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Sample Space (S):

Definition: The sample space is the complete set of all possible outcomes of a random experiment. It is usually denoted by S.

Example 1:

Experiment: Tossing a coin.

Sample Space: The possible outcomes are {Heads, Tails}, i.e., S = {Heads, Tails}.

Example 2:

Experiment: Rolling a six-sided die.

Sample Space: The possible outcomes are {1, 2, 3, 4, 5, 6}, i.e., S = {1, 2, 3, 4, 5, 6}.

Engineering Application:

In Computer Engineering, the sample space can represent all possible outcomes when running a program, such as success or failure.

Example 3:

Experiment: Running a software program that either succeeds or fails.

Sample Space: S = {Success, Failure}.

Event:

Definition: An event is a subset of the sample space. It may consist of one or more outcomes.

Example 4:

Experiment: Rolling a die.

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Sample Space: S = {1, 2, 3, 4, 5, 6}.

Event: The event of rolling an even number is {2, 4, 6}.

Example 5:

Experiment: Tossing a coin.

Sample Space: S = {Heads, Tails}.

Event: The event of getting heads is {Heads}.

Engineering Application:

In network testing, an event could be defined as a "connection failure," representing one or more failure scenarios in devices or networks.

The Three Axioms of Probability:

Axiom 1: The probability of any event is a non-negative number:

$P(E) \geq 0$ for any event E.

Example:

In a system performance test, if the probability of system failure is 0.1, this probability must be non-negative.

Axiom 2: The probability of the entire sample space is 1:

$P(S) = 1$, where S is the sample space.

Example: In the experiment of tossing a coin, the probability of getting heads is 0.5, and the probability of getting tails is also 0.5, so the sum of the probabilities is 1.

Axiom 3: If two events are mutually exclusive (i.e., they cannot occur together), the probability of their union is the sum of their probabilities:

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If A and B are mutually exclusive events, then $P(A \cup B) = P(A) + P(B)$.

Example:

In rolling a die, if event A is rolling an even number, and event B is rolling an odd number, then $P(A \cup B) = 1$, as these two events cover all possible outcomes (even and odd numbers).

Consequences of the Probability Axioms:

a. Complementary Rule:

The probability that an event A does not occur is equal to 1 minus the probability that it does occur:

$P(A') = 1 - P(A)$

Example 6: If the probability of a software program running successfully is 0.8, then the probability of failure is:

$P(\text{Failure}) = 1 - 0.8 = 0.2$.

b. Union of Events:

For any two events A and B, the probability that either A or B occurs is given by:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Example 7:

If the probability that a processor fails is 0.1 and the probability that the processor is slow is 0.2, and the probability that both happen simultaneously is 0.05, then the probability that the processor either fails or is slow is:

$P(\text{Fail or Slow}) = 0.1 + 0.2 - 0.05 = 0.25$.

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Practice Problems and Solutions:

Problem 1: In the experiment of rolling a die, what is the sample space? What is the event if the goal is to get a number greater than 4?

Solution: The sample space is {1, 2, 3, 4, 5, 6}. The event is {5, 6}.

Problem 2: If the probability of a computer program running successfully is 0.9, what is the probability of its failure?

Solution: The probability of failure = 1 - 0.9 = 0.1.

Problem 3: If the probability of a CPU failure is 0.1 and the probability of a memory failure is 0.05, with a 0.02 probability of both failures occurring together, what is the probability of either failure occurring?

Solution: P(CPU or Memory Failure) = 0.1 + 0.05 - 0.02 = 0.13.