$$P_r[blocking] = \frac{\frac{A^C}{C!}}{\sum_{k=0}^{C} \frac{A^k}{k!}} = GOS$$

where C is the number of trunked channels offered by a trunked radio system and A is the total offered traffic.

While it is possible to model trunked systems with finite users, the resulting expressions are much more complicated than the Erlang B result, and the added complexity is not warranted for typical trunked systems which have users that outnumber available channels by orders of magnitude.

Furthermore, the Erlang B formula provides a conservative estimate of the *GOS*, as the finite user results always predict a smaller likelihood of blocking. The capacity of a trunked radio system where blocked calls are lost is tabulated for various values of *GOS* and numbers of channels in Table below

Capacity of an Erlang B system

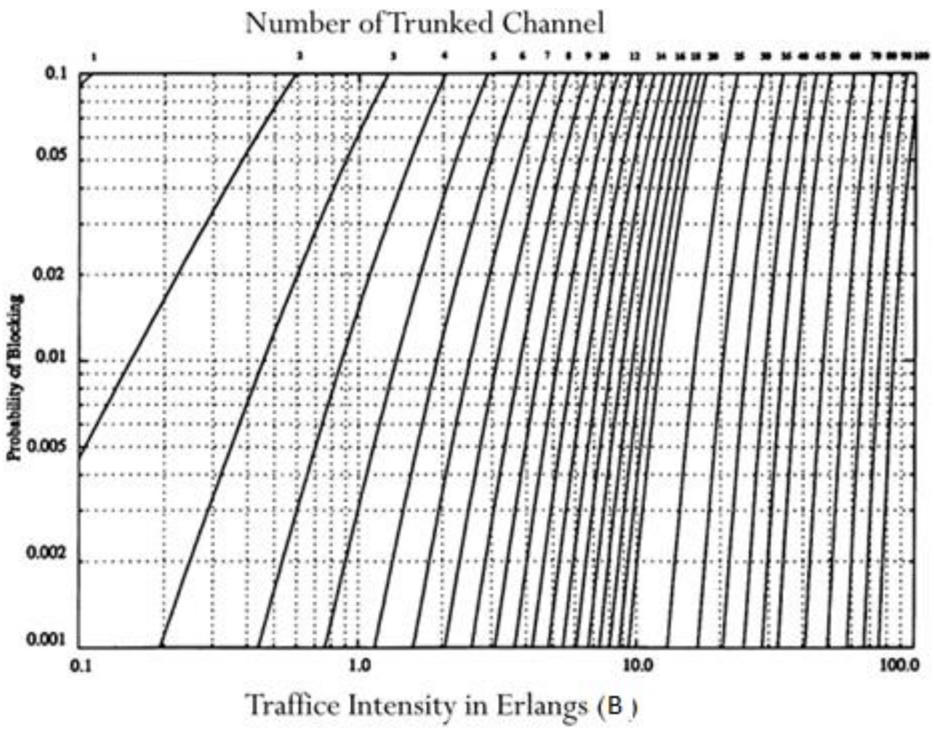
Number of channels *C*	**Capacity (*Erlangs*) for *GOS***			
	0.01	**0.005**	**0.002**	**0.001**
2	0.153	0.105	0.065	0.046
4	0.869	0.701	0.535	0.439
5	1.36	1.13	0.9	0.762
10	4.46	3.96	3.43	3.09
20	12	11.1	10.1	9.41
24	15.3	14.2	13	12.2
40	29	27.3	25.7	24.5
70	56.1	53.7	51	49.2
100	84.1	80.9	77.4	75.2

(2) The second kind of trunked system is one in which a **queue** is provided to hold calls which are blocked.

- If a channel is not available immediately, the call request may be delayed until a channel becomes available.
- This type of trunking is called **Blocked Calls Delayed**, and its measure of *GOS* is defined as the probability that a call is blocked after waiting a specific length of time in the queue.
- To find the GOS, it is first necessary to find the likelihood that a call is initially denied access to the system. The likelihood of a call not having immediate access to a channel is determined by the *Erlang C formula*

$$P_r[delay > 0] = \frac{A^C}{A^C + C!\left(1 - \frac{A}{C}\right)\sum_{k=0}^{C-1}\frac{A^k}{k!}}$$

- If no channels are immediately available the call is delayed, and the probability that the delayed call is forced to wait more than t seconds is given by the probability that a call is delayed, multiplied by the conditional probability that the delay is greater than t seconds.



