جامعة المستقبل
AL MUSTAQBAL UNIVERSITY
كلية العلوم

1st class

2024- 2025

Mathematics 1

Lecture 1

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الرياضيات الاساسية: المرحلة الاولى

مادة الرياضيات 1

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1 Matrices

A matrix is a rectangular array of numbers, symbols, or expressions arranged in rows and columns, enclosed by square or round brackets. Matrices are fundamental tools in mathematics, with applications in various fields such as physics, engineering, computer science, economics, and more. They are used to represent systems of linear equations, perform transformations in geometry, and process data in machine learning.

1.1 Definition of a Matrix

A **matrix** is a rectangular array of numbers, symbols, or expressions, arranged in rows and columns, that is used to represent linear transformations, solve systems of equations, and perform various mathematical operations.

A matrix is typically denoted by an uppercase letter (e.g., A), and its elements are enclosed within brackets. If a matrix has m rows and n columns, it is said to have dimensions $m \times n$ (read as "m by n").

A general $m \times n$ matrix is represented as:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix},$$

where a_{ij} represents the element in the i-th row and j-th column.

For example:

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}.$$

- B has 2 rows and 3 columns, so its dimensions are 2×3.
- The element in the first row and second column is $b_{12} = 2$.

- The element in the second row and first column is $b_{21} = 4$.

$$M = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}.$$

This matrix has 4 rows and 4 columns, so it is a square matrix of order 4×4. Each element m_{ij} represents the number in the i-th row and j-th column.

1.2 Types of Matrices

Matrices come in various types, each with specific properties. Below is an explanation of the most commonly encountered types of matrices:

1. Row Matrix

A **row matrix** is a matrix that has only one row and multiple columns. Its dimensions are $1 \times n$, where n is the number of columns.

$$R = \begin{bmatrix} a_1 & a_2 & \cdots & a_n \end{bmatrix}$$

Example:

$$R = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \end{bmatrix}$$

2. Column Matrix

A **column matrix** is a matrix that has only one column and multiple rows. Its dimensions are $m \times 1$, where m is the number of rows.

$$C = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix}$$

Example:

$$C = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

3. Square Matrix

A **square matrix** is a matrix that has the same number of rows and columns. The dimensions of a square matrix are $n \times n$, where n is the number of rows (or columns).

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

Example:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

4. Zero Matrix (Null Matrix)

A **zero matrix** is a matrix in which all the elements are zero. It can be of any size $m \times n$.

$$O = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}$$

5. Diagonal Matrix

A **diagonal matrix** is a square matrix in which all elements off the principal diagonal are zero. The elements on the diagonal can be any value.

$$D = \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}$$

Example:

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

6. Identity Matrix

An **identity matrix** is a square matrix in which all the elements of the principal diagonal are 1, and all other elements are 0. It is denoted by I_n, where n is the order of the matrix.

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

7. Scalar Matrix

A **scalar matrix** is a diagonal matrix where all the diagonal elements are equal. It is a special case of a diagonal matrix.

$$S = k \begin{bmatrix} 1 & 5 & 7 \\ 0 & 2 & 0 \\ -6 & 0 & 3 \end{bmatrix} = \begin{bmatrix} k & 5k & 7k \\ 0 & 2k & 0 \\ -6k & 0 & 3k \end{bmatrix}$$

H.W. If $k = 3$, find S above.

8. Triangular Matrix

A **triangular matrix** is a square matrix in which all the elements either above or below the main diagonal are zero.

Upper Triangular Matrix

In an upper triangular matrix, all elements below the main diagonal are zero.

$$U = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

Lower Triangular Matrix

In a lower triangular matrix, all elements above the main diagonal are zero.

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 5 & 0 \\ 7 & 8 & 9 \end{bmatrix}$$

1.3 Matrix Operations

Matrix operations are a set of mathematical operations that can be performed on matrices. These operations include matrix addition, subtraction and multiplication. Below are matrix operations.

