Optical fiber index profile:

Index profile is the refractive index distribution across the core and the cladding of a fiber. Some optical fiber has a step index profile, in which the core has one uniformly distributed index and the cladding has a lower uniformly distributed index. Other optical fiber has a graded index profile, in which refractive index varies gradually as a function of radial distance from the fiber center. Graded-index profiles include power-law index profiles and parabolic index profiles. The following figure shows some common types of index profiles for single mode and multimode fibers.

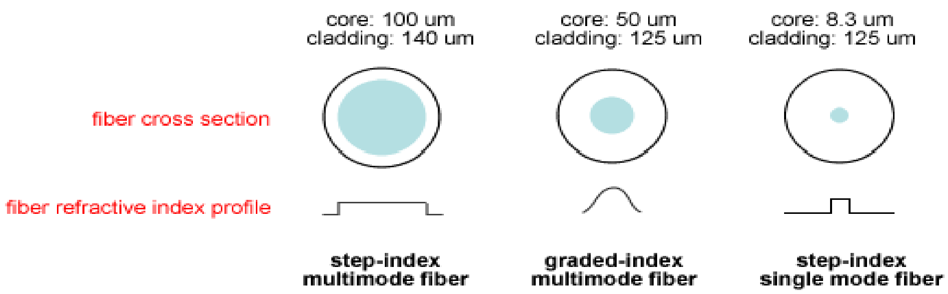

Fig.(8): Types of index profiles for single mode and multimode fibers.

Optical Fiber's Numerical Aperture (NA):

Multimode optical fiber will only propagate light that enters the fiber within a certain cone, known as the acceptance cone of the fiber. The half-angle of this cone is called the acceptance angle, ***θmax***, which depends on the material of the fiber and core diameter. For step-index multimode fiber, the acceptance angle is determined only by the indices of refraction:

$$NA = n \sin\theta_{\max} = \sqrt{n_f^{\,2} - n_c^{\,2}}$$

$$\theta_{Max} = sin^{-1} N_A$$

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Where

$\boldsymbol{n}$ is the refractive index of the medium light is traveling before entering the fiber.
n_f is the refractive index of the fiber core.
n_c is the refractive index of the cladding.

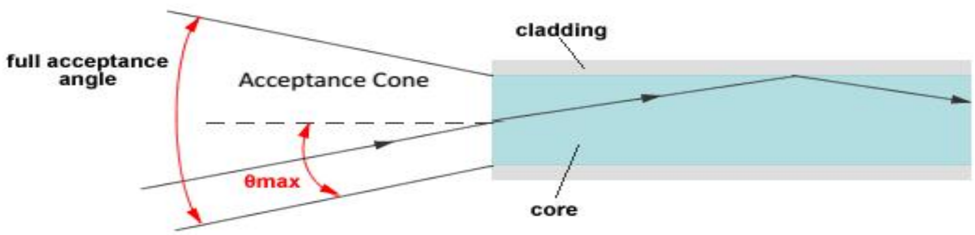

Fig.(9): The acceptance cone of optical fiber.

Consider an optical fiber having a core of refractive index n_1 and cladding of refractive index n_2. Let the incident light makes an angle i with the core axis as shown in figure (10). Then the light gets refracted at an angle θ and fall on the core-cladding interface at an angle where,

$$\theta' = (90 - \theta) \quad \text{.....................} (1)$$

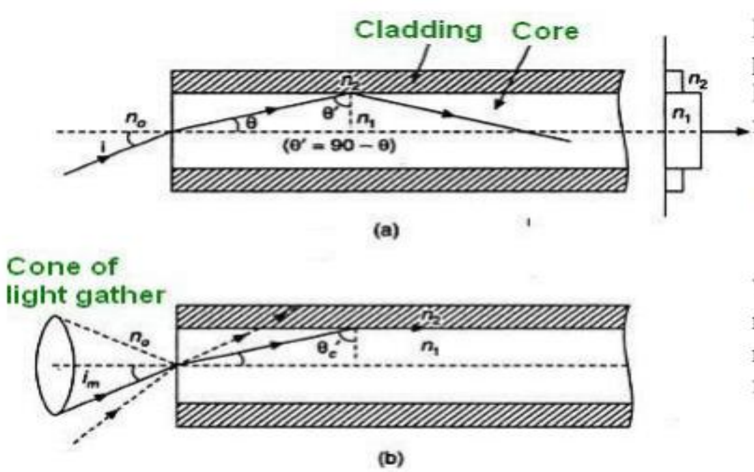

Fig.(10)

By Snell's law at the point of entrance of light in to the optical fiber we get,

$$n_0 \sin i = n_1 \sin\theta \quad \text{.......................} (2)$$

Where n_0 is refractive index of medium outside the fiber. For air $n_0 = 1$.