Example 1

How many users can be supported for 0.5% blocking probability for the following number of trunked channels in a blocked calls cleared system?

a. 5
b. 10

Assume each user generates $A_U = 0.1$ *Erlangs* of traffic.

Solution

(a) From Erlang B chart, we obtain $A \approx 1$

Therefore, total number of users, $U = A/A_U = 1/0.1 = 10$ users.

(b) From Erlang B chart, we obtain $A \approx 4$

Therefore, total number of users, $U = A/A_U = 4/0.1 = 40$ users.

Example 2

How many users can be supported for 0.5% blocking probability for the following number of trunked channels in a blocked calls cleared system? (a) 1, (b) 5, (c) 10, (d) 20, and (e) 100. Assume each user generates 0.1 *Erlangs* of traffic.

Solution

From the table of the capacity of an Erlang B system, we can find the total capacity in *Erlangs* for the 0.5% *GOS* for different numbers of channels.

By using the relation $A = UA_U$, we can obtain the total number of users that can be supported in the system.

(a) Given $C = 1$, $A_U = 0.1$, $GOS = 0.005$
From Erlang B chart, we obtain $A = 0.005$
Therefore, total number of users, $U = A/A_U = 0.005/0.1 = 0.05$ users.
But, actually one user could be supported on one channel. So, $U = 1$

(b) Given $C = 5$, $A_U = 0.1$, GOS = 0.005
From Erlang B chart, we obtain $A = 1.13$
Therefore, total number of users, $U = A/A_U = 1.13/0.1 = 11$ users.

(c) Given $C = 10$, $A_U = 0.1$, GOS = 0.005
From Erlang B chart, we obtain $A = 3.96$ *Erlangs*
Therefore, total number of users, $U = A/A_U = 3.96/0.1 \approx 39$ users.

(d) Given $C = 20$, $A_U = 0.1$, GOS = 0.005
From Erlang B chart, we obtain $A = 11.10$
Therefore, total number of users, $U = A/A_U = 11.1/0.1 = 110$ users.

(e) Given $C = 100$, $A_U = 0.1$, GOS = 0.005
From Erlang B chart, we obtain $A = 80.9$
Therefore, total number of users, $U = A/A_U = 80.9/0.1 = 809$ users.

3.2 Trunking efficiency

- It's a measure of the number of users which can be offered a particular GOS with a particular configuration of fixed channels. The way in which channels are grouped can substantially alter the number of users handled by a trunked system.

  For example, from the table of the capacity of an Erlang B system, 10 trunked channels at a *GOS* of 0.01 can support 4.46 *Erlangs* of traffic, whereas 2 groups of 5 trunked channels can support 2×1.36 *Erlangs*, or 2.72 *Erlangs* of traffic.

- Clearly, 10 channels trunked together support 60% more traffic at a specific *GOS* than do two 5 channel trunks! It should be clear that the allocation of channels in a trunked radio system has a major impact on overall system capacity.

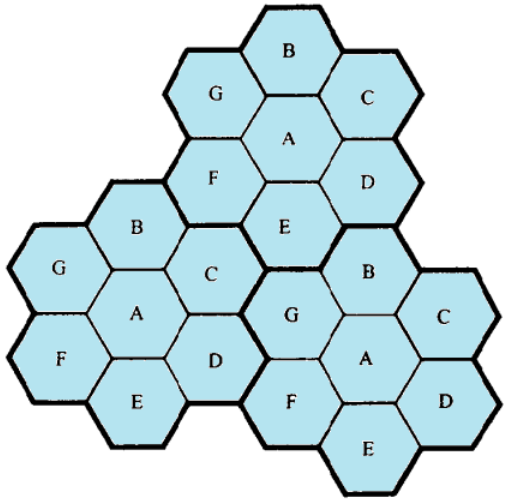
3.3 Improving Capacity in Cellular Systems

- As the demand for wireless service increases, the number of channels assigned to a cell eventually becomes insufficient to support the required number of users. At this point, cellular design techniques are needed to provide more channels per unit coverage area.
- Techniques such as cell splitting and sectoring approaches are used in practice to expand the capacity of cellular systems.

3.3.1 Cell Splitting

- It is the process of subdividing a congested cell into smaller cells, each with its own base station and a corresponding reduction in antenna height and transmitter power.
- Cell splitting increases the capacity of a cellular system since it increases the number of times that channels are reused.
- By defining new cells which have a smaller radius than the original cells and by installing these smaller cells (called microcells) between the existing cells, capacity increases due to the additional number of channels per unit area.

