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FUNCTION AND THEIR GRAPHS

Mathematics

المرحلة الاولى

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المحاضرة الثالثة

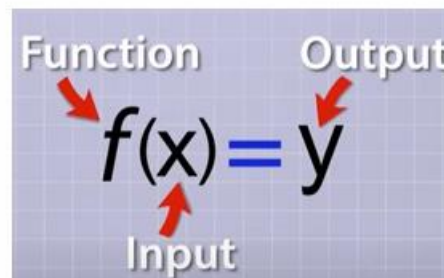
Function and their graphs

What is Function?

- Functions are a tool for describing the real world in mathematical terms.
- A function can be represented by an equation, a graph, a numerical table, or a verbal description.

Definition

A function f consists of a set of inputs, a set of outputs, and a rule for assigning each input to exactly one output. The set of inputs is called **Domain** of the function. The set of output is called the **Range** of the function.



Example

For the function $f(x) = 3x^2 + 2x - 1$, evaluate

- a. $f(-2)$
- b. $f(\sqrt{2})$
- c. $f(a + h)$

Solution

Substitute the given value for x in the formula for $f(x)$.

- a. $f(-2) = 3(-2)^2 + 2(-2) - 1 = 12 - 4 - 1 = 7$
- b. $f(\sqrt{2}) = 3(\sqrt{2})^2 + 2\sqrt{2} - 1 = 6 + 2\sqrt{2} - 1 = 5 + 2\sqrt{2}$
- c.
$$\begin{aligned} f(a + h) &= 3(a + h)^2 + 2(a + h) - 1 = 3(a^2 + 2ah + h^2) + 2a + 2h - 1 \\ &= 3a^2 + 6ah + 3h^2 + 2a + 2h - 1 \end{aligned}$$

Example: If $f(x) = 2x^2 - 5x + 1$ and $h \neq 0$, evaluate $\frac{f(a + h) - f(a)}{h}$

Solution: We first evaluate $f(a + h)$ by replacing x by $a + h$ in the expression for $f(x)$:

$$\begin{aligned} f(a + h) &= 2(a + h)^2 - 5(a + h) + 1 \\ &= 2(a^2 + 2ah + h^2) - 5(a + h) + 1 \\ &= 2a^2 + 4ah + 2h^2 - 5a - 5h + 1 \end{aligned}$$

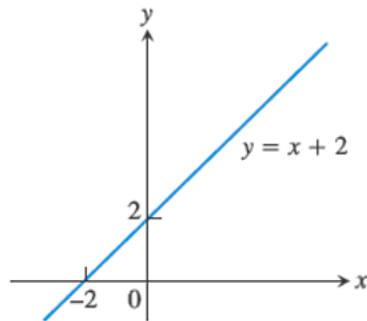
Then we substitute into the given expression and simplify:

$$\begin{aligned} \frac{f(a + h) - f(a)}{h} &= \frac{(2a^2 + 4ah + 2h^2 - 5a - 5h + 1) - (2a^2 - 5a + 1)}{h} \\ &= \frac{2a^2 + 4ah + 2h^2 - 5a - 5h + 1 - 2a^2 + 5a - 1}{h} \\ &= \frac{4ah + 2h^2 - 5h}{h} = 4a + 2h - 5 \end{aligned}$$

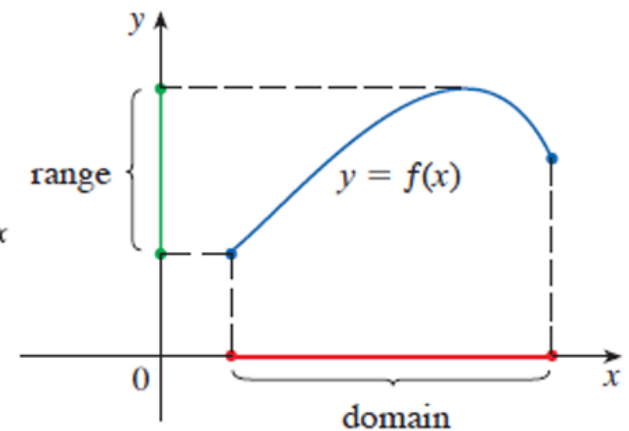
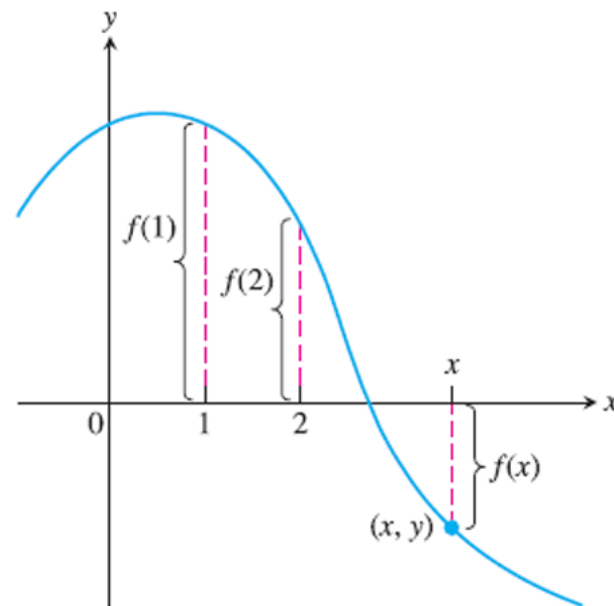
Graphs of Function:

If f is a function with domain D , its graph consists of the points in the Cartesian plane whose coordinates are the input-output pairs for f . In set notation, the graph is:

$$\{(x, f(x)) \mid x \in D\}$$



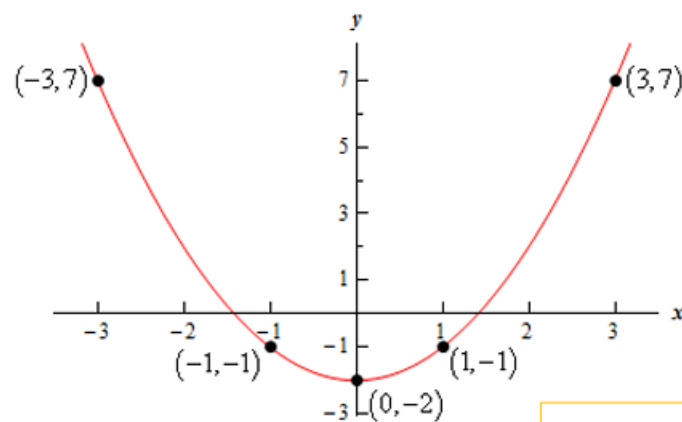
The graph of $f(x) = x + 2$ is the set of points (x, y) for which y has the value $x + 2$.



Example: Sketch the graph of the following function.

$$f(x) = x^2 - 2$$

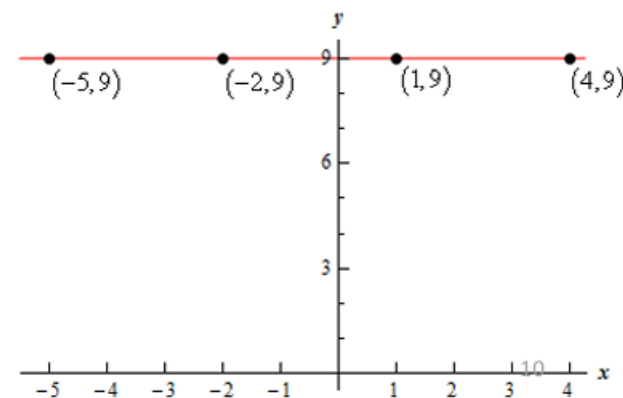
x	$f(x)$	(x, y)
-3	7	$(-3, 7)$
-1	-1	$(-1, -1)$
0	-2	$(0, -2)$
1	-1	$(1, -1)$
3	7	$(3, 7)$



