جامعة المستقبل
AL MUSTAQBAL UNIVERSITY

كلية العلوم
قسم الكيمياء الحياتية

FUNCTION AND THEIR GRAPHS

Mathematics

المرحلة الاولى

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المحاضرة الثالثة

Function and their graphs

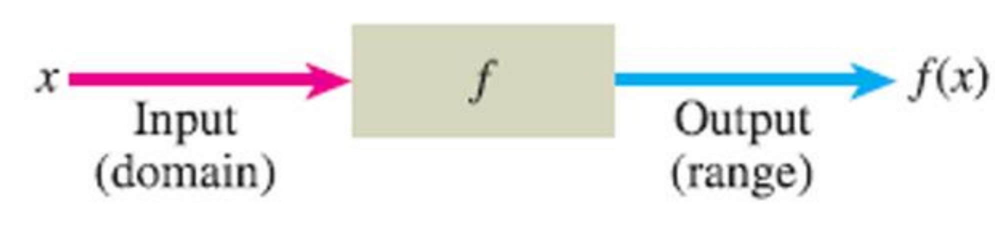
What is Function?

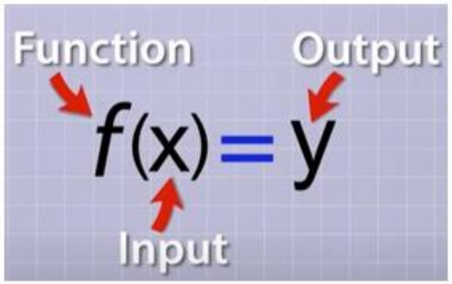
• Functions are a tool for describing the real world in mathematical terms.

• A function can be represented by an equation, a graph, a numerical table, or a verbal description.

Definition

A function f consists of a set of inputs, a set of outputs, and a rule for assigning each input to exactly one output. The set of inputs is called Domain of the function. The set of output is called the Range of the function.





Example

For the function $f(x) = 3x^2 + 2x - 1$, evaluate

a. $f(-2)$

b. $f(\sqrt{2})$

c. $f(a+h)$

Solution

Substitute the given value for x in the formula for $f(x)$.

a. $f(-2) = 3(-2)^2 + 2(-2) - 1 = 12 - 4 - 1 = 7$

b. $f(\sqrt{2}) = 3(\sqrt{2})^2 + 2\sqrt{2} - 1 = 6 + 2\sqrt{2} - 1 = 5 + 2\sqrt{2}$

c. $$\begin{aligned} f(a+h) = 3(a+h)^2 + 2(a+h) - 1 &= 3\left(a^2 + 2ah + h^2\right) + 2a + 2h - 1 \\ &= 3a^2 + 6ah + 3h^2 + 2a + 2h - 1 \end{aligned}$$

Example: If $f(x) = 2x^2 - 5x + 1$ and $h \neq 0$, evaluate $\dfrac{f(a+h) - f(a)}{h}$

Solution: We first evaluate $f(a+h)$ by replacing x by $a+h$ in the expression for $f(x)$:

$$\begin{aligned} f(a+h) &= 2(a+h)^2 - 5(a+h) + 1 \\ &= 2(a^2 + 2ah + h^2) - 5(a+h) + 1 \\ &= 2a^2 + 4ah + 2h^2 - 5a - 5h + 1 \end{aligned}$$

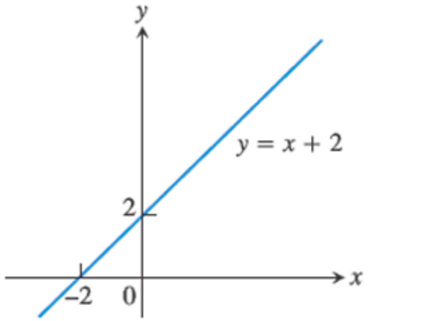
Then we substitute into the given expression and simplify:

$$\begin{aligned} \frac{f(a+h) - f(a)}{h} &= \frac{(2a^2 + 4ah + 2h^2 - 5a - 5h + 1) - (2a^2 - 5a + 1)}{h} \\ &= \frac{2a^2 + 4ah + 2h^2 - 5a - 5h + 1 - 2a^2 + 5a - 1}{h} \\ &= \frac{4ah + 2h^2 - 5h}{h} = 4a + 2h - 5 \end{aligned}$$

