Department of biology

((GENERAL MATHEMATICS))

First Stage

Week 1- lecture 1

Real Numbers and Inequalities

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By

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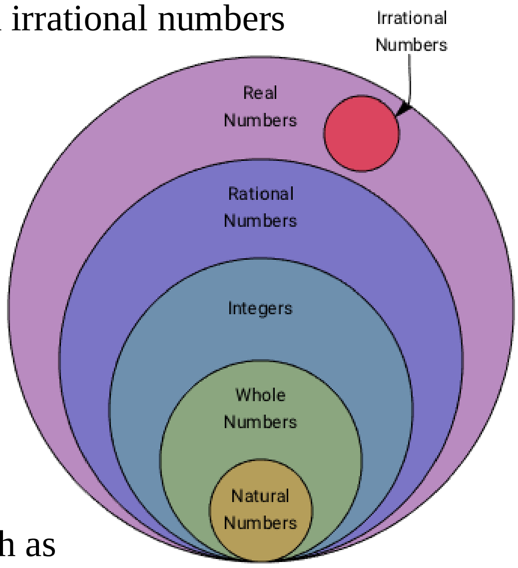
Introduction to Real Numbers

Real numbers are an essential concept in mathematics, forming the basis for many advanced topics including calculus, algebra, and real-world problem-solving. Real numbers include all the numbers that can be represented on a number line. They unify rational and irrational numbers under one comprehensive category.



1. Classification of Real Numbers

1. Natural Numbers (N):

-These are the counting numbers starting from 1, such as

{1, 2, 3, 4, ...}

2. Whole Numbers (W):

- Includes all natural numbers and the number 0, such as

{0, 1, 2, 3, ...}

3. Integers (Z):

- Extends whole numbers to include negatives, such as

{..., -3, -2, -1, 0, 1, 2, 3, ...}

4. Rational Numbers (Q):

- Numbers that can be expressed as fractions p/q, where p and q are integers and $q \neq 0$.
- Examples include 1/2, -3/4.

5. Irrational Numbers (I):

- Numbers that cannot be expressed as fractions, such as $\sqrt{2}$, π, and e.

2. Properties of Real Numbers

1. Commutative Property:

- Addition:

$a + b = b + a \longrightarrow 3 + 5 = 5 + 3$

- Multiplication:

$a \times b = b \times a \longrightarrow 3 \times 5 = 5 \times 3$

2. Associative Property:

- Addition:

$(a + b) + c = a + (b + c) \longrightarrow (3 + 5) + 2 = 3 + (5 + 2)$

$8 + 2 = 3 + 7$

$10 = 10$

- Multiplication:

$(a \times b) \times c = a \times (b \times c) \longrightarrow (3 \times 5) \times 2 = 3 \times (5 \times 2)$

$15 \times 2 = 3 \times 10$

$30 = 30$

3. Distributive Property:

- Multiplication over addition:

$a \times (b + c) = a \times b + a \times c \longrightarrow 3 \times (5 + 2) = 3 \times 5 + 3 \times 2$

$= 15 + 6$

$= 21$

4. Inverse Elements:

- Additive Inverse:

$a + (-a) = 0 \longrightarrow 3 + (-3) = 0$

- Multiplicative Inverse:

$a \times (1/a) = 1 \quad (a \neq 0) \longrightarrow 3 \times (1/3) = 1$

Table of Properties

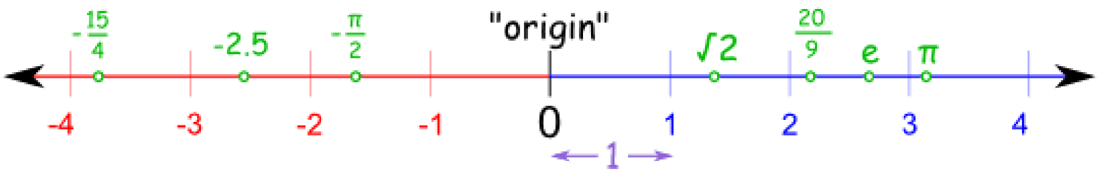
Let *a*, *b*, and *c* be real numbers, variables, or algebraic expressions.
(These are properties you need to know.)

