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Mathematics and Biostatistics **First Stage**

LECTURE 1
Introduction to Mathematical Concepts

BY

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1. General Concepts

Mathematics is built on fundamental operations and principles used in solving equations, analyzing structures, and modeling problems.

Example 1: Solving an Equation

Solve $5x + 3 = 18$.

Solution:

$$5x + 3 = 18 \implies 5x = 18 - 3 \implies 5x = 15 \implies x = 3.$$

Example 2: Simplifying Expressions

Simplify $2(x + 3) + 4(x - 2)$.

Solution:

$$2x + 6 + 4x - 8 = 6x - 2.$$

Example 3: Solving an Equation with Fractions

Solve $\frac{x}{2} + \frac{x}{3} = 5$.

Solution:

1. Find the least common denominator (LCD) of 2 and 3, which is 6.
2. Multiply through by 6 to eliminate fractions:
$$6 \cdot \frac{x}{2} + 6 \cdot \frac{x}{3} = 6 \cdot 5 \implies 3x + 2x = 30.$$
3. Combine terms:
$$5x = 30 \implies x = 6.$$

Example 4: Word Problem with an Equation

A total of $50 is divided between two people. One person gets $10 more than the other. Find how much each person receives.

Solution:

Let x be the amount the first person gets. Then the second person gets $x + 10$.

$$x + (x + 10) = 50 \implies 2x + 10 = 50 \implies 2x = 40 \implies x = 20.$$

The first person gets $20, and the second person gets $30.

2. Coordinate System and Graph in the Plane

The coordinate plane consists of two axes: the x-axis (horizontal) and the y-axis (vertical). Points are located using ordered pairs (x, y).

Example 1: Plotting Points

Plot $(-2, 4)$, $(0, 0)$, and $(3, -5)$:

- $(-2, 4)$: Move 2 units left and 4 units up.
- $(0, 0)$: Origin, no movement needed.
- $(3, -5)$: Move 3 units right and 5 units down.

Example 2: Finding Distance Between Two Points

The distance d between two points (x_1, y_1) and (x_2, y_2) is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Find the distance between $(1, 2)$ and $(4, 6)$:

$$d = \sqrt{(4 - 1)^2 + (6 - 2)^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5.$$

Example 3: Midpoint Formula

The midpoint of a line segment joining (x_1, y_1) and (x_2, y_2) is:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Find the midpoint of the segment joining $(1, 4)$ and $(-3, 6)$:

$$M = \left(\frac{1 + (-3)}{2}, \frac{4 + 6}{2}\right) = \left(\frac{-2}{2}, \frac{10}{2}\right) = (-1, 5).$$

Example 4: Equation of a Circle

The equation of a circle with center (h, k) and radius r is:

$$(x - h)^2 + (y - k)^2 = r^2.$$

Find the equation of a circle with center $(2, -3)$ and radius 5:

$$(x - 2)^2 + (y + 3)^2 = 25.$$

3. Inequalities

Inequalities describe relationships between numbers or expressions. Solutions are often intervals or sets of numbers.

Example 1: Solving a Linear Inequality

Solve $3x + 2 \leq 8$.

Solution:

$$3x + 2 \leq 8 \implies 3x \leq 6 \implies x \leq 2.$$

The solution is $x \leq 2$.

Example 2: Graphing Inequalities on a Number Line

Graph the inequality $x > -1$:

- Open circle at -1 (not included).
- Shade all points to the right of -1.

Example 3: Systems of Inequalities

Solve $x + y \leq 4$ and $x - y > 1$.

Graph each inequality in the coordinate plane and find the overlap.

Example 4: Compound Inequalities

Solve $2 < 3x - 4 \leq 8$.

Solution:

Break it into two inequalities:

1. $2 < 3x - 4$:

$$6 < 3x \implies x > 2.$$

2. $3x - 4 \leq 8$:

$$3x \leq 12 \implies x \leq 4.$$

Combine the results: $2 < x \leq 4$.

The solution is $x \in (2, 4]$.

Example 5: Graphing Systems of Linear Inequalities

Graph $y > 2x - 1$ and $y \leq -x + 3$.

- For $y > 2x - 1$, graph the line $y = 2x - 1$ with a dashed line and shade above.
- For $y \leq -x + 3$, graph the line $y = -x + 3$ with a solid line and shade below.

  The solution is the overlapping shaded region.

4. Absolute Value or Magnitude

Absolute value measures the distance from zero, irrespective of direction.

