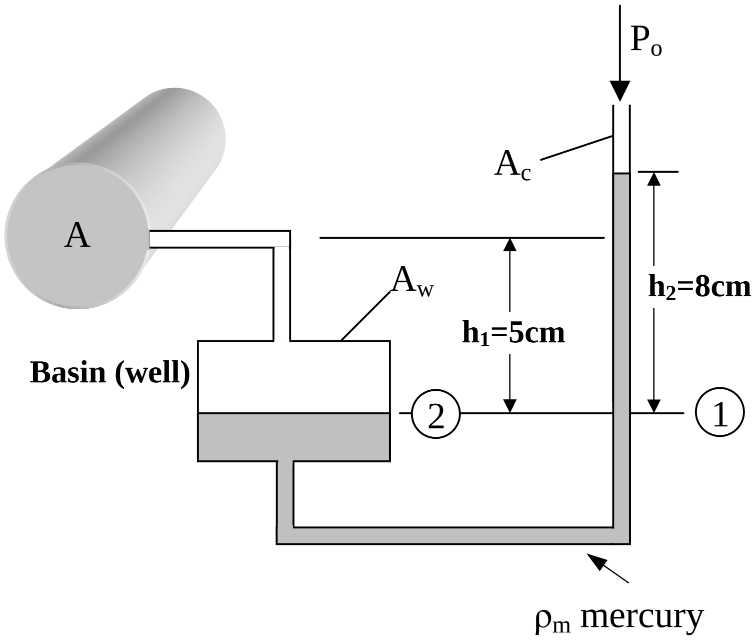
$= 9.81 \times 10^4$ Pa

$\therefore P_A = 9.81 \times 10^4 \text{ Pa} + 0.5\,(1000)\,9.81 - 0.1\,(900)\,9.81 - 0.05\,(13600)\,9.81$

$= 9.54513 \times 10^4$ Pa

Example -3.10-

A Micromanometer, having ratio of basin to limb areas as 40, was used to determine the pressure in a pipe containing water. Determine the pressure in the pipe for the manometer reading shown in Figure.



Solution:

$P_1 = P_2$

$P_1 = P_o + h_2\, \rho_m\, g$

$P_2 = P_A + h_1\, \rho_w\, g$

$\Rightarrow\ P_A = P_o + h_2\, \rho_m\, g - h_1\, \rho_w\, g$

$= 1.01325 \times 10^5 + 0.08\ (13600)\ 9.81 - 0.05\ (1000)\ 9.81$

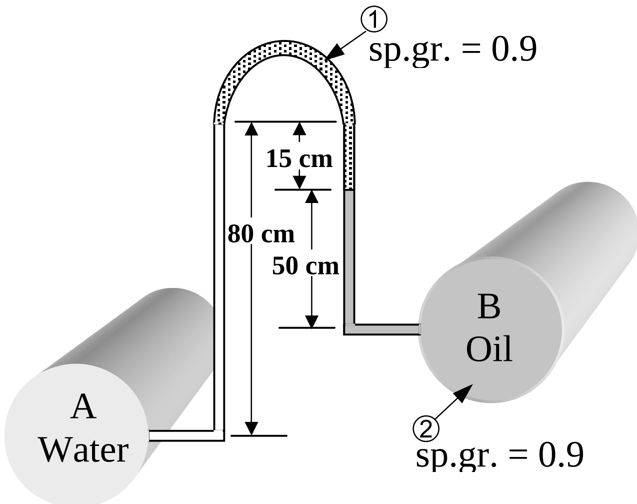
$= 1.11507 \times 10^5$ Pa

Note:

If h_2 and h_1 are the heights from initial level, the ratio (A_w/A_c) will enter in calculation.

Example -3.11-

An inverted manometer, when connected to two pipes A and B, gives the readings as shown in Figure. Determine the pressure in tube B, if the pressure in pipe A 1.0 kg/cm^2.



Solution:

$P_A - 0.8\, \rho_w\, g + 0.15\, \rho_1\, g + 0.5\, \rho_2\, g - P_B = 0$

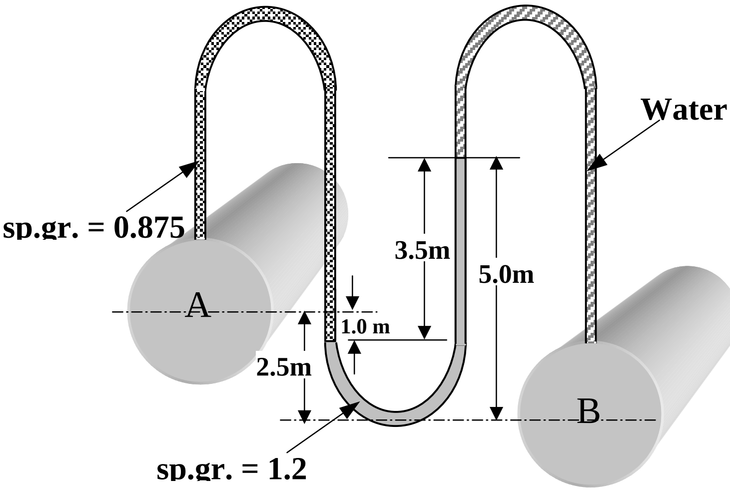
$\Rightarrow\ P_B = P_A - [0.8\ (1000) - 0.15\ (800) - 0.5\ (900)]\ 9.81$

