**Al-Mustaqbal University**

**College of Engineering and Technologies**

**Prosthetics and Orthotics Engineering Department**

**Class: 2nd**

**Lecture: 3 Kirchhoff's laws,**

## *Lecture 3:*

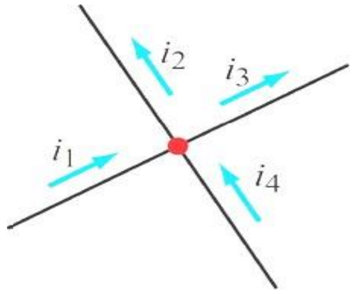
# Kirchhoff's Laws

## Kirchhoff's Laws

Ohm's law by itself is not adequate to analyze circuits. So, when it is coupled with Kirchhoff's two laws, we have an adequate, active set of tools for analyzing a large variety of electric circuits. Kirchhoff's laws were first introduced in 1847 by the German physical G. Robert Kirchhoff (1824–1887). These laws are formally known as *Kirchhoff's current law (KCL)* and *Kirchhoff's voltage law (KVL)*.

**1- Kirchhoff's current law (KCL):** states that the algebraic sum of currents entering a node is zero.

Mathematically, KCL is represented as below:




$$\sum_{n=1}^{N} i_n = 0$$

Where n is the number of branches connected to the node and $i_n$ is the $n^{th}$ current entering (or leaving) the node.

Based on this law, currents entering a node may be positive, while currents leaving the node may be taken as negative or vice versa.

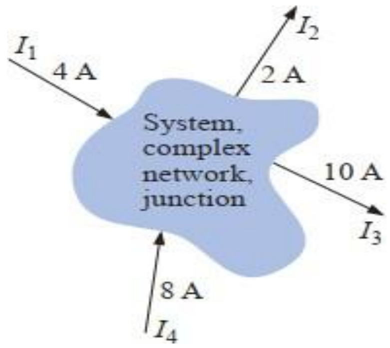



Fig.(1)

For figure(1),

$$I_1 + (-I_2) + (-I_3) + I_4 = 0$$

**So, Kirchhoff's current law (KCL) states that the sum of the** currents entering a node is equal to the sum of the currents leaving the node.

Thus,

$$\sum I_{\ entering} = \sum I_{\ leaving}$$

$$I_1 + I_4 = I_2 + I_3$$

$$4\text{A} + 8\text{A} = 2\text{A} + 10\text{A}$$

$$4\text{A} + 8\text{A} - 2\text{A} - 10\text{A} = 0$$

12A=12A

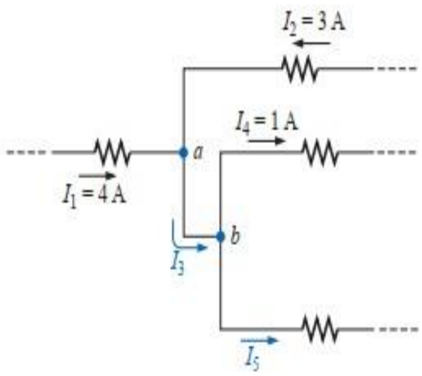



**FIG. 6.29**
*Example 6.15.*

**EXAMPLE 6.15** Determine the currents $I_3$ and $I_5$ of Fig. 6.29 through applications of Kirchhoff's current law.

***Solution:*** Note that since node $b$ has two unknown quantities and node $a$ has only one, we must first apply Kirchhoff's current law to node $a$. The result can then be applied to node $b$.

For node $a$,

$$I_1 + I_2 = I_3$$

$$4\text{ A} + 3\text{ A} = I_3$$

and

$$I_3 = \mathbf{7\ A}$$

For node $b$,

$$I_3 = I_4 + I_5$$

$$7\text{ A} = 1\text{ A} + I_5$$

and

$$I_5 = 7\text{ A} - 1\text{ A} = \mathbf{6\ A}$$

**2- Kirchhoff's voltage law (KVL): states that the algebraic sum of all voltages around a closed path (or loop) is zero.**

Mathematical expression for "KVL" states that;

of voltages in the loop (or th number of branches in the loop)and $Vm$ is the $m^{th}$ voltage.