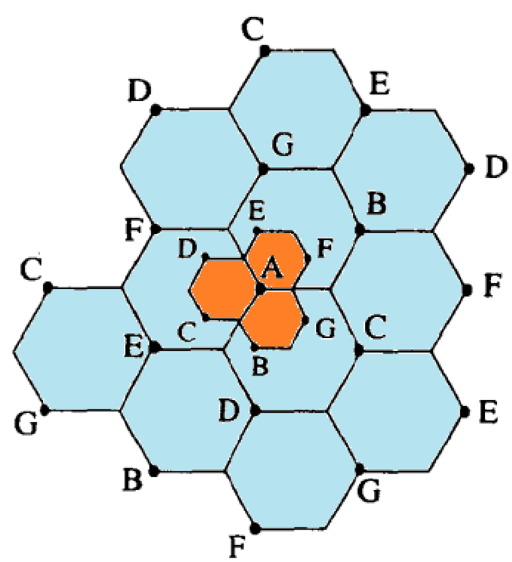
➢ Imagine if every cell in the Figure below were reduced in such a way that the radius of every cell was cut in half.



- ➢ In order to cover the entire service area with smaller cells, approximately four times as many cells would be required.
- ➢ This can be easily shown by considering a circle with radius *R*.
- ➢ The area covered by such a circle is four times as large as the area covered by a circle with radius *R/2*.
- ➢ The increased number of cells would increase the number of clusters over the coverage region, which in turn would increase the number of channels, and thus capacity, in the coverage area.
- ➢ Cell splitting allows a system to grow by replacing large cells with smaller cells, while not upsetting the channel allocation scheme required to maintain the minimum co-channel reuse ratio between co-channel cells.

Example of cell splitting: is shown in the Figure below.

- ❖ The base stations are placed at corners of the cells, and the area served by base station A is assumed to be saturated with traffic (i.e., the blocking of base station A exceeds acceptable rates).
- ❖ New base stations are therefore needed in the region to increase the number of channels in the area and to reduce the area served by the single base station. Note in the figure that the original base station A has been surrounded by six new microcell base stations.
- ❖ In the example shown in Figure, the smaller cells were added in such a way as to preserve the frequency reuse plan of the system.



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- For example, the microcell base station labeled G was placed half way between two larger stations utilizing the same channel set G. This is also the case for the other microcells in the figure.
- As can be seen from Figure, cell splitting merely scales the geometry of the cluster. In this case, the radius of each new microcell is half that of the original cell.

- For the new cells to be smaller in size, the transmit power of these cells must be reduced.
- The transmit power of the new cells with radius half that of the original cells can be found by examining the received power P_r at the new and old cell boundaries and setting them equal to each other.
- This is necessary to ensure that the frequency reuse plan for the new microcells behaves exactly as for the original cells.

$$P_r[at\,old\,cell\,boundry] \propto P_{t1}R^{-n}$$

and

$$P_r[at\,new\,cell\,boundry] \propto P_{t2}\left(\frac{R}{2}\right)^{-n}$$

where

P_{t1} an P_{t2} are the transmit powers of the larger and smaller cell base stations, respectively, n is the path loss exponent.

If we take n = 4 and set the received powers equal to each other, then

$$P_{t2} = \frac{P_{t1}}{16}$$

In other words, the transmit power must be reduced by 12 dB in order to fill in the original coverage area with microcells, while maintaining the *S/I* requirement.

- In practice, not all cells are split at the same time. It is often difficult for service providers to find real estate that is perfectly situated for cell splitting.
- Therefore, different cell sizes will exist simultaneously. In such situations, special care needs to be taken to keep the distance between co-channel cells at the required minimum, and hence channel assignments become more complicated.
- Handoff issues must be addressed so that high speed and low speed traffic can be simultaneously accommodated (the umbrella cell approach is commonly used).
- When there are two cell sizes in the same region, the last equation shows that one cannot simply use the original transmit power for all new cells or the new transmit power for all the original cells.

a. If the larger transmit power is used for all cells, some channels used by the smaller cells would not be sufficiently separated from co-channel cells.
b. If the smaller transmit power is used for all the cells, there would be parts of the larger cells left unserved.

