Al-Mustaqbal University
College of Engineering and Technologies
Class (2)
Subject (Electrical circuits)
Lecturer (Assist . lec. Ali Imad)
1st term – Lect. (nodal analysis)

Lecture 4:

Nodal Analysis

The nodal analysis is a general procedure for analyzing circuits using node voltages as the circuit variables. This methodology reduces the number of equations that must solve it together.

Steps to Determine Node Voltages :

1. Select a node as the reference node (it's voltage equal zero). Assign voltages V_1, V_2, …, V_{n-1} to the remaining n-1 nodes. The voltages are referenced with respect to the reference node.
2. Apply KCL to each of the n-1 non reference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
3. Solve the resulting equations together to obtain the unknown node voltages.

Example 1: Using nodal analysis, determine the current in (3Ω) of the circuit shown in Figure (1).

Solution:

At node (1) :

$$\frac{V_1-(2+4)}{3+2}+\frac{V_1}{2}+\frac{V_1-4}{2}=0$$

$$\frac{V_1}{5}-\frac{6}{5}+\frac{V_1}{2}+\frac{V_1}{2}-\frac{4}{2}=0$$

$$\left\{\frac{V_1}{5}-\frac{6}{5}+2\left(\frac{V_1}{2}\right)-2=0\right\}\times 5$$

$$V_1-6+5V_1-10=0$$

$$6V_1-16=0 \rightarrow V_1=\frac{16}{6}$$

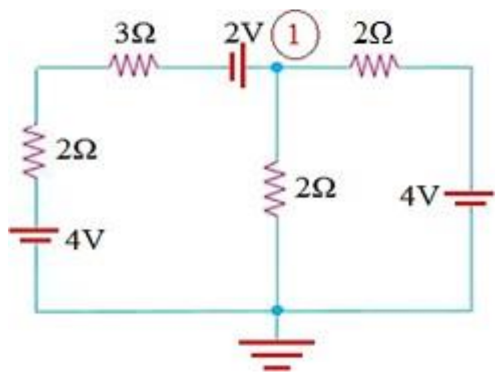


Fig.(1)

∴ V1 = 2.667 V

So the current through (3Ω)

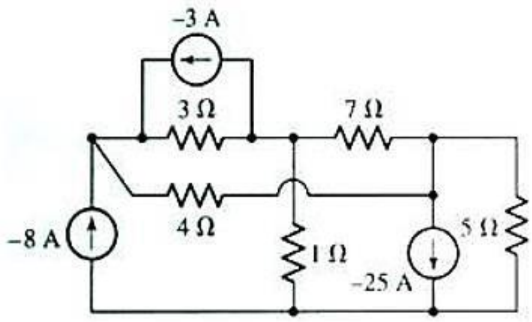
$$I_{3\Omega} = \frac{V_1 - 2 - 4}{3 + 2} = \frac{2.67 - 2 - 4}{5}$$

$$I_{3\Omega} = -0.666A$$

Example 2: Determine the current in (7Ω) & (1Ω) resistors using nodal analysis of the circuit shown in Figure (2).

Solution:

Redraw the circuit and indicate the nodes and reference node as shown in Figure (3).



Fig(2)

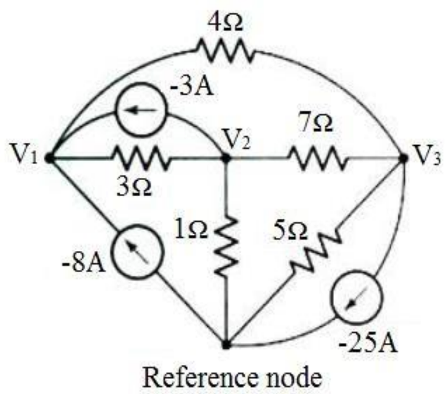


Fig.(3)

At node (1) :

$$\left(\frac{V_1 - V_2}{3}\right) + \left(\frac{V_1 - V_3}{4}\right) = -8 - 3$$

$$\left(\frac{V_1 - V_2}{3}\right) + \left(\frac{V_1 - V_3}{4}\right) + 8 + 3 = 0$$

$0.583V_1 - 0.333V_2 - 0.25V_3 = -11$ …(1)

At node (2) :

$$\frac{V_2 - V_1}{3} + \frac{V_2}{1} + \frac{V_2 - V_3}{7} - 3 = 0$$

$-0.333V_1 + 1.4762V_2 - 0.1429V_3 = 3$ …(2)