Example 1: Solving Absolute Value Equations

Solve $|3x - 4| = 7$:

Solution:

$$3x - 4 = 7 \quad \text{or} \quad 3x - 4 = -7.$$

$$3x = 11 \implies x = \frac{11}{3}, \quad 3x = -3 \implies x = -1.$$

The solutions are $x = \frac{11}{3}$ and $x = -1$.

Example 2: Solving Absolute Value Inequalities

Solve $|x+2| \leq 5$:

Solution:

$$-5 \leq x+2 \leq 5 \implies -7 \leq x \leq 3.$$

The solution is $-7 \leq x \leq 3$.

Example 3: Absolute Value Word Problem

A car travels in two directions from its starting point, 7 miles north and 3 miles south. What is its net displacement and total distance traveled?

Solution:

- **Net Displacement:** $|7 - 3| = 4\,\text{miles}$ (north).
- **Total Distance:** $7 + 3 = 10\,\text{miles}$.

Example 4: Absolute Value Inequalities

Solve $|2x+1| > 5$.

Solution:

Split into two cases:

$$2x + 1 > 5 \quad \text{or} \quad 2x + 1 < -5.$$

Solve each:

$$2x > 4 \implies x > 2 \quad \text{and} \quad 2x < -6 \implies x < -3.$$

The solution is $x \in (-\infty, -3) \cup (2, \infty)$.

5. Functions and Their Graphs

A **function** is a relationship where each input x maps to exactly one output y.

Example 1: Linear Function

Graph $f(x) = 2x + 1$:

- Table of values:

$$x = -1, \quad f(-1) = 2(-1) + 1 = -1.$$

$$x = 0, \quad f(0) = 2(0) + 1 = 1.$$

$$x = 1, \quad f(1) = 2(1) + 1 = 3.$$

Plot points $(-1, -1)$, $(0, 1)$, $(1, 3)$, and connect them.

Example 2: Quadratic Function

Graph $f(x) = x^2 - 4x + 3$:

- Factorize: $f(x) = (x-1)(x-3)$.
- Roots: $x = 1, 3$.
- Vertex: $x = \frac{-b}{2a} = \frac{4}{2} = 2$, $f(2) = 2^2 - 4(2) + 3 = -1$.

  Plot points and sketch the parabola.

Example 3: Piecewise Function

A piecewise function is defined as:

$$f(x) = \begin{cases} x^2, & \text{if } x \geq 0, \\ -x, & \text{if } x < 0. \end{cases}$$

Sketch the graph:

- For $x \geq 0$, plot $y = x^2$ (a parabola).
- For $x < 0$, plot $y = -x$ (a line).

Example 4: Exponential Function

Graph $f(x) = 2^x$:

- $f(-2) = \frac{1}{4}$, $f(-1) = \frac{1}{2}$, $f(0) = 1$, $f(1) = 2$, $f(2) = 4$.
  The graph is an increasing curve passing through $(0, 1)$ and approaching 0 as $x \to -\infty$.

6. Displacement Function

Displacement functions are often polynomial functions describing motion over time.

Example: Velocity from Displacement

Given $s(t) = t^3 - 6t^2 + 9t$:

- Displacement at $t = 2$:

$$s(2) = 2^3 - 6(2^2) + 9(2) = 8 - 24 + 18 = 2.$$

- Velocity is the derivative $v(t) = s'(t) = 3t^2 - 12t + 9$. At $t = 2$:

$$v(2) = 3(2^2) - 12(2) + 9 = 12 - 24 + 9 = -3.$$

Example 2: Acceleration from Velocity

Given $v(t) = t^2 - 4t + 3$, find the acceleration $a(t)$:

$$a(t) = v'(t) = 2t - 4.$$

At $t = 3$:

$$a(3) = 2(3) - 4 = 6 - 4 = 2\,\mathrm{m/s^2}.$$

7. Slope

The slope measures the steepness and direction of a line.

Example: Parallel and Perpendicular Slopes

- Parallel lines have equal slopes: $m_1 = m_2$.
- Perpendicular lines have slopes that are negative reciprocals: $m_1 \cdot m_2 = -1$.

Find the slope of the line perpendicular to $y = 3x + 5$:

The slope of the given line is $m = 3$. A perpendicular slope is $m = -\frac{1}{3}$.

Example 3: Horizontal and Vertical Lines

- A horizontal line has slope $m = 0$ and equation $y = c$.
- A vertical line has undefined slope and equation $x = c$.

Find the equations of the lines through $(3, 5)$:

- Horizontal: $y = 5$.
- Vertical: $x = 3$.