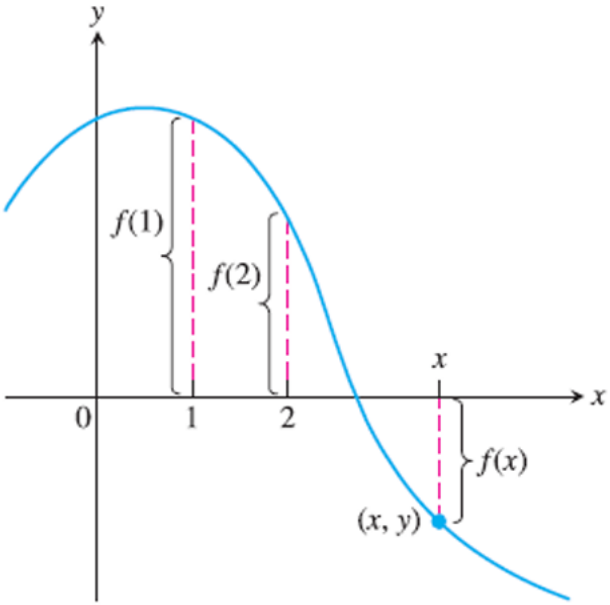
Graphs of Function:

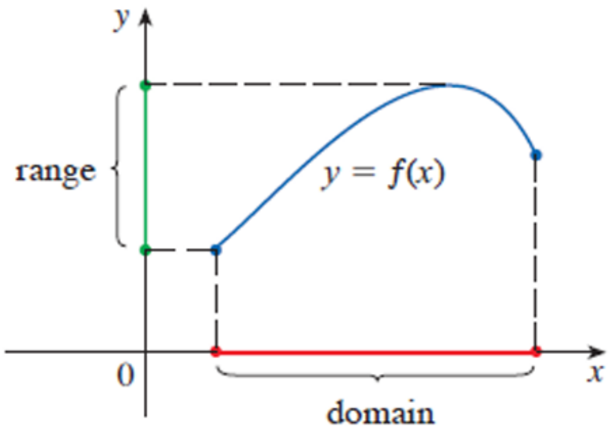
If f is a function with domain D, its graph consists of the points in the Cartesian plane whose coordinates are the input-output pairs for f. In set notation. the graph is:

$$\{(x, f(x)) \mid x \in D\}$$



The graph of $f(x) = x + 2$ is the set of points (x, y) for which y has the value $x + 2$.

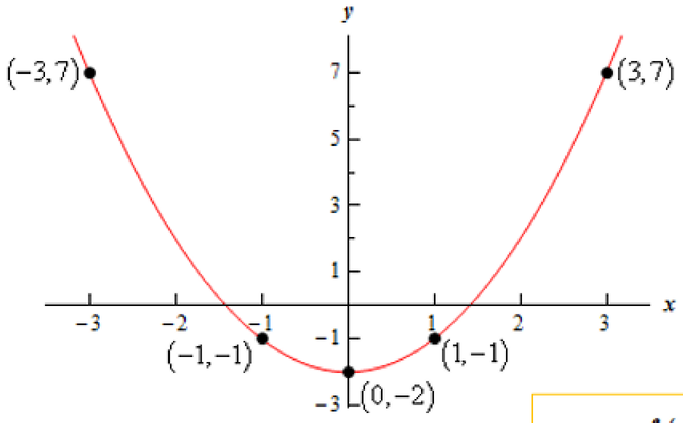




Example: Sketch the graph of the following function.

$f(x) = x^2 - 2$

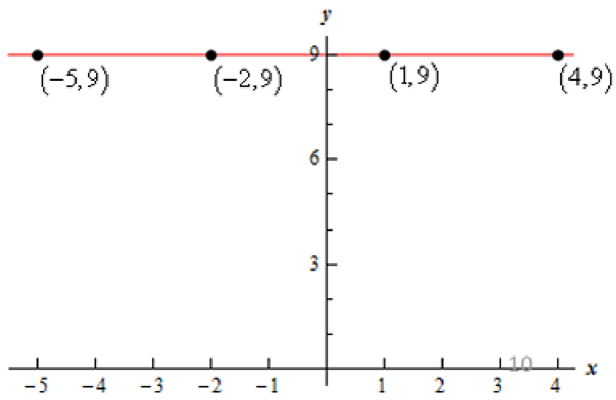
x	$f(x)$	(x, y)
-3	7	$(-3, 7)$
-1	-1	$(-1, -1)$
0	-2	$(0, -2)$
1	-1	$(1, -1)$
3	7	$(3, 7)$