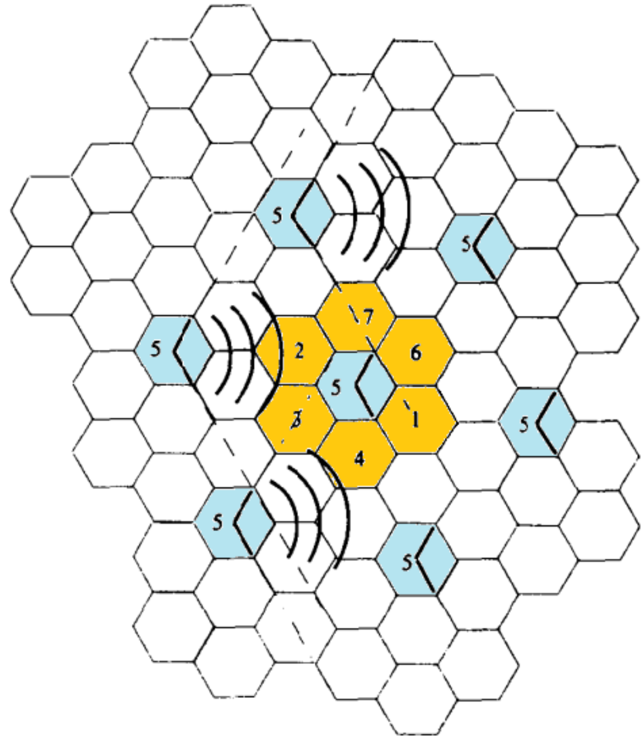
- For this reason, channels in the old cell must be broken down into two channel groups, one that corresponds to the smaller cell reuse requirements and the other that corresponds to the larger cell reuse requirements.
- The larger cell is usually dedicated to high speed traffic so that handoffs occur less frequently.

3.3.2 Sectoring

Cell splitting achieves capacity improvement by essentially rescaling the system.

- By decreasing the cell radius *R* and keeping the co-channel reuse ratio *D/R* unchanged, cell splitting increases the number of channels per unit area.
- Another way to increase capacity is to keep the cell radius unchanged and seek methods to decrease the *D/R* ratio.
- In this approach, capacity improvement is achieved by reducing the number of cells in a cluster and thus increasing the frequency reuse. In order to do this, it is necessary to reduce the relative interference without decreasing the transmit power.
- The co-channel interference in a cellular system may be decreased by replacing a single omnidirectional antenna at the base station by several directional antennas, each radiating within a specified sector.
- By using directional antennas, a given cell will receive interference and transmit with only a fraction of the available co-channel cells.
- The technique for decreasing co-channel interference and thus increasing system capacity by using directional antennas is called sectoring.
- The factor by which the co-channel interference is reduced depends on the amount of sectoring used.
- A cell is normally partitioned into three 120° sectors or six 60° sectors as shown in Figures below.
- When sectoring is employed, the channels used in a particular cell are broken down into sectored groups and are used only within a particular sector.
- Assuming 7-cell reuse, for the case of 120° sectors, the number of interferers in the first tier is reduced from 6 to 2. This is because only 2 of the 6 co-channel cells receive interference with a particular sectored channel group.

Referring to Figure (right), consider the interference experienced by a mobile located in the right-most sector in the center cell labeled "5".



- ❖ There are 3 co-channel cell sectors labeled "5" to the right of the center cell, and 3 to the left of the center cell.
- ❖ Out of these 6 co-channel cells, only 2 cells have sectors with antenna patterns which radiate into the center cell, and hence a mobile in the center cell will experience interference on the forward link from only these two sectors.
- ❖ The resulting *S/I* for this case can be found to be 24.2 dB, which is a significant improvement over the omni-directional case, where the worst case *S/I* was shown to be 17 dB.
- ❖ In practical systems, further improvement in *S/I* is achieved by downtilting the sector antennas such that the radiation pattern in the vertical (elevation) plane has a notch at the nearest co-channel cell distance.

- The improvement in *S/I* implies that with 120° sectoring, the minimum required *S/I* of 18 dB can be easily achieved with 7-cell reuse, as compared to 12-cell reuse for the worst possible situation in the unsectored case.
- Thus, sectoring reduces interference, which amounts to an increase in capacity by a factor of 12/7, or 1.714.
- In practice, the reduction in interference offered by sectoring enable planners to reduce the cluster size N, and provides an additional degree of freedom in assigning channels.
- The penalty for improved *S/I* and the resulting capacity improvement is an increased number of antennas at each base station, and a decrease in trunking efficiency due to channel sectoring at the base station.