$$\sum_{n=1}^{M} V_m = 0$$

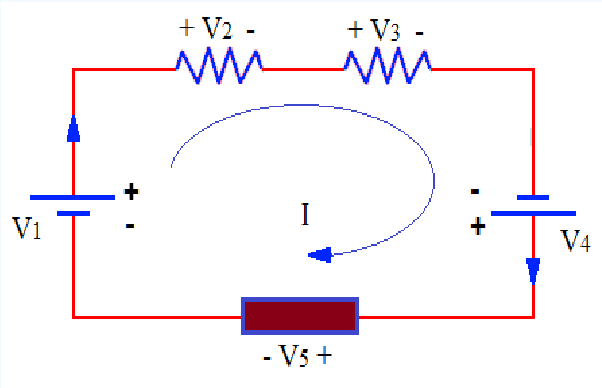



Fig.(2)

So for Figure (2):

$$-V_1 + V_2 + V_3 - V_4 + V_5 = 0$$

And

## Determination of signs for sources and elements for KVL

**1-For voltage source, If we start the loop from negative terminal to positive terminal, then the voltage should be given negative sign and vas versa.**

**2-For elements (resistances),** if we start the loop through the resistor in the same direction as the current through it, then the voltage should be given positive sign vas versa.

To illustrate this rule , see the two examples below:

**Example 1:** Find the voltage between terminals a & b in Figure (3)?

**Solution:**

$- V_{ab} + V_1 + V_2 - V_3 = 0$

$\therefore V_{ab} = V_1 + V_2 - V_3$

If the sources have the followingvalues;

$V_1 = 50\ V$

$V_2 = 30\ V$

$V_3 = 45\ V$

Then $V_{ab} = 50 + 30 - 45 = 35\ V$

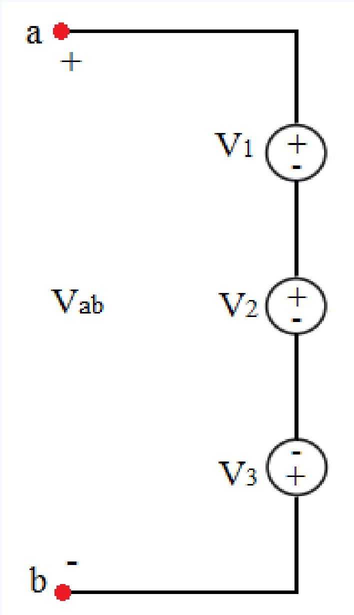



Fig.(3)

So we can replace the three series sources by a single source

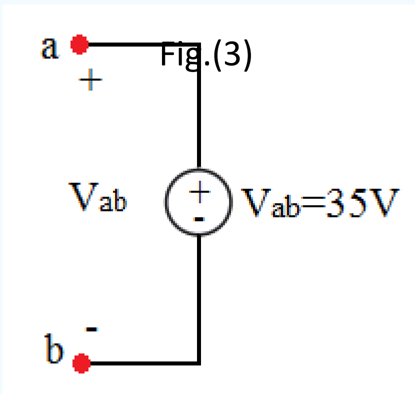
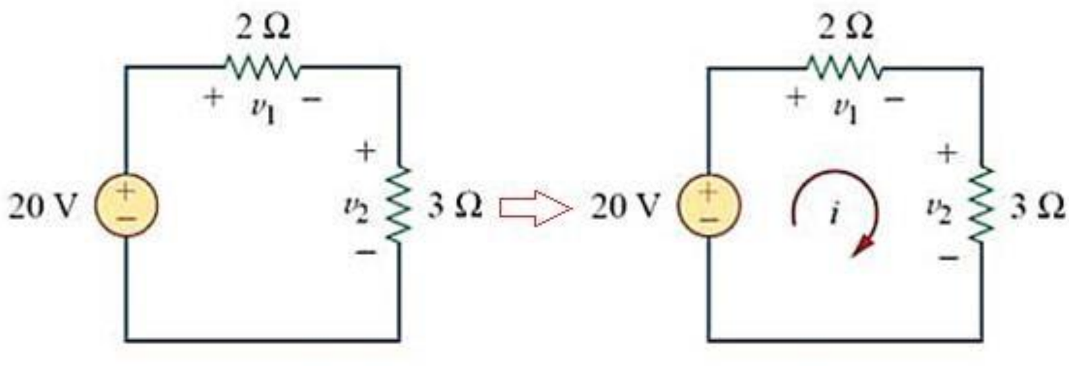



