Department of biology

((GENERAL MATHEMATICS))

First Stage

Week 2- lecture 2

Solution of Inequality with Absolute Value

حل المتباينة ذات القيمة المطلقة

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The Absolute Value

The absolute value of a number x, denoted as |x|, is the distance of from zero on the number line, regardless of direction, it is defined by the formula:

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

|3|=3 , |0|=0 , |-5|=5

1. Properties of Absolute Values

Properties of absolute values

1. $|-a| = |a|$. A number and its negative have the same absolute value.
2. $|ab| = |a||b|$ and $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$. The absolute value of a product (or quotient) of two numbers is the product (or quotient) of their absolute values.
3. $|a \pm b| \leq |a| + |b|$ (the **triangle inequality**). The absolute value of a sum of or difference between numbers is less than or equal to the sum of their absolute values.

2. Equations and Inequalities Involving Absolute Values

$$\begin{aligned} |x| = D &\iff \text{either } x = -D \text{ or } x = D \\ |x| < D &\iff -D < x < D \\ |x| \leq D &\iff -D \leq x \leq D \\ |x| > D &\iff \text{either } x < -D \text{ or } x > D \end{aligned}$$

- Where D is the distance from the origin and (D>0)

More generally,

$$\begin{aligned} |x-a| = D &\iff \text{either } x = a - D \text{ or } x = a + D \\ |x-a| < D &\iff a - D < x < a + D \\ |x-a| \leq D &\iff a - D \leq x \leq a + D \\ |x-a| > D &\iff \text{either } x < a - D \text{ or } x > a + D \end{aligned}$$

Example 1: Solve the equation $|x| = 6$

Solution: $x = +6$ or $x = -6x$

So $6x = \pm 6$

Example 2: Solve the equation $|x-4| = 10$

Solution: 1. $x - 4 = 10 \rightarrow x = 14$

2. $x - 4 = -10 \rightarrow x = -6$

So $x = 14$ or $x = -6$

Example 3: Solve $|2x-3| = 7$

Solution: 1. $2x - 3 = 7 \rightarrow 2x = 10 \rightarrow x = 5$

2. $2x - 3 = -7 \rightarrow 2x = -4 \rightarrow x = -2$

So $x = 5$ or $x = -2$

Example 4: Solve $|x+2| \leq 5$

Solution:

- Rewrite as: $-5 \leq x+2 \leq 5$
- Subtract 2 from all parts: $-7 \leq x \leq 3$

The solution is $x \in [-7, 3]$

Example 5: Solve $|3x+1| \leq 8$

Solution

- $-8 \leq 3x+1 \leq 8$
- Subtract 1 from all sides: $-9 \leq 3x \leq 7$
- Divide by 3: $-3 \leq x \leq 7/3$

The solution is $x \in [-3, 7/3]$

Example 6: Solve

(a) $|2x+5| = 3$ (b) $|3x-2| \leq 1$

Solution:

(a) $|2x+5| = 3 \iff 2x+5 = \pm 3$. Thus, either $2x = -3-5 = -8$ or $2x = 3-5 = -2$. The solutions are $x = -4$ and $x = -1$.

(b) $|3x-2| \le 1 \iff -1 \le 3x-2 \le 1$. We solve this pair of inequalities:

$$\left\{\begin{array}{r} -1 \le 3x-2 \\ -1+2 \le 3x \\ 1/3 \le x \end{array}\right\} \quad \text{and} \quad \left\{\begin{array}{r} 3x-2 \le 1 \\ 3x \le 1+2 \\ x \le 1 \end{array}\right\}.$$

Thus the solutions lie in the interval $[1/3, 1]$.