Lecture 5:

Mesh Analysis

Mesh analysis provides another general procedure for analyzing circuits, using mesh currents as the circuit variables instead of branch currents (as in Kirchhoff's laws). Using mesh currents instead of element currents as circuit variables is suitable and reduces the number of equations that must be solved simultaneously. Mesh analysis applies "KVL" to find unknown currents.

Steps to Determine Mesh Currents:

1. Specify mesh currents to the *n* meshes.

2. Apply "KVL" to each of the *n* meshes. Use Ohm's law to express the voltages in terms of the mesh currents.

3. Solve the resulting *n* simultaneous equations to get the mesh currents.

For the **Figure(6)** below:

For the first loop :

(clockwise from point *f*)

$-V_1+I_1R_1+(I_1-I_2)R_3=0 \quad \dots(1)$

Or

$I_1(R_1+R_3)-I_2R_3=V_1 \quad \dots(2)$

For the second loop :

(clockwise from point *b*)

$+I_2R_2+V_2+(I_2-I_1)R_3=0 \quad \dots(3)$

Or

$-I_1R_3+I_2(R_2+R_3)=-V_2 \quad \dots(4)$

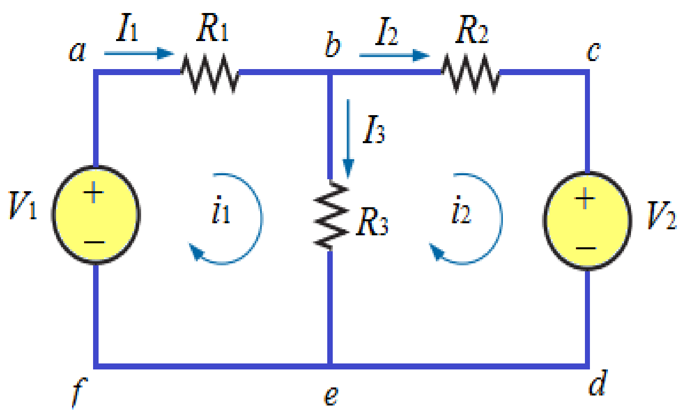

Fig.(6)

Note: **A mesh is a loop that does not contain any other loop within it.**

Equations (2) & (4) can be solved either simultaneously or by matrix.

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix}$$

So,

$$\Delta = \begin{vmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{vmatrix} = [(R_1 + R_3) \times (R_2 + R_3)] - [(-R_3) \times (-R_3)]$$

$$\Delta_1 = \begin{vmatrix} V_1 & -R_3 \\ -V_2 & R_2 + R_3 \end{vmatrix} = [V_1 \times (R_2 + R_3)] - [(-R_3) \times (-V_2)]$$

$$\Delta_2 = \begin{vmatrix} R_1 + R_3 & V_1 \\ -R_3 & -V_2 \end{vmatrix} = [(R_1 + R_3) \times (-V_2)] - [(V_1) \times (-R_3)]$$

$i_1 = \frac{\Delta_1}{\Delta}$ and , $i_2 = \frac{\Delta_2}{\Delta}$

Example 1: For the circuit in **Figure (7)**, find the branch currents using mesh analysis.

Solution:

We first obtain the mesh currents using KVL. For mesh 1,

$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$$

or

$$3i_1 - 2i_2 = 1 \qquad \textbf{(1)}$$

For mesh 2,

$$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0$$

or

$$i_1 = 2i_2 - 1 \qquad \textbf{(2)}$$

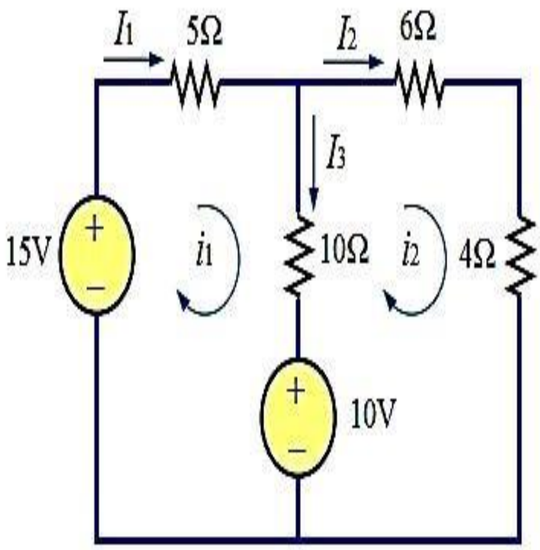

Fig.(7)

METHOD 1 Using the substitution method, we substitute Eq. (2) into Eq. (1), and write

$$6i_2 - 3 - 2i_2 = 1 \quad \Rightarrow \quad i_2 = 1\text{ A}$$

From Eq. (2), $i_1 = 2i_2 - 1 = 2 - 1 = 1$ A. Thus,

$$I_1 = i_1 = 1\text{ A}, \quad I_2 = i_2 = 1\text{ A}, \quad I_3 = i_1 - i_2 = 0$$

METHOD 2 To use Cramer's rule, we cast Eqs. (1) and (2) in matrix form as

$$\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

We obtain the determinants

$$\Delta = \begin{vmatrix} 3 & -2 \\ -1 & 2 \end{vmatrix} = 6 - 2 = 4$$

$$\Delta_1 = \begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix} = 2 + 2 = 4, \qquad \Delta_2 = \begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} = 3 + 1 = 4$$

Thus,

$$i_1 = \frac{\Delta_1}{\Delta} = 1\text{ A}, \qquad i_2 = \frac{\Delta_2}{\Delta} = 1\text{ A}$$

Example 2 : Use mesh analysis to find the current **Io** in the circuit of **Figure (8)**?