Fig.(4)

**Example:-** for the circuit in **Figure(a & b),** find voltages $v_1$ & $v_2$ ?




(a) (b)

**Solution:**

**Assume that current *i* flows through the loop as shown in Figure**

Applying KVL around the loop gives :

$-20 + v_1 + v_2 = 0$ ---------------------------------------------- (1)

$v_1 = 2i$, & $v_2 = 3i$ ---------------------------------------- (2)

**Substituting Eq. (2) into Eq. (1), we obtain:**

$-20 + 2i + 3i = 0 \Rightarrow -20 = -2i - 3i \Rightarrow 20 = (2+3)\, i \Rightarrow 5i = 20 \Rightarrow$

$i = 4$ A

**Substituting *i* in Eq. (1) finally gives,**

$v_1 = 8V$ , $v_2 = 12$ V

**Example** 2: Find the unknown voltage $V_1$ in the circuit of Figure (5), use Kirchhoff's laws?

**Solution:**

Apply KVL through loop AFEDCBA,

$-V_1 + 40 - V_2 - V_3 = 0$

$V_2 = 2I_2 = 2 \times 4 = 8$ V

$V_3 = 3I_3 = 3 \times 16 = 48$ V

Applying "KVL"

$\therefore -V_1 + 40 - 8 - 48 = 0$

$\therefore V_1 = -16$ V

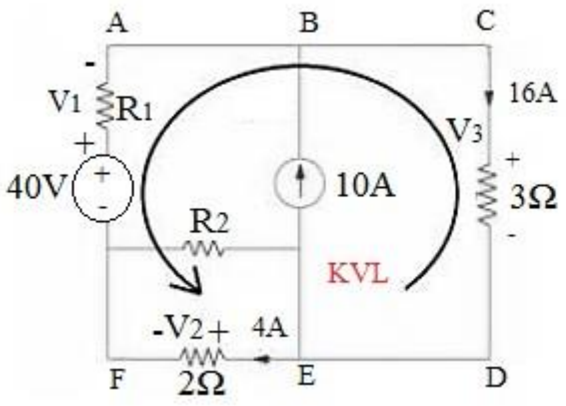



Fig.(5)

Revers loop:

$+V_1 - 40 + V_2 + V_3 = 0$

Example 3: Using Kirchhoff's Current Law and Ohm's Law, find the magnitude and polarity of voltage V in Figure (6)?

Solution:

- Choose the directions of $I_1$, $I_2$ and $I_3$ and polarity of V as shown in Figure (7).
- Applying (KCL) to node A, we have

$+30 + I_2 - I_1 - I_3 - 8 = 0$

<u>Or</u> ➔ $I_1 - I_2 + I_3 = 22$ --------- (1)

Applying Ohm's law to the three resistive branches in Figure (7), we have

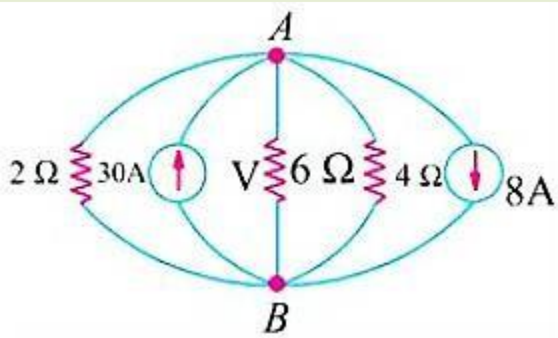



Fig.(6)

$I_1 = \frac{V}{2}$ , $I_2 = -\frac{V}{6}$ , $I_3 = \frac{V}{4}$

Substituting these values in (1)

above, we get:

$\frac{V}{2} + \frac{V}{6} + \frac{V}{4} = 22$ ➔ $\therefore V = 24V$

$$\therefore I_1 = \frac{V}{2} = \frac{24}{2} = 12A$$

$$I_2 = -\frac{24}{6} = -4A$$

$$I_3 = \frac{24}{4} = 6\ A$$

$$\therefore 30 - 4 - 12 - 6 - 8 = 0$$

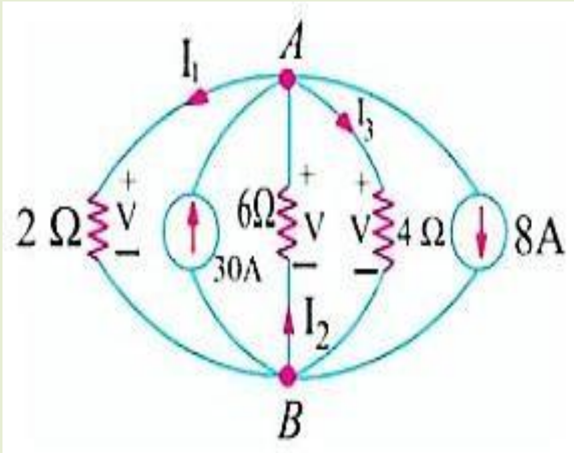



Fig.(7)

**EXAMPLE 8** Use Kirchhoff's voltage law to determine the unknown voltage for the circuit in Fig. 27.

***Solution:***

Application of Kirchhoff's voltage law to the circuit in Fig. 27 in the clockwise direction results in

$$+E_1 - V_1 - V_2 - E_2 = 0$$

and
$$V_1 = E_1 - V_2 - E_2$$
$$= 16\ V - 4.2\ V - 9\ V$$

so
$$V_1 = 2.8\ V$$

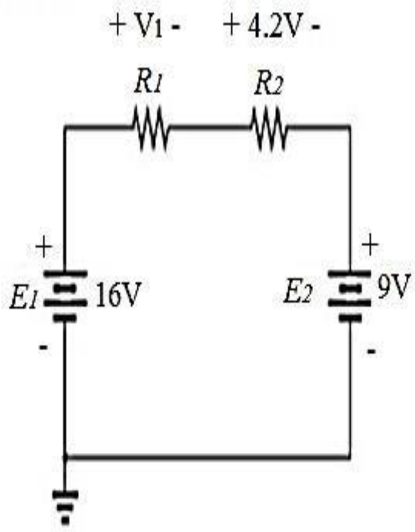



**FIG. 27**

2nd solution

$$-E_1 + V_1 + V_2 + E_2 = 0$$

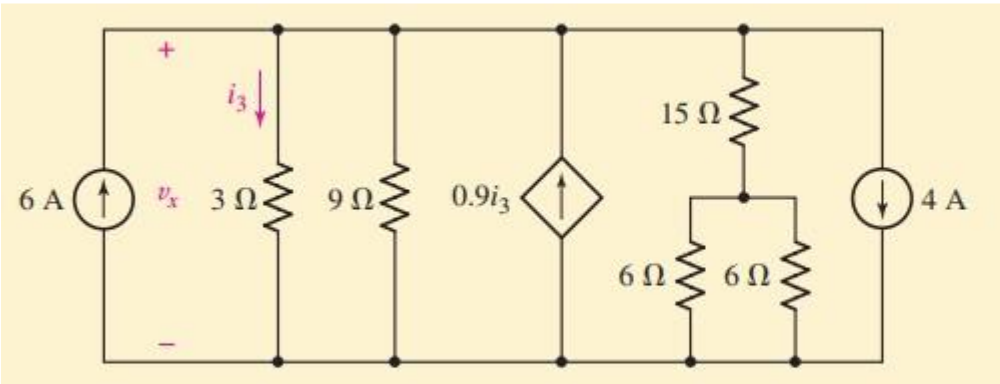
$$V_1 = +E_1 - V_2 - E_2$$

$V_1 = 16 - 4.2 - 9$

$V_1 = 2.8\ \text{V}$

EXAMPLE 3.12: page 59-60 in chapter 3 [Hayt Engineering Circuit Analysis 8th edt.txt-book ]

Calculate the power and voltage of the dependent source in Fig. 9-a.?