1. Matrix Addition

Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two matrices of the same size, say $m \times n$. The sum of A and B, denoted $A + B$, is a matrix $C = [c_{ij}]$, where each element c_{ij} is the sum of the corresponding elements from A and B:

$$C = A + B = [c_{ij}] = [a_{ij} + b_{ij}].$$

Example 1.1. Consider the two matrices:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix}.$$

The sum of A and B is:

$$A + B = \begin{bmatrix} 1+7 & 2+8 & 3+9 \\ 4+10 & 5+11 & 6+12 \end{bmatrix} = \begin{bmatrix} 8 & 10 & 12 \\ 14 & 16 & 18 \end{bmatrix}.$$

Example 1.2. Consider the two matrices:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

The sum of A and B is:

$$A + B = \begin{bmatrix} 1+1 & 2 & 3 \\ 4+2 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 3 \\ 6 & 5 & 6 \end{bmatrix}.$$

2. Matrix Subtraction

Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two matrices of the same size, say $m \times n$. The difference of A and B, denoted $A - B$, is a matrix $C = [c_{ij}]$, where each element c_{ij} is the difference of the corresponding elements from A and B:

$$C = A - B = [c_{ij}] = [a_{ij} - b_{ij}].$$

Example 1.3. Consider the two matrices:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix}.$$

The difference of A and B is:

$$A - B = \begin{bmatrix} 1-7 & 2-8 & 3-9 \\ 4-10 & 5-11 & 6-12 \end{bmatrix} = \begin{bmatrix} -6 & -6 & -6 \\ -6 & -6 & -6 \end{bmatrix}.$$

Properties of Matrix Addition:

1. **Commutative Property:** $A + B = B + A$.
2. **Associative Property:** $(A + B) + C = A + (B + C)$.
3. **Existence of Additive Identity:** There exists a matrix O (the zero matrix) such that: $A + O = A$.
4. **Existence of Additive Inverse:** For each matrix A, there exists a matrix $-A$ such that: $A + (-A) = O$.

3. Matrix Multiplication

Matrix multiplication is an operation that takes two matrices and produces a new matrix. To multiply two matrices, the number of columns in the first matrix must equal the number of rows in the second matrix.

Let A be a matrix of size $m \times n$ and B be a matrix of size $n \times p$. The product of A and B, denoted $A \times B$, is a matrix C of size $m \times p$. The element c_{ij} in the resulting matrix is obtained by taking the dot product of the i-th row of matrix A and the j-th column of matrix B:

$$c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}.$$

This means that the i-th row of matrix A is multiplied element-wise by the j-th column of matrix B, and the results are summed.

Example 1.4. Consider the two matrices:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}.$$

The product $A \times B$ is computed as follows:

$$A \times B = \begin{bmatrix} (1 \times 5 + 2 \times 7) & (1 \times 6 + 2 \times 8) \\ (3 \times 5 + 4 \times 7) & (3 \times 6 + 4 \times 8) \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}.$$

Example 1.5 (Multiply a 3×2 Matrix by a 2×3 Matrix)**.** Let the matrices A and B be:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix}$$

The product $A \times B$ is calculated as:

$$A \times B = \begin{bmatrix} (1 \times 7 + 2 \times 10) & (1 \times 8 + 2 \times 11) & (1 \times 9 + 2 \times 12) \\ (3 \times 7 + 4 \times 10) & (3 \times 8 + 4 \times 11) & (3 \times 9 + 4 \times 12) \\ (5 \times 7 + 6 \times 10) & (5 \times 8 + 6 \times 11) & (5 \times 9 + 6 \times 12) \end{bmatrix} = \begin{bmatrix} 27 & 30 & 33 \\ 61 & 68 & 75 \\ 95 & 106 & 117 \end{bmatrix}.$$

Properties of Matrix Multiplication:

1. **Associative Property:**

$$(A \times B) \times C = A \times (B \times C).$$

2. **Distributive Property:**

$$A \times (B + C) = A \times B + A \times C.$$

$$(A + B) \times C = A \times C + B \times C.$$

3. **Existence of Multiplicative Identity:** There exists an identity matrix I such that:

$$A \times I = A \quad \text{and} \quad I \times A = A.$$

where I is the identity matrix of appropriate size.

4. **Non-Commutative Property:**

$$A \times B \neq B \times A \quad \text{(in general)}.$$

Properties of Scalar Multiplication:

1. Distributive Property over Matrix Addition: $\alpha(A + B) = \alpha A + \alpha B$.
2. Distributive Property over Scalar Addition: $(\alpha + \beta)A = \alpha A + \beta A$.
3. Associative Property with Scalars: $\alpha(\beta A) = (\alpha\beta)A$.
4. Existence of Multiplicative Identity: $1 \times A = A$.