Domain is $(-\infty, \infty)$ *and*
Range is $[-2, \infty)$

$f(x) = 9$

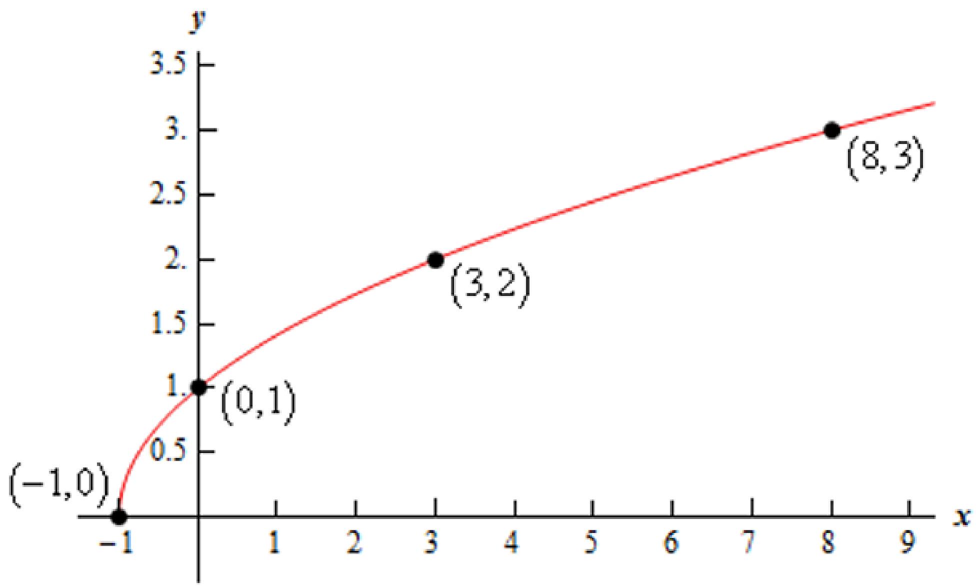
x	$f(x)$	(x, y)
-5	9	$(-5, 9)$
-2	9	$(-2, 9)$
1	9	$(1, 9)$
4	9	$(4, 9)$



Example: Sketch the graph of the following function.

$$f(x) = \sqrt{x+1}$$

x	$f(x)$	(x, y)
-1	0	$(-1, 0)$
0	1	$(0, 1)$
3	2	$(3, 2)$
8	3	$(8, 3)$



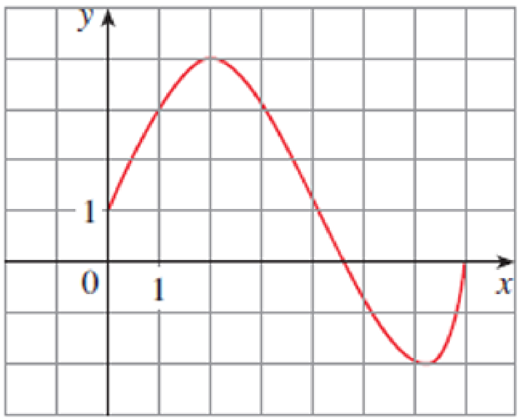
Domain is $[-1, \infty)$ *and*

Range is $[0, \infty)$

EXAMPLE // The graph of a function f is shown in Figure shown.

(a) Find the values of $f(1)$ and $f(5)$.

(b) What are the domain and range of f?



SOLUTION

(a) We see from Figure the value of f at 1 is $f(1) = 3$.

(the point on the graph that lies above $x = 1$ is 3 units above the x-axis.)

When $x = 5$, the graph lies about 0.7 units below the x-axis,

so we estimate that $f(5) \approx -0.7$.

(b) We see that $f(x)$ is defined when $0 \leqslant x \leqslant 7$, so the domain of f is the closed interval $[0, 7]$. Notice that f takes on all values from -2 to 4, so the range of f is

$$\{y \mid -2 \leqslant y \leqslant 4\} = [-2, 4]$$

Example: Find the Domains and Ranges of each of all of the following

(a) $y = x^3 \quad -5 \le x < 4$ (b) $y = x^4$ (c) $y = \dfrac{1}{(x-1)(x+2)} \quad 0 \le x \le 6$

Solution

(a) $y = x^3 \quad -5 \le x < 4$

domain $-5 \le x < 4$, range $-125 \le y < 64$

(b) $y = x^4$

domain $-\infty < x < \infty$, range $0 \le y < \infty$

(c) $y = \dfrac{1}{(x-1)(x+2)}, \quad 0 \le x \le 6$

domain $0 \le x < 1$ and $1 < x \le 6$,

range $-\infty < y \le -0.5,\ 0.25 \le y < \infty$

$$\boxed{\begin{gathered}(x-1)(x+2) \ge 0\\ x^2 + x - 2 \ge 0\end{gathered}}$$

H.W: Find the Domain and Range of each function.

1. $f(x) = 2x + 3$
2. $f(x) = x^2+4$
3. $f(x) = \frac{1}{x}$
4. $f(x) = \sqrt{x - 4}$
5. $f(x) = \sqrt{4 - x}$
6. $f(x) = \frac{1}{\sqrt{x-4}}$
7. $f(x) = x^2 + 3x + 1$
8. $f(x) = \frac{2x+1}{x^2+5x+6}$
9. $f(x) = \frac{2}{x^2+3}$
10. $f(x) = \sqrt{x^2 + 5x + 6}$
11. $f(x) = \frac{x^2+2x+3}{\sqrt{x+1}}$
12. $f(x) = \sqrt{1 - x^2}$