8. Equation of a Line

Lines can be expressed in various forms:

- **Slope-intercept form:** $y = mx + b$.
- **Point-slope form:** $y - y_1 = m(x - x_1)$.
- **Standard form:** $Ax + By = C$.

Example: Finding an Equation

Find the equation of a line passing through $(2, 3)$ with slope $m = -2$:

Using point-slope form:

$$y - 3 = -2(x - 2) \implies y = -2x + 4 + 3 \implies y = -2x + 7.$$

Example: Converting Between Forms

Convert $2x + 3y = 6$ to slope-intercept form:

$$3y = -2x + 6 \implies y = -\frac{2}{3}x + 2.$$

Example 4: Finding Intersection of Two Lines

Find the intersection of $y = 2x - 3$ and $y = -x + 4$:

Solution:

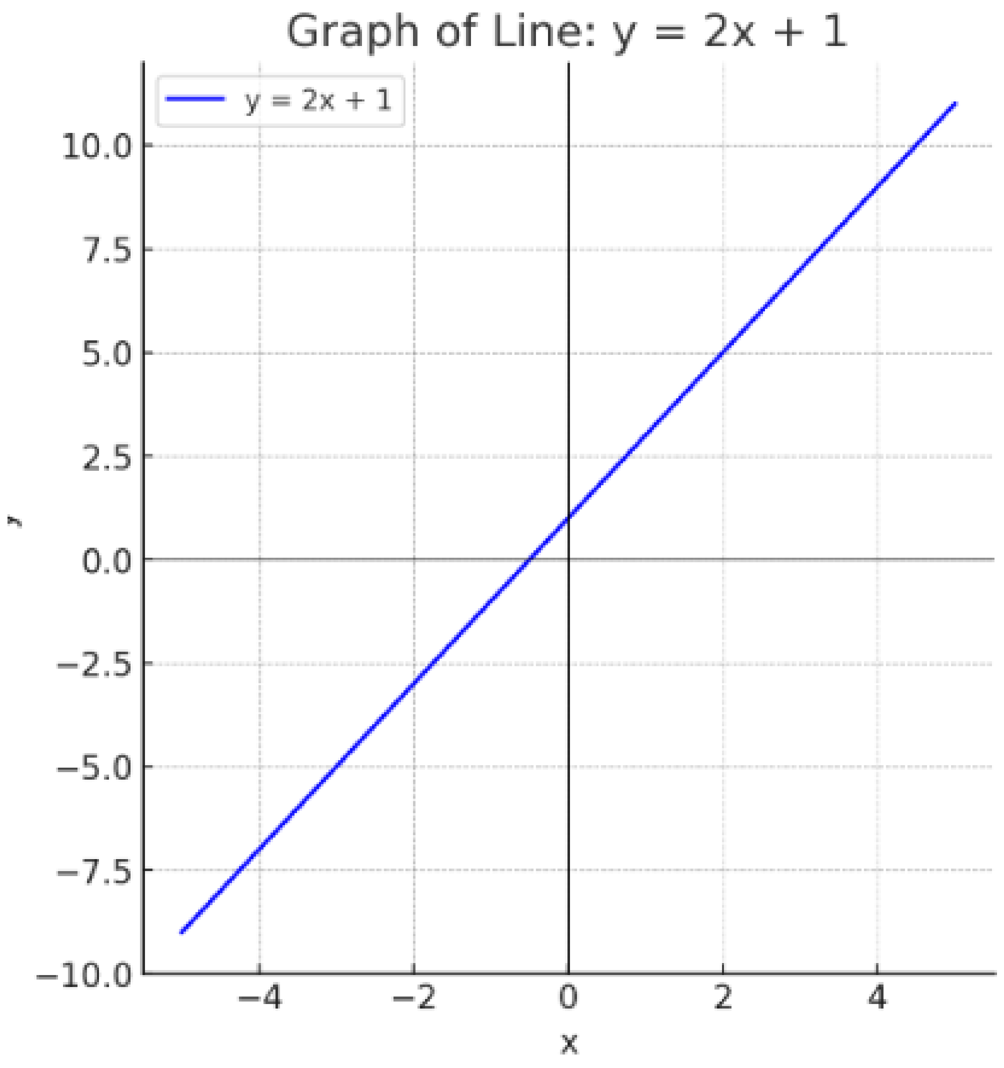
Set the equations equal:

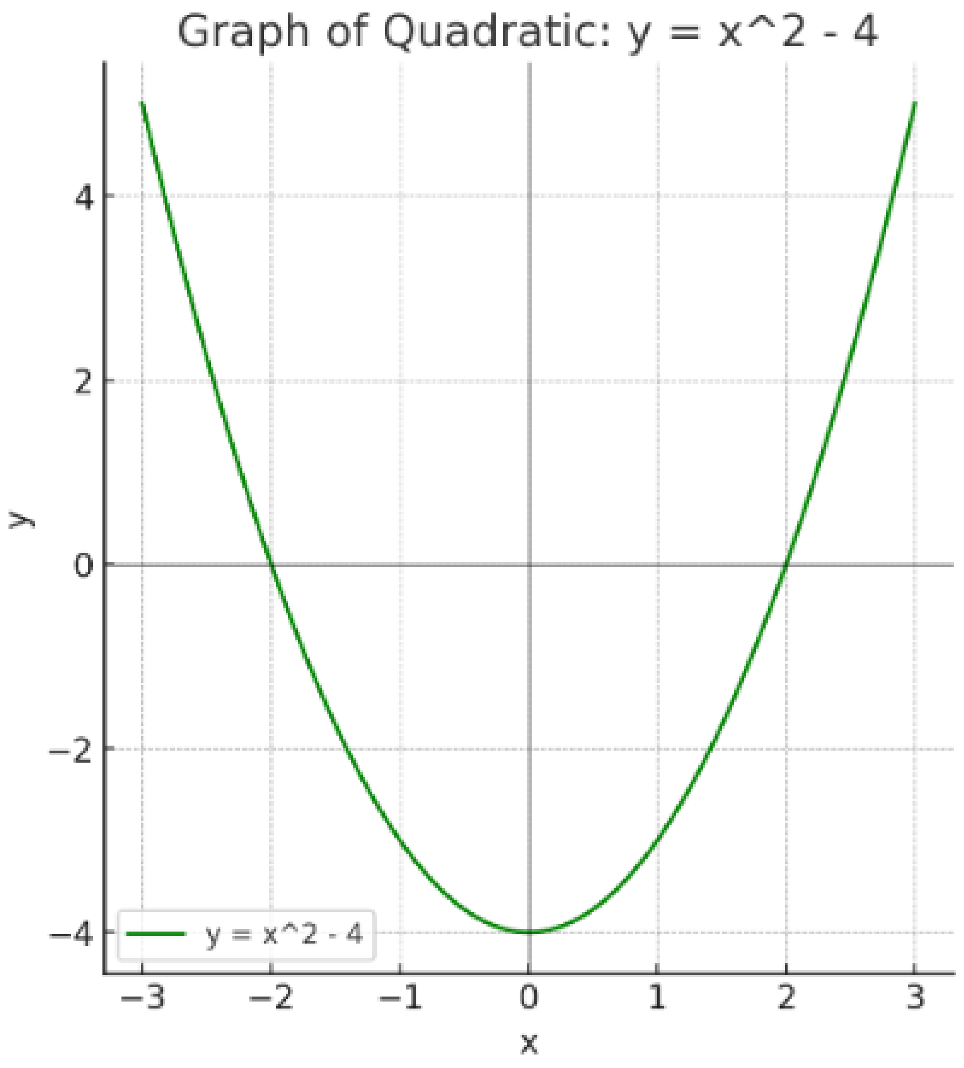
$$2x - 3 = -x + 4 \implies 3x = 7 \implies x = \frac{7}{3}.$$