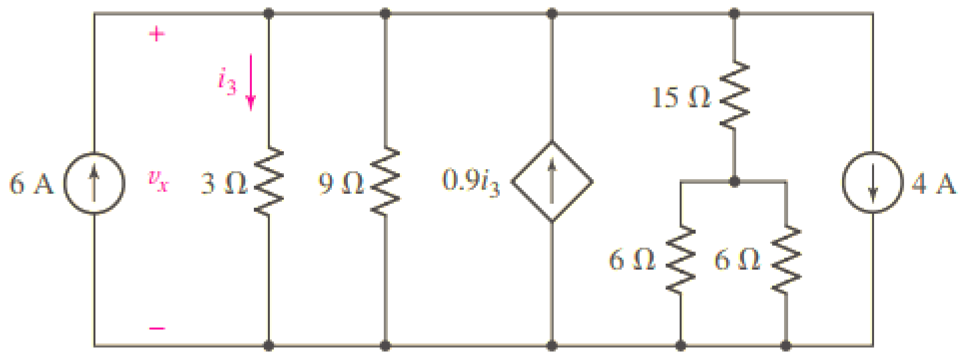




Fig.(9-a)

The two independent current sources are in fact in parallel, so we replace them with a (2A) source.

The two 6Ω resistors are in parallel and can be replaced with a single 3Ω resistor in series with the 15Ω resistor. Thus, the two 6Ω resistors and the 15Ω resistor are replaced by an 18Ω resistorFig. (9-b).

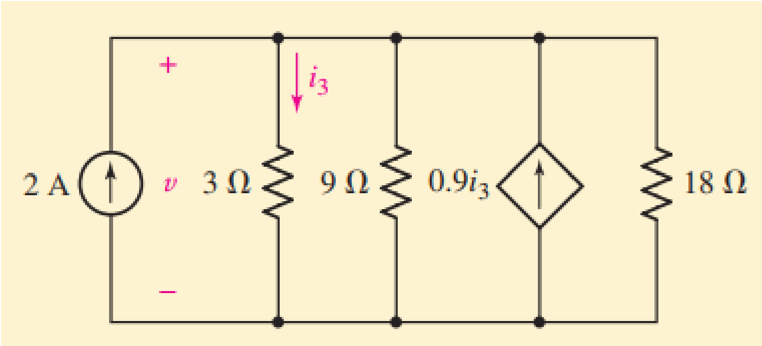



Fig.9-b

we should not integrate the remaining three resistors; the controlling variable $i_3$ depends on the 3Ω resistor, therefore, must remain this resistor. then, is (9Ω×18Ω) / (9Ω+18Ω) = 6Ω , as shown in Fig. 9*c*.

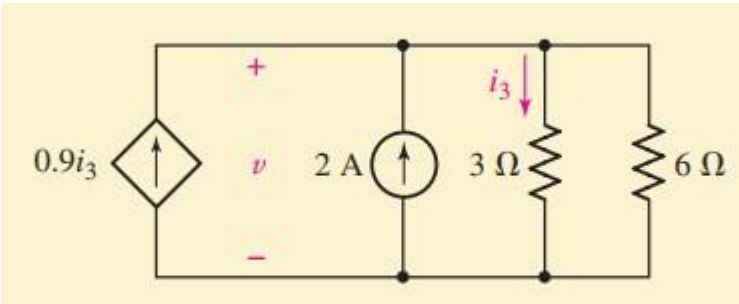



Fig.9-c

Applying KCL at the top node of Fig. 9-c, we have :

$$-0.9i_3 - 2 + i_3 + \frac{v}{6} = 0 \qquad ------------(1)$$

Employing Ohm's law in 3Ω branch;

$$v = 3i_3 \qquad ------------(2)$$

Put equation 2 in 1;

$$-0.9i_3 - 2 + i_3 + \frac{3i_3}{6} = 0$$

$$+\frac{3}{6})\,i_3 = 2$$

$$i_3 = \frac{2}{1 - 0.9 + \frac{3}{6}}$$

$i_3 = 3.3333A$

Thus, the voltage across the dependent source (which is the same asthe voltage across the 3Ω resistor) is

$v = 3i_3 = 3 \times (3.333\text{A}) = 10\text{V}$

The dependent source, provide power as,

$P = V \times I$

$p = v \times 0.9 i_3$

$p = 10 \times [(0.9)(3.333A)] = 29.9997W \cong 30$ W to the circuit