3 – First order : linear

It is found in two forms

1- $\frac{dy}{dx}$ + *P(x). y = Q (x)*

The *P(x)* and *Q(x)* are function for (*x*)

The solution is

$\rho = e^{\int P(x)dx}$

$\rho.y = \int \rho\, Q(x)dx + c$

2- $\frac{dx}{dy}$ + *P(y) . x = Q (y)*

The *P(y)* and *Q(y)* are function for (*y*)

$\rho = e^{\int P(y)dy}$

$\rho.x = \int \rho\, Q(y)dy + c$

Ex. 1.: Solve the differential equation:-

$$x\frac{dy}{dx} + 3y = x^2$$

Solution

$$\frac{x}{x}\frac{dy}{dx} + \frac{3y}{x} = \frac{x^2}{x}$$

$$\frac{dy}{dx} + \frac{3}{x}y = x$$

Let $P(x) = \frac{3}{x}$, $Q(x) = x$

$$\rho = e^{\int P(x)dx}$$

$$\rho = e^{\int \frac{3}{x}dx} = e^{3\ln x} = x^3$$

Email (tayser.sumer.gaaz@uomus.edu.iq)

ρ . y $=\int \rho\, Q(x)$dx + c

$x^3.y = \int x^3.x\, dx$ + c

$$x^3.y = \int x^4\, dx + c$$

$$x^3.y = \frac{x^5}{5} + c$$

$$y = \frac{x^5}{5x^3} + \frac{c}{x^3}$$

y $= \frac{x^2}{5} + cx^{-3}$

Ex. 2.: Solve the differential equation:-

$$\boldsymbol{x\frac{dy}{dx} - 3y = x^2}$$

Solution

$$\frac{x}{x}\frac{dy}{dx} - \frac{3y}{x} = \frac{x^2}{x}$$

$$\frac{dy}{dx} - \frac{3y}{x} = x$$

Let $P(x) = -\frac{3}{x}$, $Q(x) = x$

$$\rho = e^{\int P(x)dx}$$

$$\rho = e^{\int \frac{-3}{x}dx} = e^{-3\ln x} = x^{-3}$$

ρ . y $=\int \rho\, Q(x)$dx + c

$x^{-3}.y = \int x^{-3}.x\, dx$ + c

$$x^{-3}.y = \int x^{-2}\, dx + c$$

$$x^{-3}.y = \frac{x^{-1}}{-1} + c$$

$$\frac{x^{-3}}{x^{-3}}y = \frac{-x^{-1}}{x^{-3}} + \frac{c}{x^{-3}}$$

$$y = -x^2 + cx^3$$

H.W.: **Find a general solution of each the following equations:-**

1) $\frac{dy}{dx} + 2y = e^{-x}$

2) $2\frac{dy}{dx} - y = e^{x/2}$

3) $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

4) $(x - 2y)\,dy + y\,dx = 0$

4 - First order: Exact

The general formula of the equation $M(x,y)\,dx + N(x,y)\,dy = 0$

If $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ then the equation is ***Exact***

If the equation was Exact the solve:

$$df(x,y) = 0$$

$$\int df(x,y) = \int 0$$

$f(x,y) = c$

$$M(x,y)\,dx + N(x,y)\,dy = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = df$$

Ex. 1.: Show that differential equation:-

$(x^2 + y^2)dx + (2xy + \cos y)dy = 0$ **is exact and solve it?**

Solution

$$\frac{\partial M}{\partial y} = 2y \,, \quad \frac{\partial N}{\partial x} = 2y$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$ **Exact D.E.**

To find f(x,y) such that

$$\frac{\partial f}{\partial x} = x^2 + y^2 \,, \quad \frac{\partial f}{\partial y} = 2xy + \cos y$$

f(x,y) = $\int M(x,y)dx + K(y)$

$$f(x,y) = \int (x^2 + y^2)dx + K(y)$$

$$f(x,y) = \frac{x^3}{3} + xy^2 + K(y)$$

To find K(y)

$$\frac{\partial f}{\partial y} = 2xy + K'y = 2xy + \cos y$$

$K'(y) = \cos y$ تكامل

K(y) = siny

$$f(x,y) = \frac{x^3}{3} + xy^2 + siny$$

f(x,y) = c

$$\frac{x^3}{3} + xy^2 + siny = c$$

Ex. 2.: State the type of D.E. (y sec^{2}x + secx tanx) dx + (tanx + 2y) dy = 0

Solution

M = ysec^{2}x + secx tanx

$\frac{\partial M}{\partial y} = sec^2x$

N = tanx +2y

$\frac{\partial N}{\partial x} = sec^2x$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ **Exact D.E.**

$$\frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = df = (ysec^2x + secxtanx)dx + (tanx + 2y)dy$$

$\frac{\partial f}{\partial x} = ysec^2x + secxtanx$

df = $\int(ysec^2x + secxtanx)\,dx$

f(x,y) = y tanx + secx + K(y)

$\frac{\partial f}{\partial y} = tanx + 0 + K'(y) = tanx + 2y$

$K'(y) = 2y$

K(y) = $\int 2ydy$

K(y) = $\frac{2y^2}{2}$

f(x,y) = y tanx + secx + y^2

f(x,y) = c

y tanx + secx + y^2 = c

H.W.: Solve equation $\frac{dx}{x^2y} + \frac{dy}{xy^2} = 0$

Email (tayser.sumer.gaaz@uomus.edu.iq)