At node (3) :

$$\frac{V_3}{5} + \frac{V_3 - V_2}{7} + \frac{V_3 - V_1}{4} - 25 = 0$$

$-0.25V_1 - 0.1429V_2 + 0.5929V_3 = 25$ …(3)

Solving Equ (1), (2) & (3) we get,

V_1 = 5.41 V,

V_2 = 7.736 V,

V_3 = 46.3109 V

Now, Current in (7Ω) is,

$$I_{7\Omega} = \frac{V_3 - V_2}{7} = \frac{46.32 - 7.736}{7} = 5.512A$$

Current in (1Ω) is,

$$I_{1\Omega} = \frac{V_2 - V_0}{1} = \frac{V_2 - 0}{1}$$

$$I_{1\Omega} = \frac{7.736}{1} = 7.736A$$

Example 3: Determine the power supplied by the dependent source of the Figure (3) using nodal analysis.

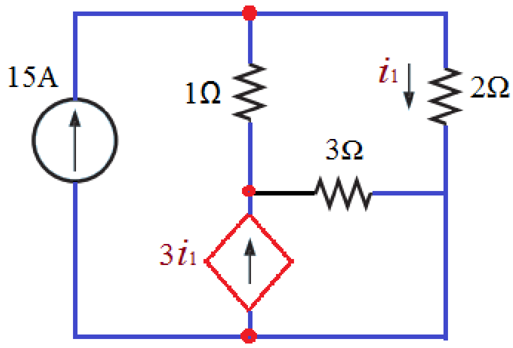


Fig.(3)

Solution:

Redraw the circuit and indicate the nodes and reference node as shown in Figure (4).

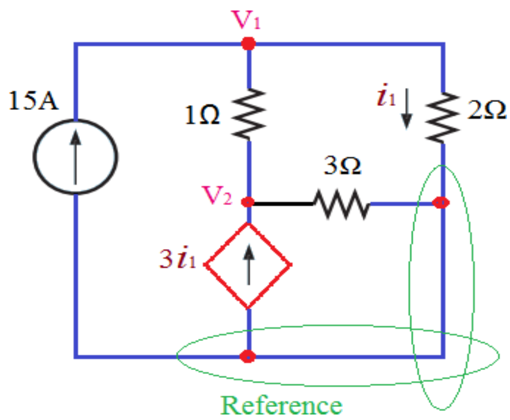


Fig.(4)

At node (1) :

$$\frac{V_1 - V_2}{1} + \frac{V_1}{2} = 15$$

$$\frac{V_1 - V_2}{1} + \frac{V_1}{2} - 15 = 0$$

∴ $1.5\ V_1 - V_2 = 15$ …(1)

At node (2) :

$$\left\{ \frac{V_2 - V_1}{1} + \frac{V_2}{3} = 3i_1 \right\} \times 3$$

$3V_2 - 3V_1 + V_2 - 9i_1 = 0$

$-3V_1 + 4V_2 - 9i_1 = 0$

Since, $i_1 = \frac{V_1}{2}$

$-3V_1 + 4V_2 - 9\left(\frac{V_1}{2}\right) = 0$

$\therefore \quad -7.5V_1 + 4V_2 = 0 \quad \ldots(2)$

Solve Equ.s (1) & (2) we get,

$V_1 = -40\ V$ & $V_2 = -75\ V$

$$i_1 = \frac{V_1}{2} = \frac{-40}{2} = -20A$$

Now the power supplied by the dependent source is,

$P = I \times V$

➔ $P = (3i1)\, v2 = (3\times(-20)) \times (-75)$

$P = 4500\ W = 4.5\ kW$

H.W.:- Find V_1 , V_2 and V_3 in the circuit of Figure (5) using nodal analysis.

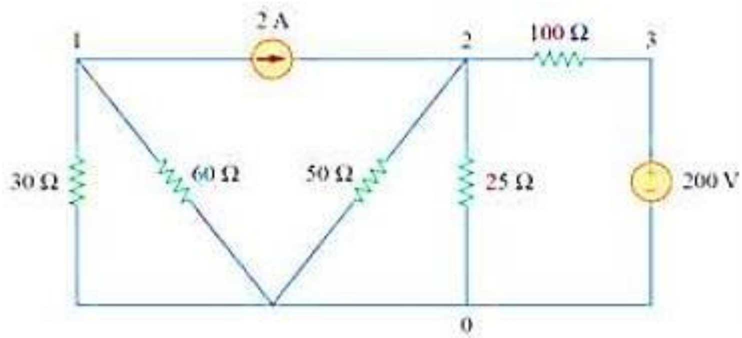


Fig.(5)