	Property	Example
1.	Commutative Property of Addition $a + b = b + a$	$2 + 3 = 3 + 2$
2.	Commutative Property of Multiplication $a \cdot b = b \cdot a$	$2 \cdot (3) = 3 \cdot (2)$
3.	Associative Property of Addition $a + (b + c) = (a + b) + c$	$2 + (3 + 4) = (2 + 3) + 4$
4.	Associative Property of Multiplication $a \cdot (b \cdot c) = (a \cdot b) \cdot c$	$2 \cdot (3 \cdot 4) = (2 \cdot 3) \cdot 4$
5.	Distributive Property $a \cdot (b + c) = a \cdot b + a \cdot c$	$2 \cdot (3 + 4) = 2 \cdot 3 + 2 \cdot 4$
6.	Additive Identity Property $a + 0 = a$	$3 + 0 = 3$
7.	Multiplicative Identity Property $a \cdot 1 = a$	$3 \cdot 1 = 3$
8.	Additive Inverse Property $a + (-a) = 0$	$3 + (-3) = 0$
9.	Multiplicative Inverse Property $a \cdot \left(\frac{1}{a}\right) = 1$ **Note:** *a* cannot = 0	$3 \cdot \left(\frac{1}{3}\right) = 1$
10.	Zero Property $a \cdot 0 = 0$	$5 \cdot 0 = 0$

Example: Classify the following numbers: 0, -1, √2, π, 3.5

3. The Real Line

The real numbers can be represented geometrically as points on a number line, which we call the Real Line.



Inequalities

An inequality is a mathematical statement that compares two values using symbols such as $<$, $>$, $\leq$, or $\geq$. Inequalities describe a range of possible solutions rather than a single value.

Example: Solve $2x + 3 > 7$

1. Subtract 3 from both sides: $2x > 4$
2. Divide both sides by 2: $x > 2$

Example: Solve the inequality: $5x - 7 \leq 18$

$$5x \leq 18 + 7$$

$$x \leq 25/5$$

$$x \leq 5$$

Rules for Inequalities

Rules for inequalities

If a, b, and c are real numbers, then:

1. $a < b \implies a + c < b + c$
2. $a < b \implies a - c < b - c$
3. $a < b$ and $c > 0 \implies ac < bc$
4. $a < b$ and $c < 0 \implies ac > bc$; in particular, $-a > -b$
5. $a > 0 \implies \dfrac{1}{a} > 0$
6. $0 < a < b \implies \dfrac{1}{b} < \dfrac{1}{a}$

Rules 1–4 and 6 (for $a > 0$) also hold if $<$ and $>$ are replaced by $\leq$ and $\geq$.

Intervals

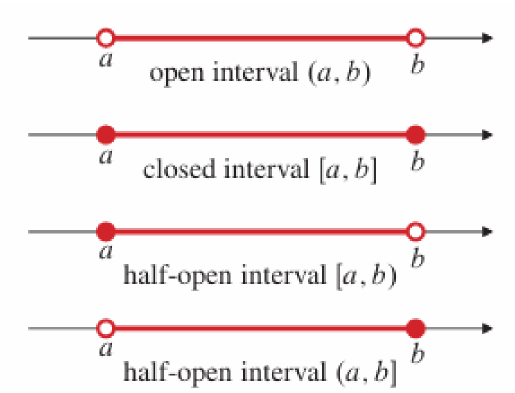
An **interval** is a way to represent the possible values of a variable that satisfy a given inequality. Intervals use parentheses ( ) or brackets [ ] to indicate the range of values.

Types of Intervals:

1. Open Interval:

Does not include the endpoints, denoted with parentheses ( )

$$a < x < b \Longrightarrow (a , b)$$



2. Closed Interval:

Includes the endpoints, denoted with brackets [ ]

$$a \le x \le b \Longrightarrow [a , b]$$

3. Half-Open Interval:

Includes only one endpoint

$$a \le x < b \Longrightarrow [a , b)$$

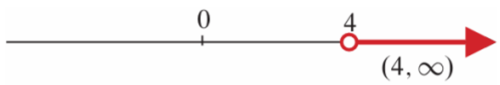
Example: Solve The Following Inequalities. Express the solution sets in terms of intervals and graph them.

(a) $2x - 1 > x + 3$ (b) $-\frac{x}{3} \ge 2x - 1$

Solution:

(a) $2x - 1 > x + 3$	Add 1 to both sides.
$2x > x + 4$	Subtract x from both sides.
$x > 4$	The solution set is the interval $(4, \infty)$.
(b) $-\frac{x}{3} \ge 2x - 1$	Multiply both sides by -3.
$x \le -6x + 3$	Add $6x$ to both sides.
$7x \le 3$	Divide both sides by 7.
$x \le \frac{3}{7}$	The solution set is the interval $(-\infty, 3/7]$.

For (a)



For (b)