جامعة المستقبل
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كلية العلوم
قسم الانظمة الطبية الذكية

REAL AND COMPLEX NUMBERS

Mathematics

المرحلة الاولى

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REAL AND COMPLEX NUMBERS

Real number representation

Real number represent graphically in one dimension (either horizontal or vertical) as shown.

Due to *Quadratic* Algebric Equation ;

$$ax^2 + bx + C = 0$$

The solution will be x_1 and x_2. The square root of $(\sqrt{b^2 - 4ac})$ may be (posative ,negative or zero).

The *negative* value will be expressed as ***(complex number)***

Complex Numbers represent by:

1. ***Rectangular*** **coordinate representation**

$$z = x + iy \implies \quad as\ a\ point\ with\ \ (x\ \ , y)$$

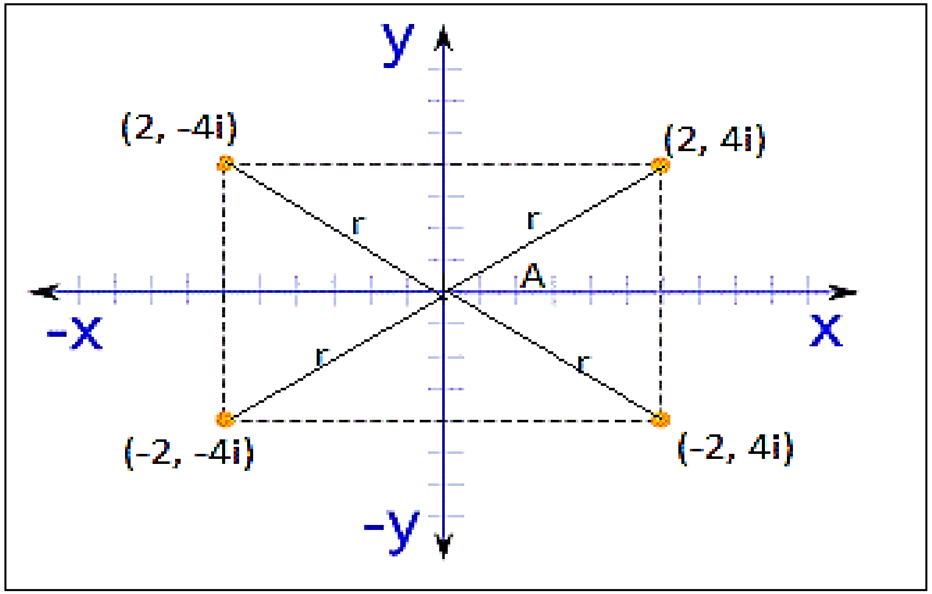
x and y are real numbers.