1.3.1 Exercises of Matrix Operations

$$A = \begin{bmatrix} 3 & 5 & 0 \\ 7 & 2 & -3 \\ 2 & 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 1 & 5 \\ 0 & 6 & 6 \\ 2 & 9 & 0 \end{bmatrix},$$

Find $A + B$, $A - B$, A^2, $-3B$, AB, BA, $3B + 2A$.

1.4 The Determinant of a Matrix:

The determinant is a scalar value associated with a square matrix. It is useful in various areas of linear algebra, including solving systems of linear equations, finding matrix inverses, and understanding matrix properties.

Definition of Determinant

Let A be a square matrix of order $n \times n$. The determinant of A, denoted by det(A) or $|A|$, is computed using specific rules depending on the size of the matrix.

Determinant of a 2×2 Matrix:

For a 2×2 matrix:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix},$$

the determinant is given by:

$$\det(A) = |A| = ad - bc.$$

Example 1.6. For a 2×2 matrix:

$$A = \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix}$$

calculate the determinant of A:

$$\det(\mathbf{A}) = \begin{vmatrix} 1 & -2 \\ 4 & 5 \end{vmatrix} = 1 \cdot 5 - (-2) \cdot 4 = 5 + 8 = 13$$

Determinant of an $n \times n$ Matrix

Let A be an $n \times n$ matrix:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}.$$

The determinant of A, denoted as $\det(A)$ or $|A|$, is computed by expanding along the first row (or any row/column):

$$\det(A) = \sum_{j=1}^{n} (-1)^{1+j} a_{1j} \det(M_{1j}),$$

where: a_{1j} is the element in the first row and j-th column. M_{1j} is the matrix obtained by removing the first row and the j-th column from A. $(-1)^{1+j}$ is the sign factor.

Example 1.7.

$$A = \begin{bmatrix} 3 & -6 & 7 \\ 4 & 0 & -5 \\ 5 & -8 & 6 \end{bmatrix}$$

The determinant of A is:

$$\det(A) = 3 \cdot \begin{vmatrix} 0 & -5 \\ -8 & 6 \end{vmatrix} - (-6) \cdot \begin{vmatrix} 4 & -5 \\ 5 & 6 \end{vmatrix} + 7 \cdot \begin{vmatrix} 4 & 0 \\ 5 & -8 \end{vmatrix}$$

$$= 3(0 \cdot 6 - (-5) \cdot (-8)) + 6(4 \cdot 6 - (-5) \cdot 5) + 7(4 \cdot (-8) - 0 \cdot 5)$$

$$= 3(0 - 40) + 6(24 + 25) + 7(-32 - 0)$$

$$= 3(-40) + 6(49) + 7(-32)$$

$$= -120 + 294 - 224$$

$$= -50$$

1.4.1 Determinant of a Matrix Exercises

1. Given the matrix

$$A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix},$$

find the determinant of matrix A.

2. Given the matrix

$$B = \begin{bmatrix} 1 & 4 & 6 \\ 2 & 5 & 8 \\ 3 & 7 & 9 \end{bmatrix},$$

find the determinant of matrix B.

3. Given the matrix

$$J = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix},$$

find the determinant of matrix J.

1.5 Matrix Transpose

The **transpose** of a matrix is an operation that flips a matrix over its diagonal, turning the rows into columns and the columns into rows.

If A is a matrix, its transpose is denoted as A^T.

Definition of Transpose

Let A be a matrix of size $m \times n$:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}.$$

The transpose of A, denoted A^T, is a matrix of size $n \times m$ where the rows of A

become the columns:

$$A^T = \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{bmatrix}.$$

Example 1.8 (Transpose of a 2×3 Matrix)**.** Let:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}.$$

The transpose of A is:

$$A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}.$$

Example 1.9 (Transpose of a 3×3 Matrix)**.** Let:

$$B = \begin{bmatrix} 7 & 8 & 9 \\ 10 & 11 & 12 \\ 13 & 14 & 15 \end{bmatrix}.$$

The transpose of B is:

$$B^T = \begin{bmatrix} 7 & 10 & 13 \\ 8 & 11 & 14 \\ 9 & 12 & 15 \end{bmatrix}.$$

Example 1.10 (Transpose of a 1×4 Row Vector)**.** Let:

$$C = \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}.$$

The transpose of C is:

$$C^T = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}.$$

1.5.1 Properties of the Transpose

1. $(A^T)^T = A$ (Double transpose gives the original matrix).
2. $(A + B)^T = A^T + B^T$ (The transpose of a sum is the sum of the transposes).
3. $(cA)^T = cA^T$, where c is a scalar.
4. $(AB)^T = B^T A^T$ (The transpose of a product reverses the order).