Domain is $(-\infty, \infty)$ and
Range is $[-2, \infty)$

$$f(x) = 9$$

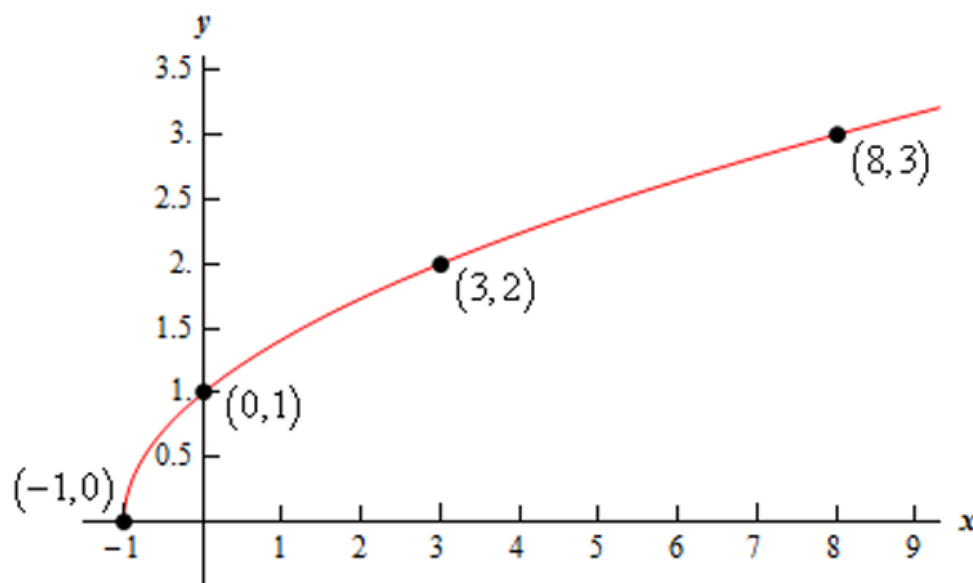
x	$f(x)$	(x, y)
-5	9	$(-5, 9)$
-2	9	$(-2, 9)$
1	9	$(1, 9)$
4	9	$(4, 9)$



Example: Sketch the graph of the following function.

$$f(x) = \sqrt{x+1}$$

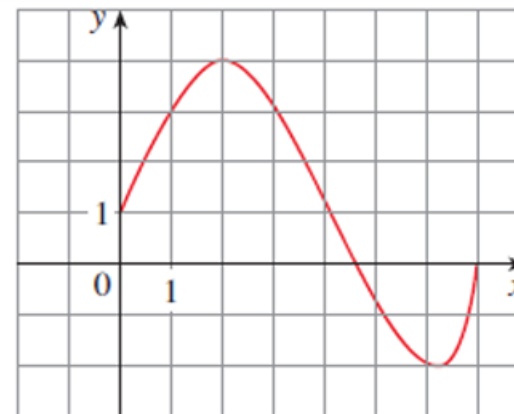
x	$f(x)$	(x, y)
-1	0	$(-1, 0)$
0	1	$(0, 1)$
3	2	$(3, 2)$
8	3	$(8, 3)$



Domain is $[-1, \infty)$ and
Range is $[0, \infty)$

EXAMPLE // The graph of a function f is shown in Figure shown.

- (a) Find the values of $f(1)$ and $f(5)$.
- (b) What are the domain and range of f ?



SOLUTION

- (a) We see from Figure the value of f at 1 is $f(1) = 3$.
(the point on the graph that lies above $x = 1$ is 3 units above the x -axis.)

When $x = 5$, the graph lies about 0.7 units below the x -axis,
so we estimate that $f(5) \approx -0.7$.

- (b) We see that $f(x)$ is defined when $0 \leq x \leq 7$, so the domain of f is the closed interval $[0, 7]$. Notice that f takes on all values from -2 to 4 , so the range of f is

$$\{y \mid -2 \leq y \leq 4\} = [-2, 4]$$

Example: Find the Domains and Ranges of each of all of the following

$$(a) y = x^3 \quad -5 \leq x < 4 \quad (b) y = x^4 \quad (c) y = \frac{1}{(x-1)(x+2)} \quad 0 \leq x \leq 6$$

Solution

$$(a) y = x^3 \quad -5 \leq x < 4$$

domain $-5 \leq x < 4$, range $-125 \leq y < 64$

$$(b) y = x^4$$

domain $-\infty < x < \infty$, range $0 \leq y < \infty$

$$(c) y = \frac{1}{(x-1)(x+2)}, \quad 0 \leq x \leq 6$$

domain $0 \leq x < 1$ and $1 < x \leq 6$,

range $-\infty < y \leq -0.5$, $0.25 \leq y < \infty$

$$\begin{aligned} (x-1)(x+2) &\geq 0 \\ x^2 + x - 2 &\geq 0 \end{aligned}$$

H.W: Find the Domain and Range of each function.

1. $f(x) = 2x + 3$

2. $f(x) = x^2 + 4$

3. $f(x) = \frac{1}{x}$

4. $f(x) = \sqrt{x - 4}$

5. $f(x) = \sqrt{4 - x}$

6. $f(x) = \frac{1}{\sqrt{x-4}}$

7. $f(x) = x^2 + 3x + 1$

8. $f(x) = \frac{2x+1}{x^2+5x+6}$

9. $f(x) = \frac{2}{x^2+3}$

10. $f(x) = \sqrt{x^2 + 5x + 6}$

11. $f(x) = \frac{x^2+2x+3}{\sqrt{x+1}}$

12. $f(x) = \sqrt{1 - x^2}$