x are the real part of $z \implies x = Re(z)$

y are the imaginary part of $z \implies y = Im(z)$

$$z = Re(z) + Im(z).\boldsymbol{i}$$

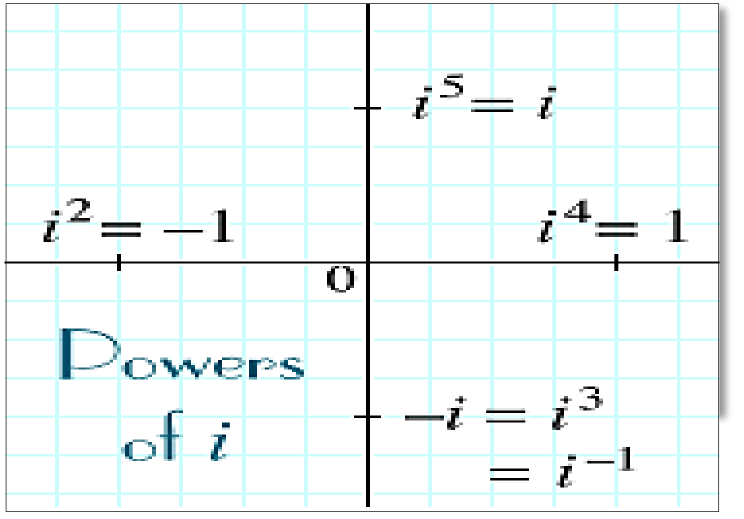
Argand Plane **or** ***Complex Plane***



Complex unit

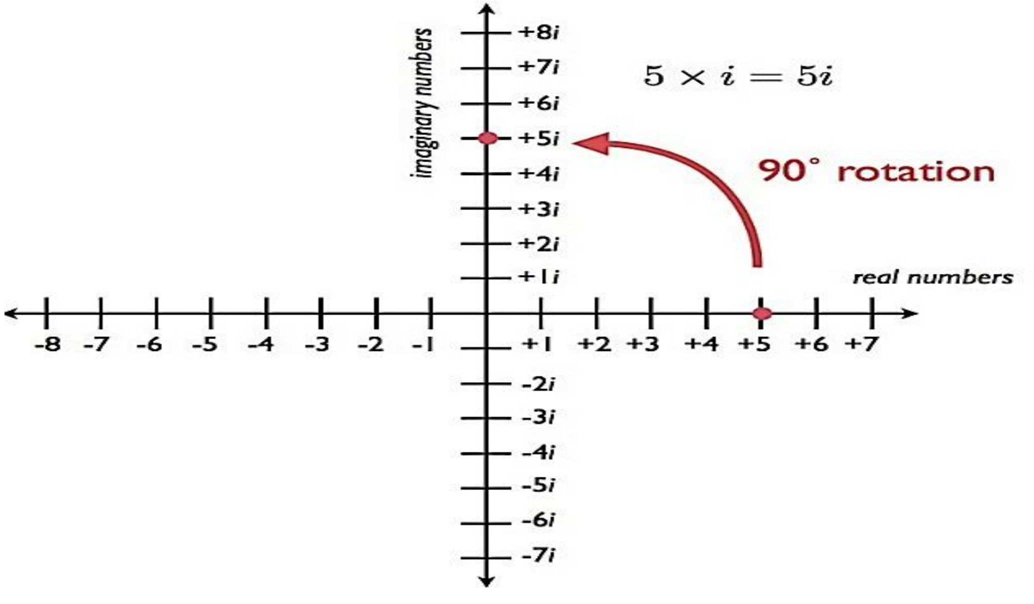
Now,

$$i = \sqrt{-1}$$
$$i^2 = -1$$



$$i^5 = i^2 . i^2 . i = (i^2)^2 . i = (-1)^2 . i = +i$$

$$i^{101} = (i^2)^{50} . i = (-1)^{50} . i = +i$$



2. Polar coordinate representation

$z = re^{i\theta}$ specified by (r, θ)

$z = r[cos\theta + isin\theta]$ specified by (r, θ) *known as Euler's formula*

$z = r \angle \theta$

$z = r \angle \theta$

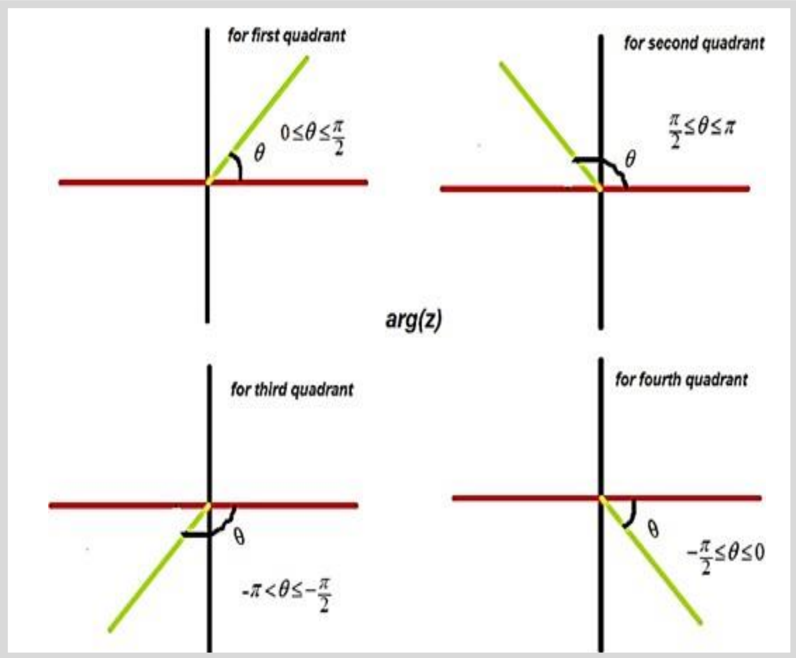
- **Length** of z
- **Amplitude** z
- **Modulus** z
- **Absolute value**