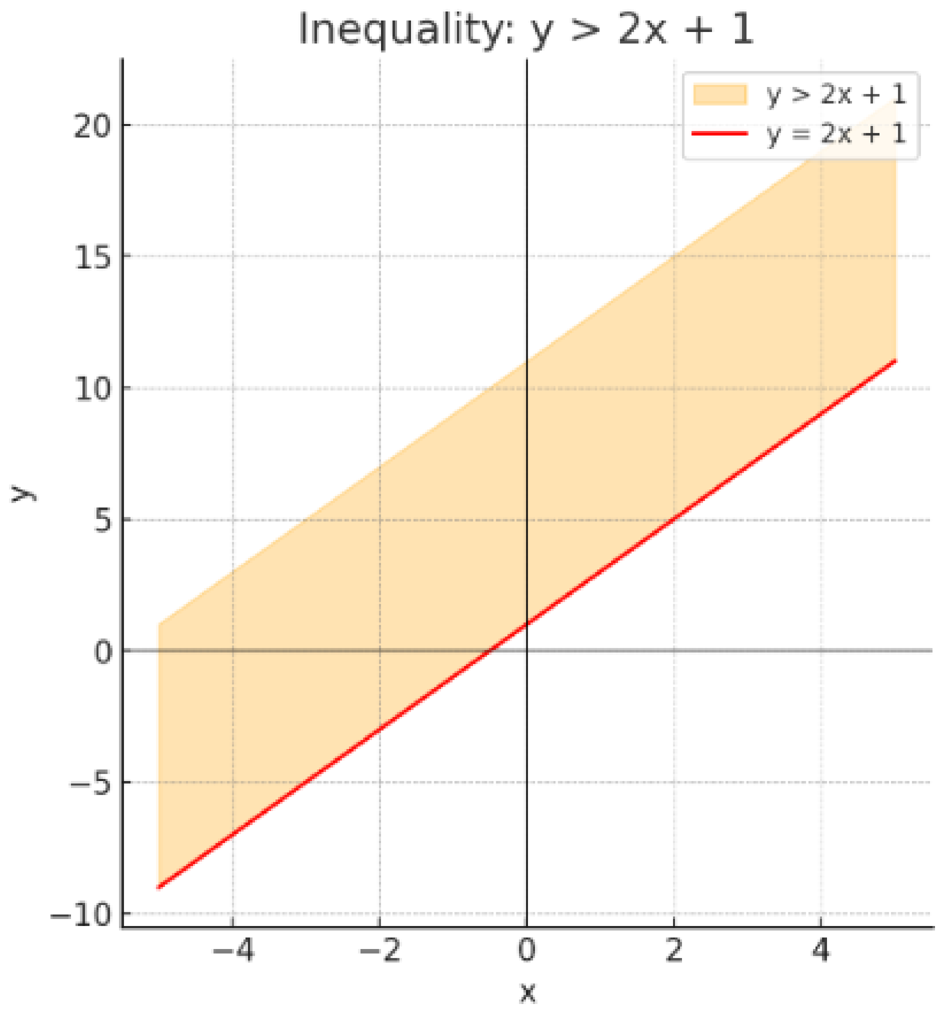
Substitute $x = \frac{7}{3}$ into $y = 2x - 3$:

$$y = 2\left(\frac{7}{3}\right) - 3 = \frac{14}{3} - 3 = \frac{5}{3}.$$

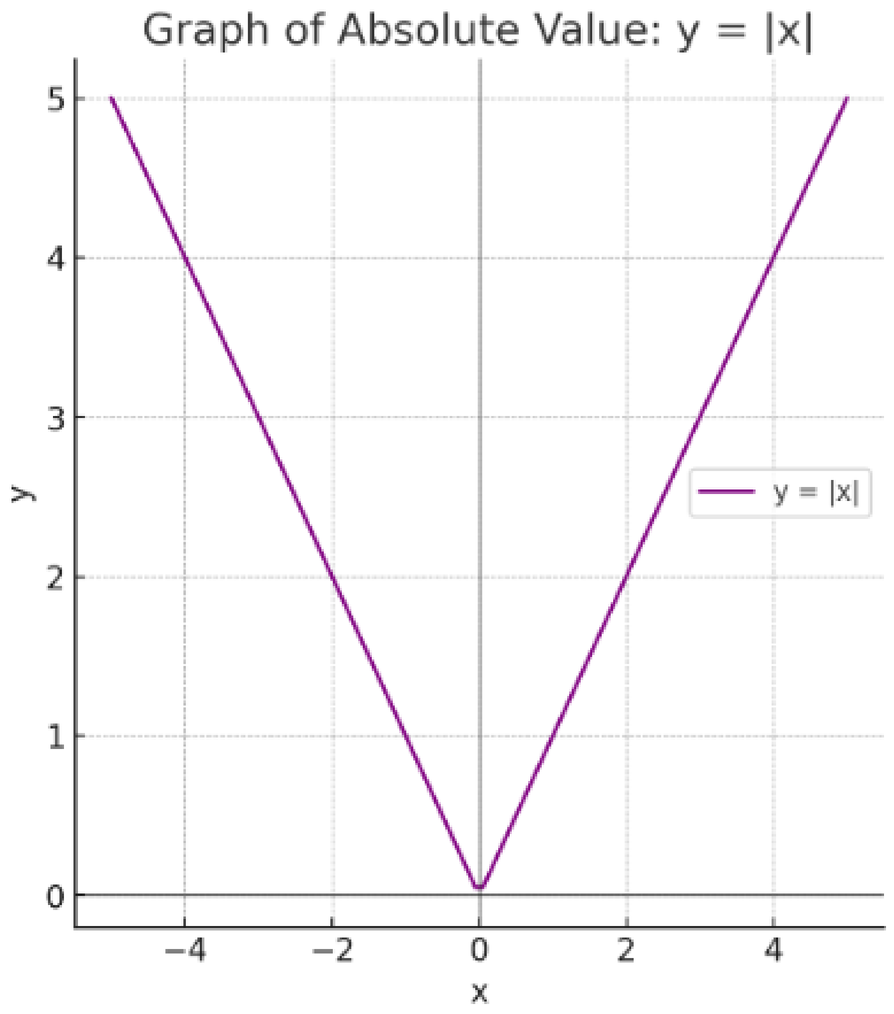
The intersection point is $\left(\frac{7}{3}, \frac{5}{3}\right)$.


