Al-Mustaqbal University
Chemical Engineering and Petroleum Industries Department
4th Class
Subject: Operation units
Lecturer: Abbas K. Al-Gburi
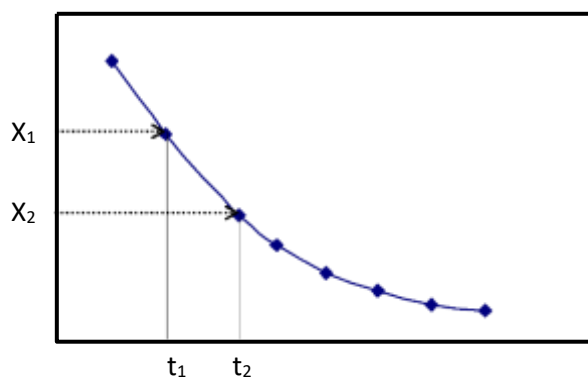1st/2nd term- Lect. 3 (Drying rate)

1.8.2 Drying rate.

1. Constant rate drying period.

• Method used drying curve.

To estimate the time of drying for a given batch of material, based on actual experimental data. The time required is determined directly from the drying curve of free moisture content vs. time.

Where t= t_2-t_1 and X_1, X_2 > Xc (critical moisture content).

• Method used Drying Rate Equation.

$$R = -\frac{L_s}{A}\frac{dX}{dt} \dots \dots \dots \dots 10$$

Rearranged (10) and integrated over the time interval to dry from X_1 at t_1=0 to X_2 at t_2=t

$$t = \int_{t_1=0}^{t_2=t} dt = -\frac{L_s}{A} \int_{X_1}^{X_2} \frac{dX}{R} \dots \dots \dots \dots 11$$

Drying takes place within the constant rate period, then R=constant=Rc.

$$t = \frac{L_s}{A}(X_1 - X_2) \dots \dots \dots \dots 12$$

In the constant rate period of drying, the surface of grains of solid in contact with drying air flow remain completely wetted. Drying of material occurs by mass transfer vapor from the saturated surface of the material through

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the air film to the bulk gas phase. The rate of moisture movement within the solid is sufficient to keep the surface saturated. The rate removal of water vapor (drying) is controlled by the rate of heat transfer to the evaporating surface which furnishes the latent heat of evaporation for the liquid. At steady state the rate of mass transfer balances the rate of heat transfer.

Assuming only heat transfer to the solid surface by convection from the hot gas to the surface of the solid and mass transfer from the surface to the hot gas.

⦿ No heat transfer by conduction from metal pans or surface, neglect heat transfer by radiation.
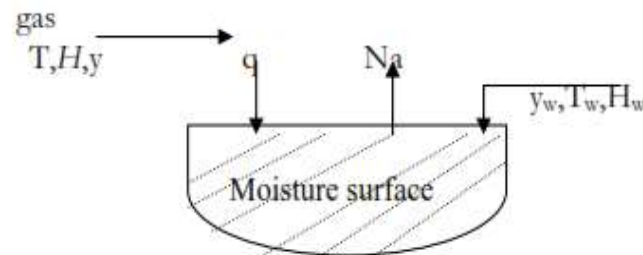
The rate of heat transfer by convection (q) from the gas at T to the surface of the solid at Tw is:

$$q = hA(t - t_w) \ldots \ldots \ldots \ldots 13$$

The flux of water vapor from the surface is:

$$Na = k_y(y_w - y) \ldots \ldots \ldots \ldots 14a$$

$$Na = k_y \frac{M_B}{M_A}(\mathcal{H}_w - \mathcal{H}) \ldots \ldots \ldots \ldots 14b$$

$$y = \frac{\frac{\mathcal{H}}{M_A}}{\frac{1}{M_B} + \frac{\mathcal{H}}{M_A}}$$, the term $(\frac{\mathcal{H}}{M_A})$ is neglected due H very mall … so:

$$y = \frac{M_B \mathcal{H}}{M_A} \ , \ y_w = \frac{M_B H_w}{M_A}$$

The amount of heat needed $\quad q = m.\lambda_w$

$$= Na.M_A.A.\lambda_w \ \ldots \ldots \ldots \ldots 15$$

Where λw latent heat of vaporization at T_w. Equating equation (13) and (15) and sub into (14b):