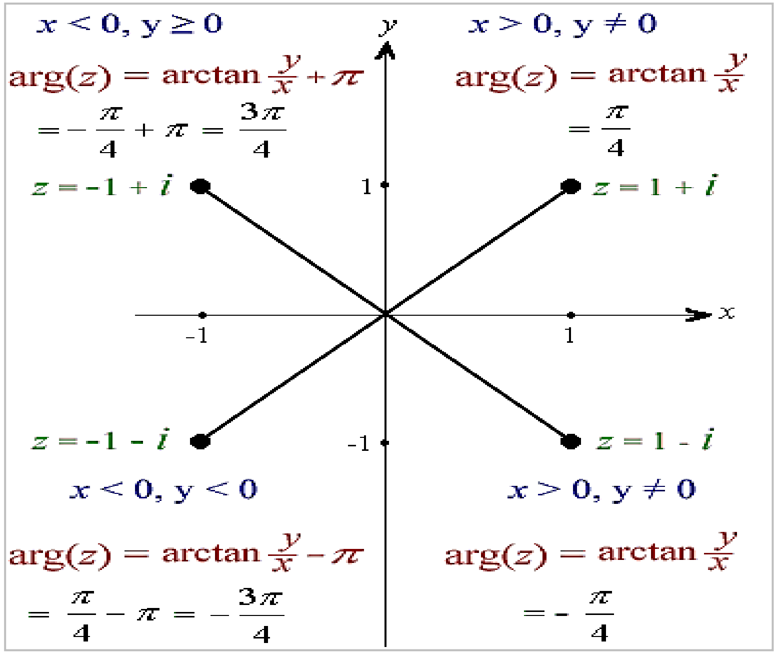
$|z| = \sqrt{x^2 + y^2} = r$

r is the " ***radius of circle*** " **centered** at orign.

θ is called '***angle***" or , "***argument***' or '***phase***" represent the direction of Z and can be evaluated by :