Solution:

We apply KVL to the three meshes in turn. For mesh 1,

$$-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0$$

or

$$11i_1 - 5i_2 - 6i_3 = 12 \qquad (1)$$

For mesh 2,

$$24i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0$$

or

$$-5i_1 + 19i_2 - 2i_3 = 0 \qquad (2)$$

For mesh 3,

$$4I_o + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

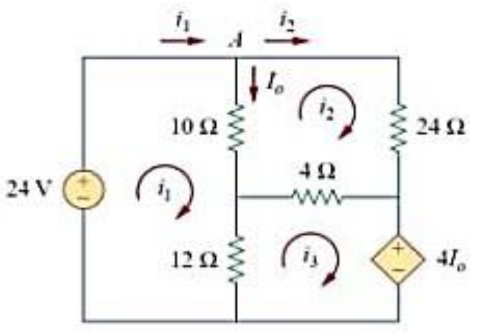

Fig.(8)

But at node A, $I_o = i_1 - i_2$, so that

$$4(i_1 - i_2) + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

or

$$-i_1 - i_2 + 2i_3 = 0 \qquad (3)$$

In matrix form, Eqs. (1) to (3) become

$$\begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

We obtain the determinants as

$$\Delta = \begin{vmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \\ 11 & -5 & -6 \\ -5 & 19 & -2 \end{vmatrix}$$

$$= 418 - 30 - 10 - 114 - 22 - 50 = 192$$

$$\Delta_1 = \begin{vmatrix} 12 & -5 & -6 \\ 0 & 19 & -2 \\ 0 & -1 & 2 \\ 12 & -5 & -6 \\ 0 & 19 & -2 \end{vmatrix} = 456 - 24 = 432$$

$$\Delta_2 = \begin{vmatrix} 11 & 12 & -6 \\ -5 & 0 & -2 \\ -1 & 0 & 2 \\ 11 & 12 & -6 \\ -5 & 0 & -2 \end{vmatrix} = 24 + 120 = 144$$

$$\Delta_3 = \begin{vmatrix} 11 & -5 & 12 \\ -5 & 19 & 0 \\ -1 & -1 & 0 \\ 11 & -5 & 12 \\ -5 & 19 & 0 \end{vmatrix} = 60 + 228 = 288$$

We calculate the mesh currents using Cramer's rule as

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{432}{192} = 2.25 \text{ A}, \qquad i_2 = \frac{\Delta_2}{\Delta} = \frac{144}{192} = 0.75 \text{ A},$$

$$i_3 = \frac{\Delta_3}{\Delta} = \frac{288}{192} = 1.5 \text{ A}$$

Thus, $I_o = i_1 - i_2 = 1.5$ A.

EXAMPLE 3 Find the branch currents of the network of **Fig. 9**.

Solution:

Steps 1: is as indicated in the circuit.

Step 3: Kirchhoff's voltage law is applied around each closed loop:

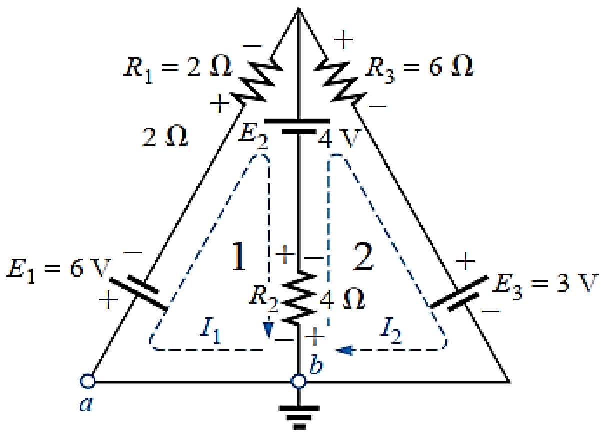

Fig. 9

loop 1: (clockwise from point *a*)

$+ E_1 + I_1R_1 + E_2 + V_2 = 0$

$6 + 2I_1 + 4 + 4(I_1 – I_2) = 0$

$2I_1 + 4I_1 – 4I_2 + 10 = 0$

$6I_1 – 4I_2 + 10 = 0$ ------------- (1)

loop 2: (clockwise from point *b*)

$+ V_2 – E_2 + V_3 + E_3 = 0$

$4(I_2 – I_1) – 4 + 6I_2 + 3 = 0$

$– 4I_1 + 4I_2 + 6I_2 – 1 = 0$

$– 4I_1 + 10I_2 – 1 = 0$------------(2)

Solving Equ. (1) & (2) we get :

$I_1 = 2.182$ A & $I_2 = 0.7727$ A