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$$R_C = \frac{q}{A\lambda_w} = \frac{h(T - T_w)}{\lambda_w} = k_y M_B (\mathcal{H}_w - \mathcal{H}) \dots \dots \dots \dots 16$$

Therefore equation 12 becomes:

$$t = \frac{L_s \lambda_w}{A\,h\,(T - T_w)}(x_1 - x_2) \dots \dots \dots \dots 17$$

$$t = \frac{L_s \lambda_w}{A\,k_y M_B (\mathcal{H}_w - \mathcal{H})}(x_1 - x_2) \dots \dots \dots \dots 18$$

To predict R_c, the heat transfer coefficient h must be known.

- If air flowing parallel to the drying surface:

$$h = 0.0204 G^{0.8} \qquad SI\ unit$$

$$h = 0.0128 G^{0.8} \qquad English\ unit$$

at T_{air} 45-150 ˚C, G=2450-2950 kg/m^2.h, U_g= 0.61-7.6 m/s

- if air flowing perpendicular to the drying surface:

$$h = 0.0204 G^{0.8} \qquad SI\ unit$$

$$h = 0.0128 G^{0.8} \qquad English\ unit$$

G=3900-19500 kg/m^2.h, U$_g$=0.9-4.6 m/s

Where gas mass velocity $G=\rho_{air}.v_{air}$

On the basis of the above equations, effects of some important parameters on the rate of drying can be determined as:

Effect of gas velocity: If conduction through the solid and radiation are neglected, the drying rate becomes proportional to G$^{0.71}$ for parallel flow and to G$^{0.37}$ for perpendicular flow.

If conduction and radiation are present, effect of air rate will be less important.

Effect of gas temperature: Increased air temperature increases the quantity $(T - T_w)$ increases drying rate. Neglecting radiation and, R$_C$ becomes directly proportional to $(T - T_w)$.

Effect of gas humidity: At moderate temperatures the rate of drying varies directly as $y_w - y$ hence increasing gas humidity reduces drying rate.

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Effect of thickness of drying solid: If conduction through solid occurs, R_c decreases as solid thickness increases, but rate of drying may sometimes increase due to higher conduction of heat through the edges.

2. Falling rate drying period.

• Graphical integration method.

In falling rate period, the rate of drying R is not constant but decreases when drying proceeds past the critical free moisture content X_c. The time for drying for any region between X_1 and X_2 given by:

$$t = -\frac{L_s}{A} \int_{X_1}^{X_2} \frac{dX}{R} \dots \dots \dots \dots 11$$

In falling rate period, R varies for any shape of falling rate drying curve, equation (8) can be integrated graphically by plotting $1/R$ vs. X and determine the area under the curve.

• Calculation Method for special cases in falling rate period.

1. Rate is a linear function of X.

Both X_1 and $X_2 < X_c$ and the rate R is linear in X over this region.

$$R = aX + b \dots \dots \dots \dots 19$$

Where a is a slope of the line and b is a constant. Differentiating equation (19) gives:

$$dR = a\,dX \quad \dots \dots \dots \text{ sub. into equation (11)}$$

$$t_f = -\frac{L_s}{aA} \int_{R_1}^{R_2} \frac{dR}{R}$$

$$t_f = \frac{L_s}{aA} Ln\frac{R_1}{R_2} \dots \dots \dots \dots 20$$

Since $R_1 = a\,X_1 + b$ and $R_2 = a\,X_2 + b$

$$a = \frac{R_1 - R_2}{x_1 - x_2} \dots \dots \dots sub.\,in\,equation\,(20)$$

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$$t_f = \frac{L_s(x_1 - x_2)}{A\,(R_1 - R_2)}\,Ln\frac{R_1}{R_2}\,\ldots\ldots\ldots\ldots 21$$