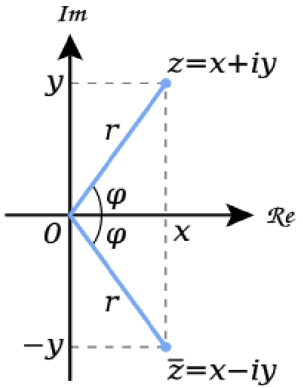
$$\theta = tan^{-1}(\frac{y}{x})$$

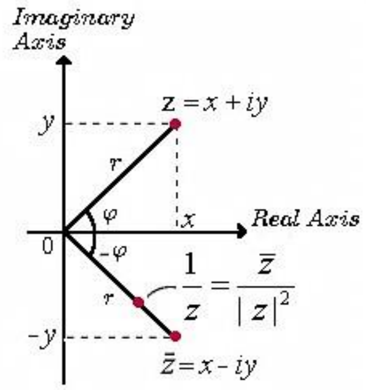


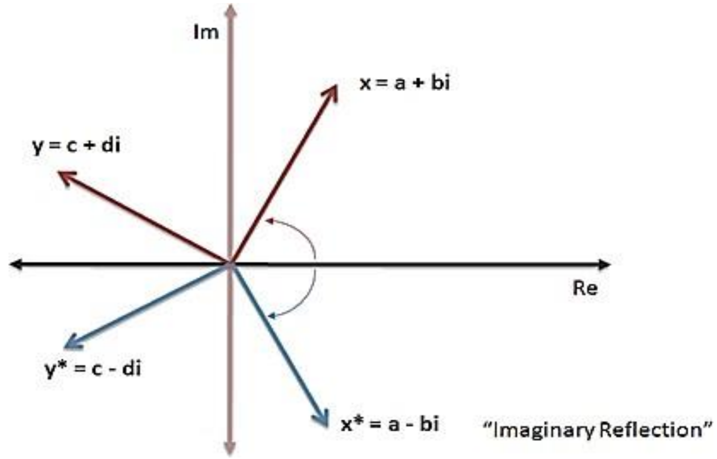


Complex conjugate of z Represented by $\bar{z}$ or z^*

$$\bar{z} = x - iy \implies \quad as\ a\ point\ with\ (x\ , -y)$$







Inflection of z

$$-z = -x - iy \implies \quad as\ a\ point\ with\ (-x\ ,-y)$$

Absolute value of z

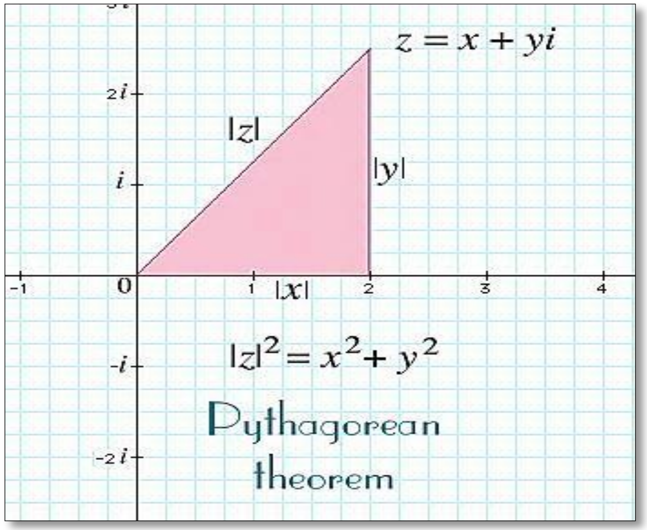
Also called , • **Length** of z

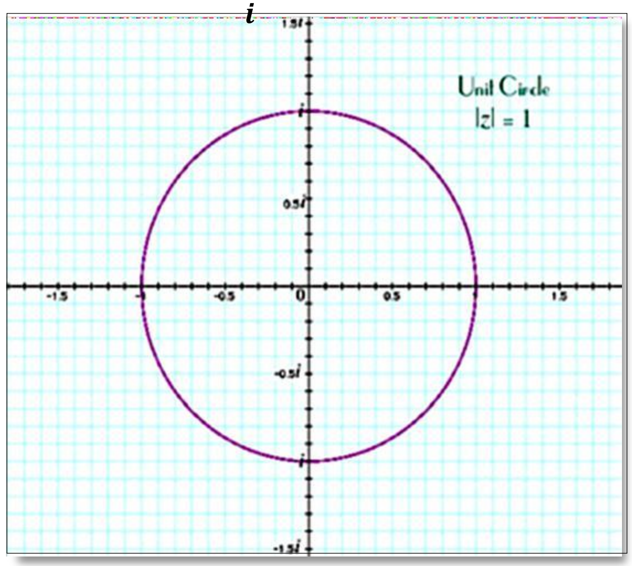
- **Amplitude** z
- **Modulus** z

r is the " ***radius of circle*** "

$$|z| = \sqrt{x^2 + y^2} = r$$

centered at $(x\ , y)$.





- **Distance between** z_1 *and* z_2

Represented by $|z_1 - z_2|$ and given by :

$$|z_1 - z_2| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

- **Power of** z

Or ,

$$z^n = r^n e^{(in\theta)}$$

$$(cos\theta + isin\theta)^n = \cos(n\theta) + isin(n\theta)$$

"*De Moivers theorem*"

PASCAL TRIANGLE

$$(a+b)^0 = 1$$

$$(a+b)^1 = 1a + 1b$$

$$(a+b)^2 = 1a^2 + 2ab + 1b^2$$

$$(a+b)^3 = 1a^3 + 3a^2b + 3ab^2 + 1b^3$$

$$(a+b)^4 = 1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$$

Roots of z

$$z^{1/n} = r^{1/n}\left[\cos\left(\frac{\theta+2k\pi}{n}\right) + i\sin\left(\frac{\theta+2k\pi}{n}\right)\right], \qquad k=0,1,2,3,\ldots,(n-1)$$