When light travels from core to cladding it moves from denser to rarer medium and so it may be totally reflected back to the core medium if θ' exceeds the critical angle θ'_c. The critical angle is that angle of incidence in denser medium (n1) for which angle of refraction become 90°. Using Snell's laws at core cladding interface,

$$n_1 \sin \theta'_c = n_2 \sin 90$$

or

$$Sin\ \theta'_c = \frac{n_2}{n_1} \quad \text{......................... (3)}$$

Therefore, for light to be propagated within the core of optical fiber as guided wave, the angle of incidence at core-cladding interface should be greater than θ'_c. As i increases, θ increases and so θ' decreases. Therefore, there is maximum value of angle of incidence beyond which, it does not propagate rather it is refracted in to cladding medium (fig: 3(b)). This maximum value of i say i_m is called maximum angle of acceptance and n_0 sin i_m is termed as the numerical aperture (NA).

From equation (2),

$$NA = n_0 \sin i_m = n_1 \sin \theta$$

$$= n_1 \sin(90 - \theta_c)$$

$$Or\ NA = n_1\ Cos\ \theta'_c$$

$$= n_1 \sqrt{1 - Sin^2\theta'_c}$$

From equation (2) $\sin \theta'_c = \frac{n_2}{n_1}$

Therefore, $NA = n1 \sqrt{1 - \frac{{n_2}^2}{n_1^2}}$

$$NA = \sqrt{{n_1}^2 - {n_2}^2}$$

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The significance of NA is that light entering in the cone of semi vertical angle i_m only propagate through the fiber. The higher the value of i_m or NA more is the light collected for propagation in the fiber. Numerical aperture is thus considered as a light gathering capacity of an optical fiber.

Numerical Aperture is defined as the Sine of half of the angle of fiber's light acceptance cone. i.e. NA= Sin θ_a where θ_a, is called acceptance cone angle.

Let the spot size of the beam at a distance d (distance between the fiber end and detector) as the radius of the spot(r). Then,

$$\sin\theta = \frac{r}{\sqrt{r^2 + d^2}}$$

How to Calculate Number of Modes in a Fiber?

Modes are sometimes characterized by numbers. Single mode fibers carry only the lowest-order mode, assigned the number 0. Multimode fibers also carry higher-order modes. The number of modes that can propagate in a fiber depends on the fiber's numerical aperture (or acceptance angle) as well as on its core diameter and the wavelength of the light. For a step-index multimode fiber, the number of such modes, N_m, is approximated by

$$Nm = 0.5\left(\frac{\pi D \times NA}{\lambda}\right)^2$$

Where

D is the core diameter
λ is the operating wavelength
NA is the numerical aperture (or acceptance angle)

Note: this formula is only an approximation and does not work for fibers carrying only a few modes.

The number of modes supported by a fiber is determined by an important parameter- called "cut-off" or V- number is

$$V = \frac{2\pi}{\lambda} a\, N_A$$

Where, a: is the core radius.

EX. (1): Compute the NA, acceptance angle, and the critical angle of an optical fiber from the following data.

$n_{1\,(core)} = 1.55$ and $n_{2\,(cladding)} = 1.50$.

Ans.:

$$NA = \sqrt{n_1^2 - n_2^2} = \sqrt{(1.55)^2 - (1.5)^2} = 0.3905$$

$$\theta_{Max} = \sin^{-1} N_A = 23.2^0$$

$$\theta_c = \sin^{-1}(n_2/n_1) = \sin^{-1}(1.5/1.55) = 75.4^0$$

EX. (2): Compute the cut-off parameter and the number of modes supported by a fiber $n_{1\,(core)} = 1.54$ and $n_{2\,(cladding)} = 1.5$, core radius 25 μm and operating wavelength is 1300nm.

Ans. :

$$V = \frac{2\pi}{\lambda} a\, N_A$$

$$NA = \sqrt{n_1^2 - n_2^2} = \sqrt{(1.54)^2 - (1.5)^2} = 0.349$$

$$V = 42.15$$

$$Nm = 0.5\left(\frac{\pi D \times NA}{\lambda}\right)^2 = 888.$$

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