$P_A = 1.0$ kg/cm$^2 \times 9.81 \times 10^4 = 9.81 \times 10^4$ Pa

$\therefore P_A = 9.58437 \times 10^4$ Pa

Example -3.12-

Two pipes, one carrying toluene of sp.gr. = 0.875, and the other carrying water are placed at a difference of level of 2.5 m. the pipes are connected by a U-tube manometer carrying liquid of sp.gr. = 1.2. The level of the liquid in the manometer is 3.5 m higher in the right limb than the lower level of toluene in the limb of the manometer. Find the difference of pressure in the two pipes.



Solution:

T ≡ Toluene, W ≡ Water, L ≡ Liquid

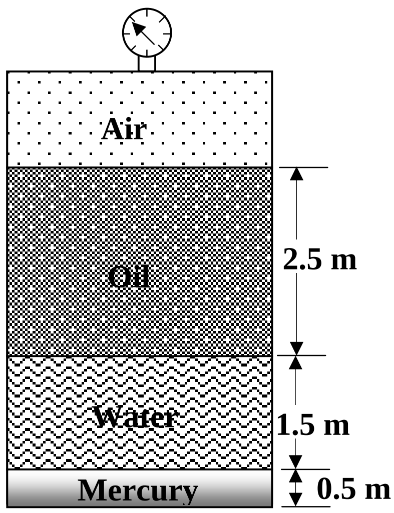
$P_A + 3.5\,\rho_T\,g - 3.5\,\rho_L\,g + 5\,\rho_W\,g - P_B = 0$

$\Rightarrow P_A - P_B = [3.5\,(1200) - 3.5\,(875) - 5\,(1000)]\,9.81$
$= -3862.5$ Pa

$\Rightarrow P_B - P_A = 3862.5$ Pa

Example -3.13-

A closed tank contains 0.5 m of mercury, 1.5 m of water, 2.5 m of oil of sp.gr. = 0.8 and air space above the oil. If the pressure at the bottom of the tank is 2.943 bar gauge, what should be the reading of mechanical gauge at the top of the tank.



Solution:

Pressure due to 0.5 m of mercury

$P_m = 0.5\,(13600)\,9.81 = 0.66708$ bar

Pressure due to 1.5 m of water

$P_w = 1.5\,(1000)\,9.81 = 0.14715$ bar

Pressure due to 2.5 m of oil

$P_O = 2.5\,(800)\,9.81 = 0.19620$ bar

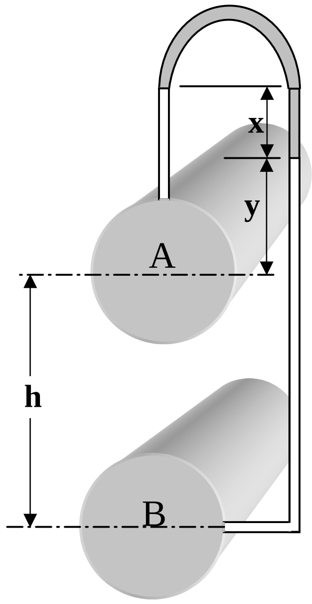
Pressure at the bottom of the tank = $P_m + P_w + P_O + P_{Air}$

$\Rightarrow 2.943 = 0.66708$ bar $+ 0.14715$ bar $+ 0.19620$ bar $+ P_{Air}$

$\Rightarrow P_{Air} = 1.93257$ bar

Home Work

P.3.1



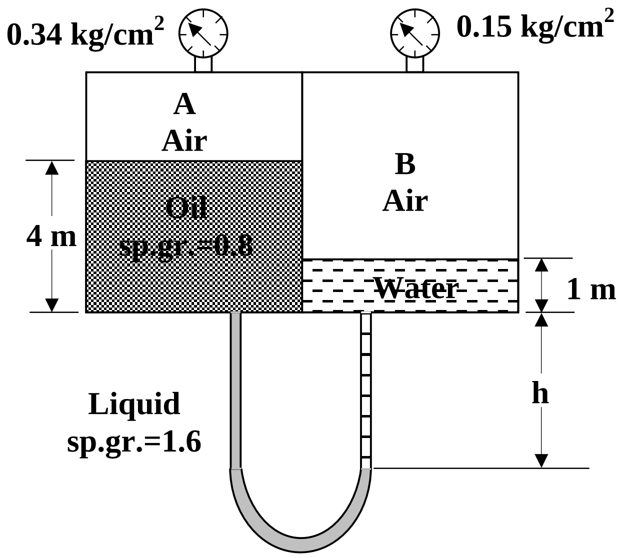
Two pipes A and B carrying water are connected by a connecting tube as shown in Figure,