2. Rate is a linear function through origin.

In some cases, a straight line from the critical moisture content passing through the origin adequately represents the whole falling rate period. Then the rate of drying is directly proportional to the free moisture content.

$$R = aX \ldots\ldots\ldots\ldots 22$$

Differentiating equation (22), $dR = adX$

$$dx = \frac{dR}{a}\,\ldots\ldots\ldots\ldots sub.\,in\,equation\,(11)$$

$$t_f = -\frac{L_s}{A}\int_{R_1}^{R_2}\frac{dR}{R} = \frac{L_s}{aA}\,Ln\frac{R_1}{R_2}\,\ldots\ldots\ldots\ldots 23$$

The slope a of the line is R_c/X_c and for $X_1=X_c$ at $R_1=R_c$

$$t_f = \frac{L_s X_c}{A\,R_c}\,ln\left(\frac{R_1}{R_2}\right)\ldots\ldots\ldots\ldots 24$$

Noting also that $R_c / R_2 = X_c / X_2$, therefore equation (24) become:

$$t_f = \frac{L_s X_c}{A\,R_c}\,ln\left(\frac{X_1}{X_2}\right)\ldots\ldots\ldots\ldots 25$$

3. Rate of drying in falling rate period can be expressed as a parabolic function.

$$R = ax + bx^2 \ldots\ldots\ldots\ldots 26$$

Sub equation 26 in equation 11, then

$$t_f = -\frac{L_s}{A}\int_{X_1}^{X_2}\frac{dX}{ax + bx^2}$$

The time for falling rate period:

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$$t_f = \frac{L_s}{aA} \ ln \left[\frac{x_c(a + bx_f)}{x_f(a + bx_c)} \right] \dots\dots\dots\dots 27a$$

$$t_f = \frac{L_s}{aA} \ ln \left[\frac{x_c^2 R_f}{x_f^2 R_c} \right] \dots\dots\dots\dots 27b$$

1.9 Mechanism of moisture movement in wet solid.

When drying takes place, moisture moves from the inner core to the external surface and evaporates. The nature of movement influences the drying during the falling rate period and the following theories have been to explain the moisture movement in solids.

Liquid diffusion: Moisture moves in liquid state through the solid on account of concentration gradient between the interior of the solid where the concentration is high and the surface where the same is low. In these cases, diffusion controlled falling rate period may start immediately after constant rate period and unsaturated surface drying may not be present. Liquid diffusion is encountered in soap, glue, gelatin, etc. as well as during last phase of drying of clay, textile, paper, wood, etc.

Capillary movement: Unbound moisture in granular or porous solids moves through the capillaries or interstices of the solid by the action of surface tension. Initially moisture moves fast enough to keep the surface wet and constant rate drying continues. As drying proceeds, a stage is reached when the surface becomes dry at some spots and falling rate period starts with unsaturated surface drying. The sub-surface water soon dries up and the surface of evaporation gradually recedes into the solid. Evaporation then takes place well within the solid and the vapour thus produced diffuses to the surface.

Vapour diffusion: If heat is supplied to one surface of the solid and drying takes place from the other, moisture may be evaporated within the solid and the vapour diffuses to the surface.

Pressure action: The solid may shrink on drying and squeeze the moisture to the surface where drying takes place.

◉ If internal diffusion controls drying during the entire falling rate period, the rate is independent of air velocity and the effect of humidity is limited to deciding the equilibrium moisture content.

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1.9.1 Drying time in falling period according liquid diffusion theory.

When liquid diffusion of moisture controls the rate of drying in the falling rate period. The equation for diffusion used Fick's second law for unsteady state diffusion using the concentrations as *X kg free moisture/kg* dry solid instead of concentrations *kg mole moisture/m³*,

$$\frac{d_X}{d_t} = D_L \frac{d^2 X}{dX^2} \dots\dots\dots\dots 28$$

Where D_L is the liquid diffusion coefficient m²/h .