1.5.2 Matrix Transpose Exercises

1. Given the matrix
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix},$$
find A^T.

2. Given the matrix
$$G = \begin{bmatrix} 6 & 2 \\ 9 & 4 \\ 1 & 7 \end{bmatrix},$$
find G^T.

1.6 Symmetric Matrix

A **symmetric matrix** is a square matrix that is equal to its transpose. In mathematical terms, a matrix A is symmetric if:

$$A = A^T,$$

where A^T is the transpose of A.

Example 1.11. Consider the matrix:

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 5 & 4 \\ 1 & 4 & 6 \end{bmatrix}.$$

The transpose of A is:

$$A^T = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 5 & 4 \\ 1 & 4 & 6 \end{bmatrix}.$$

Since $A = A^T$, A is symmetric.

Example 1.12. Consider the matrix:

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}.$$

The transpose of B is:

$$B^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}.$$

Since $B \neq B^T$, B is not symmetric.

1.7 Rank of a Matrix

The **rank** of a matrix is the maximum number of linearly independent rows or columns. It represents the dimension of the row space (or column space) of the matrix.

Definition Rank of a Matrix

Let A be an $m \times n$ matrix. The rank of A, denoted by rank(A), is:

$$\text{rank}(A) = \dim(\text{Row Space of } A) = \dim(\text{Column Space of } A).$$

The rank of a matrix can be determined using two common methods:

1. Minor Method

2. Echelon Form Method

1.7.1 Minor Method

The **minor method** involves finding the largest square submatrix of the given matrix with a non-zero determinant. The size of this submatrix determines the rank of the matrix.

Example 1.13. Consider the matrix:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}.$$

Step 1: Check the determinant of the full 3×3 matrix:

$$\det(A) = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}.$$

Expanding along the first row:

$$\det(A) = 1 \cdot \begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} - 2 \cdot \begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} + 3 \cdot \begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix}.$$

Calculate:

$$\det(A) = 1(5 \cdot 9 - 6 \cdot 8) - 2(4 \cdot 9 - 6 \cdot 7) + 3(4 \cdot 8 - 5 \cdot 7).$$

$$\det(A) = 1(45 - 48) - 2(36 - 42) + 3(32 - 35).$$

$$\det(A) = -3 + 12 - 9 = 0.$$

Since $\det(A) = 0$, the matrix is singular, so the rank is less than **3**.

Step 2: Check the determinant of 2×2 submatrices. Consider the submatrix:

$$\begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix} = 1 \cdot 5 - 2 \cdot 4 = -3.$$

Since this determinant is non-zero, the rank of A is **2**.

Example 1.14. Consider the matrix:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{bmatrix}.$$

The determinant of the full 3×3 matrix is:

$$\det(A) = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{vmatrix}.$$

Using cofactor expansion:

$$\det(A) = -3.$$

Since the determinant is non-zero, the rank is $\text{rank}(A) = 3$.

1.7.2 Row Echelon Form (REF) Method

The **REF** involves reducing the matrix to its row-echelon form using elementary row operations. The rank is equal to the number of non-zero rows in the row-reduced form.

Example 1.15. Consider the matrix:

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}.$$

Step 1: Perform Gaussian elimination to simplify the matrix: 1. Subtract 4 times Row 1 from Row 2:

$$R_2 = R_2 - 4R_1.$$

2. Subtract 7 times Row 1 from Row 3:

$$R_3 = R_3 - 7R_1.$$

This gives:

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -6 & -12 \end{bmatrix}.$$

Step 2: Simplify further. Divide Row 2 by -3:

$$R_2 = \frac{R_2}{-3}.$$

This gives:

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -6 & -12 \end{bmatrix}.$$

Step 3: Eliminate Row 3 by adding 6 times Row 2 to Row 3:

$$R_3 = R_3 + 6R_2.$$

This gives:

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}.$$

Step 4: Count the non-zero rows:

- Row 1: $[1, 2, 3]$ (non-zero),

- Row 2: $[0, 1, 2]$ (non-zero),

- Row 3: $[0, 0, 0]$ (zero row).

The rank is **2**.

1.7.3 Rank of a Matrix Exercises

Find the rank of matrix A.

1. $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$

2. $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 4 \end{bmatrix}$

3. $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{bmatrix}$