$z_0, z_1, z_2 \ldots$ *the roots of* z

COMPLEX NUMBERS

Examples

1. Verify

a) $(\sqrt{2} - i) - i(1 - \sqrt{2}i) = -2i$

b) $\left(\frac{1+2i}{3-4i}\right) + \left(\frac{2-i}{i}\right) = \frac{-2}{5}$

$$\Longrightarrow \frac{1+2i}{3-4i} * \frac{3+4i}{3+4i} + \frac{2-i}{5i} * \frac{-5i}{-5i}$$

$$= \boxed{\frac{3+4i+6i-8}{25} = \frac{-1}{5} - \frac{2i}{5}}$$

c) $\frac{5}{(1-i)(2-i)(3-i)} = \frac{1}{2}\ i$

$$\frac{5}{(1-i)(2-i)(3-i)} = \frac{5}{(1-3i)(3-i)} = \frac{5}{3-i-9i-3} = \frac{5}{-10i} = \frac{-i}{2i^2}$$

$$= +\frac{i}{2} \quad \text{since } i^2 = -1$$

2. Simplify $\left(\frac{1}{2-3i}\right)\left(\frac{1}{1+i}\right)$

Solution:

$$\left(\frac{1}{2-3i}\right)\left(\frac{1}{1+i}\right) = \frac{1}{2+3-3i+2i} = \frac{1}{5-i} * \frac{5+i}{5+i} = \frac{5+i}{25-1} = \frac{5+i}{24}$$

$$= \frac{5}{24} + \frac{1}{24}i$$

Question : Express $X, Y, and\ |z|^2$ in terms of $Re(z)$ and $Im(z)$

Solution:

$$z = x + iy \quad \dots(1)$$

$$\underline{\bar{z} = x - iy} \quad \dots(2)$$

Add (1) and (2) ,

$$z + \bar{z} = 2x \Longrightarrow \boxed{X \equiv Re(z) = \frac{z+\bar{z}}{2}}$$

Now, subtract (1) and (2),

$$z = x + iy$$

$$\bar{z} = x - iy$$

$$z - \bar{z} = 2yi \Longrightarrow \boxed{Y \equiv Im(z) = \frac{z - \bar{z}}{2i}}$$

$$|z|^2 = zz^* = \boxed{x^2 + y^2} = \left(Re(z)\right)^2 + \left(Im(z)\right)^2$$

Basic of algebraic properties of z, verify a few algebraic properties of z.

1. The commutative laws.

$$z_1 \pm z_2 = z_2 \pm z_1, \qquad z_1 z_2 = z_2 z_1$$

2. The associative laws

$$(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3), \quad (z_1 z_2) z_3 = z_1 (z_2 z_3)$$

Now,

$$\frac{1}{z_1 z_2} = \left(\frac{1}{z_1}\right)\left(\frac{1}{z_2}\right), \quad (z_1 \neq 0, z_2 \neq 0, z_1 z_2 \neq 0)$$

$$\frac{z_1 + z_2}{z_3} = \frac{z_1}{z_3} + \frac{z_2}{z_3}, \quad \frac{z_1}{z\ z_4} = \left(\frac{z_1}{z_3}\right)\left(\frac{z_2}{z_4}\right), \quad (z_3 \neq 0, z_4 \neq 0, z_3, z_4 \neq 0)$$

Absolute value of Z

With that interpretation in mind, we introduce the length, amplitude, absolute value or modulus of the complex number its the length when thinking of it as a vector:

If $z = x + iy$ then $|z| = \sqrt{X^2 + y^2}$

$|z|$ is the distance between the point (x, y) and the origin.

Example:

Compute the absolute value for each of the complex numbers:

$z_1 = 1 + i, z_2 = 2 - 3i$, find $|z_1 - z_2|$

Solution:

$$|z_1 - z_2| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(1 - 2)^2 + (1 + 3)^2} = \sqrt{1^2 + 4^2} = \sqrt{17}$$