X is the distance in the solid *m*.

This type of diffusion is often characteristic of relating slow drying in no granular materials such as soap, gelatin and others. During diffusion type drying, the resistance to mass transfer of water vapor from the surface is usually very small and the diffusion in the solid controls the rate of drying. Then the moisture content at the surface is at the equilibrium value X^*. This means that the free moisture content X at the surface is zero.

Assume that initial moisture distribution is uniform at t=0, equation (28) 8

$$\frac{X_1 - X^*}{X_{t1} - X^*} = \frac{8}{\pi^2}\left[e^{-D_L t\left[\frac{\pi}{2\eta}\right]^2} + \frac{1}{9}e^{-9D_L t\left[\frac{\pi}{2\eta}\right]^2} - \frac{1}{25}e^{-25D_L t\left[\frac{\pi}{2\eta}\right]^2} + \cdots\right] \dots\dots 29a\dots$$

Where X is the average free moisture content at time t.

X_1 is the initial free moisture content at time =0.

X^* is the equilibrium free moisture content.

$X_1 = 0.5$ is the thickness of the slab when drying occurs from the top and the bottom parallel faces, and X_1

= total thickness of slab if drying only from the top face.

From equation (29) assume that D_L is constant, but D_L is rarely constant, it varies with moisture content, temperature, and humidity. For long drying times, only the first term in the equation (29) is significant:

$$\frac{X}{X_t} = \frac{8}{\pi^2}e^{-D_L t\left[\frac{\pi}{2X_1}\right]^2} \dots\dots\dots\dots 29b$$

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$$t = \frac{4X_1^2}{\pi^2 D_L} Ln \frac{8X_1}{\pi^2 X} \dots \dots \dots \dots 30$$

In this equation if the diffusion mechanism starts at $X=X_c$. Differentiating equation (30) with respect to time an rearranging,

$$\frac{d_X}{d_t} = -\frac{\pi^2 D_l X}{4X_1^2}$$

Multiplying both sides by L_s/A:

$$R = -\frac{L_s}{A}\frac{d_X}{d_t} = -\frac{\pi^2 L_s D_L X}{4X_1^2 A} \dots \dots \dots \dots 31$$

Equation (30 and 31) state that when internal diffusion controls for long times, the rate of drying is directly proportional to the free moisture X and the liquid diffusivity and that the rate of drying is inversely proportional to the thickness squared.

1.9.2 Drying time in falling period according <u>capillary theory</u>.

Water can flow from region of high concentrations to those of low concentrations as a result of capillary action rather than by diffusion if the pore size of granular materials are suitable. The capillary theory assumes that a packed bed of nonporous spheres contains a void space between the spheres called pores. As water is evaporated, capillary forces are set up by the interfacial tension between the water and solid. These forces provide the driving force for moving the water through the pores to the drying surface. If the moisture movement follows the capillary theory rate of drying R will vary linearly with X:

$$R = -\frac{L_s}{A}\frac{dX}{dt} \dots \dots \dots \dots 10$$

For the rate R varying linearly with X; then $R= R_c (X / X_c)$; therefore:

$$t_f = \frac{L_s X_c}{A R_c} ln\left[\frac{X_1}{X_2}\right] \dots \dots \dots \dots 25$$

Where $L_s= X_1. A. \rho_s$ kg dry solid.

ρ_s: is the solid density kg/m³.

X_1: is the thickness m.

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A: is the exposed surface area m^2.

Equation (25) becomes: $t_f = \dfrac{X_1 \rho_s X_c}{R_c} \, ln \left[\dfrac{X_1}{X_2}\right] \dots \dots \dots \dots 26$

Where R_c can be calculated from equation (16):

$$R_C = \frac{q}{A\lambda_w} = \frac{h(T - T_w)}{\lambda_w} = k_y M_B (\mathcal{H}_w - \mathcal{H}) \dots \dots \dots \dots 16$$

The time of drying is directly proportional to the thickness, while the rate of drying inversely proportional to the thickness when capillary controls the falling rate period.