a- If the manometric liquid is oil of sp.gr. = 0.8, find the difference in pressure intensity at A and B when the difference in level between the two pipes be (h = 2 m) and (x = 40 cm).

b- If mercury is used instead of water in the pipes A and B and the oil used in the manometer has sp.gr. = 1.5, find the difference in pressure intensity at A and B when (h = 50 cm) and (x = 100 cm).

Ans. a- P_B-P_A =18835.2 Pa, b- P_B-P_A =51993 Pa

P.3.2



A closed vessel is divided into two compartments. These compartments contain oil and water as shown in Figure. Determine the value of (h).

Ans. h = 4.5 m

P.3.3

Oil of sp.gr. = 0.9 flows through a vertical pipe (upwards). Two points A and B one above the other 40 cm apart in a pipe are connected by a U-tube carrying mercury. If the difference of pressure between A and B is 0.2 kg/cm^2,

1- Find the reading of the manometer.

2- If the oil flows through a horizontal pipe, find the reading in manometer for the same difference in pressure between A and B.

Ans. 1- R = 0.12913 m, 2- R = 0.1575 m,

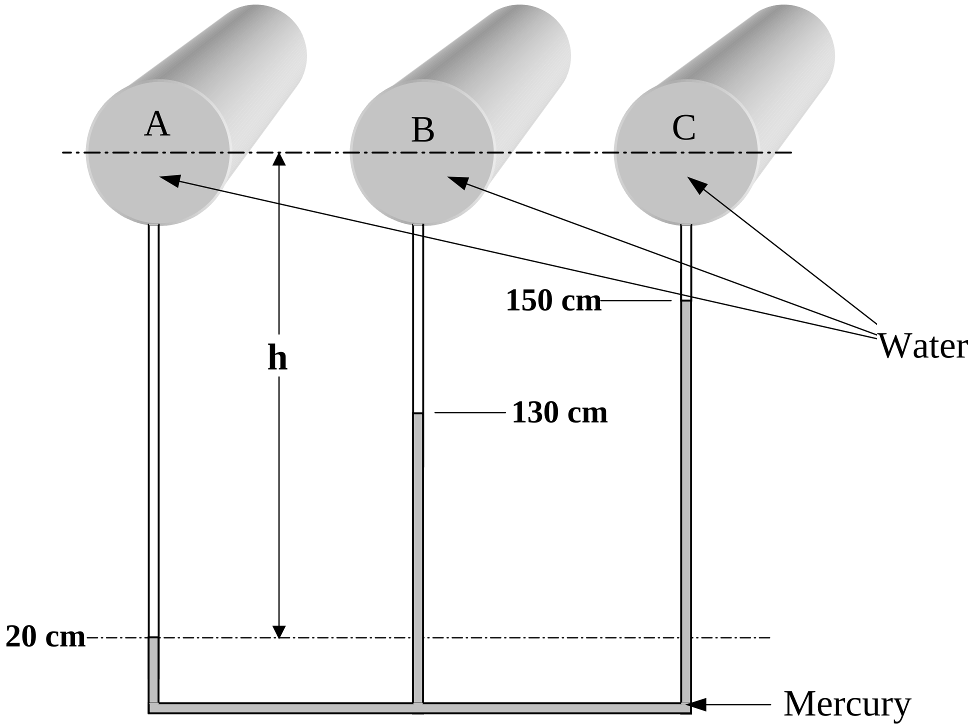
P.3.4

A mercury U-tube manometer is used to measure the pressure drop across an orifice in pipe. If the liquid that flowing through the orifice is brine of sp.gr. 1.26 and upstream pressure is 2 psig and the downstream pressure is (10 in Hg) vacuum, find the reading of manometer.

Ans. R = 394 mm Hg

P.3.5

Three pipes A, B, and C at the same level connected by a multiple differential manometer shows the readings as show in Figure. Find the differential of pressure heads in terms of water column between A and B, between A and C, and between B and C.



Ans. P_A-P_B =1.359666 bar = 13.86 m H_2o
P_A-P_C =1.606878 bar = 16.38 m H_2o
P_B-P_C =0.247212 bar = 2.52 m H_2o