Graph of Line: y = 2x + 1
y = 2x + 1
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
−4
−2
0
2
4
x

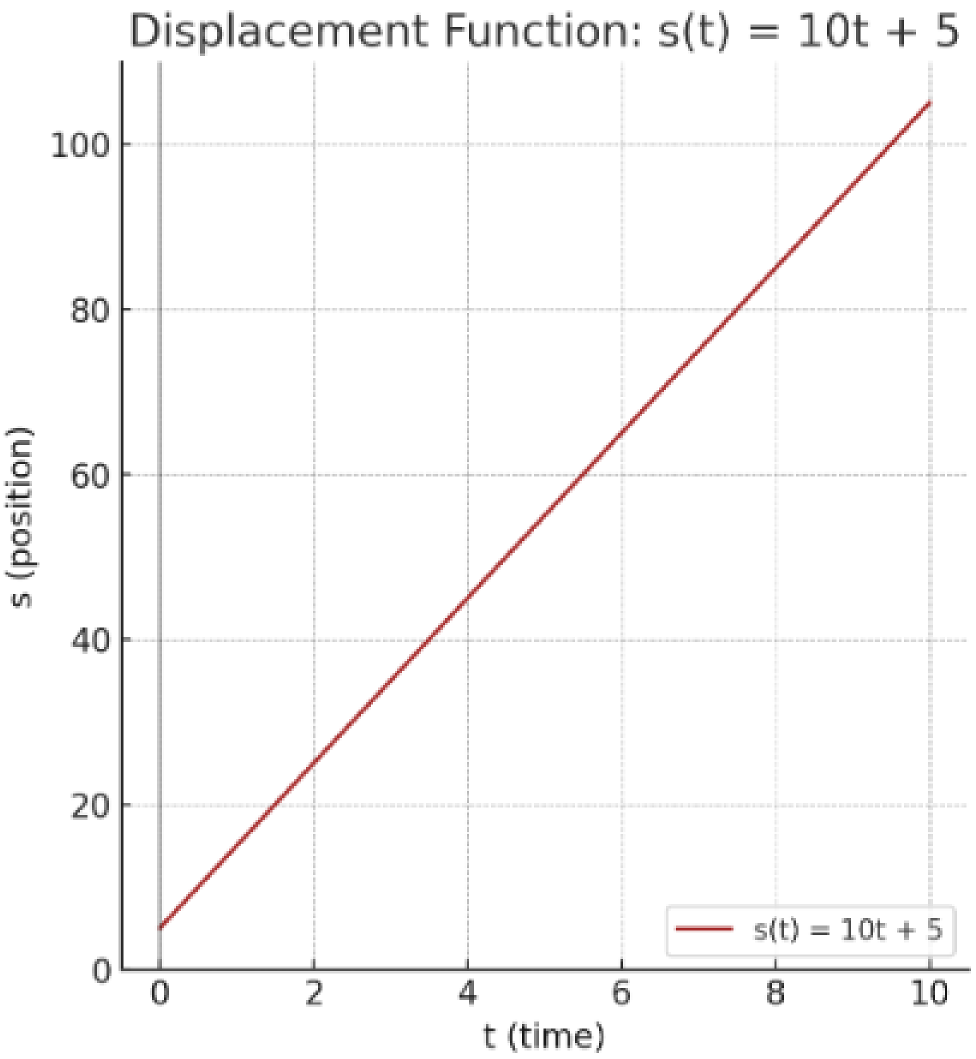

Graph of Quadratic: y = x^2 - 4
4
2
0
−2
−4
y
y = x^2 - 4
−3
−2
−1
0
1
2
3
x



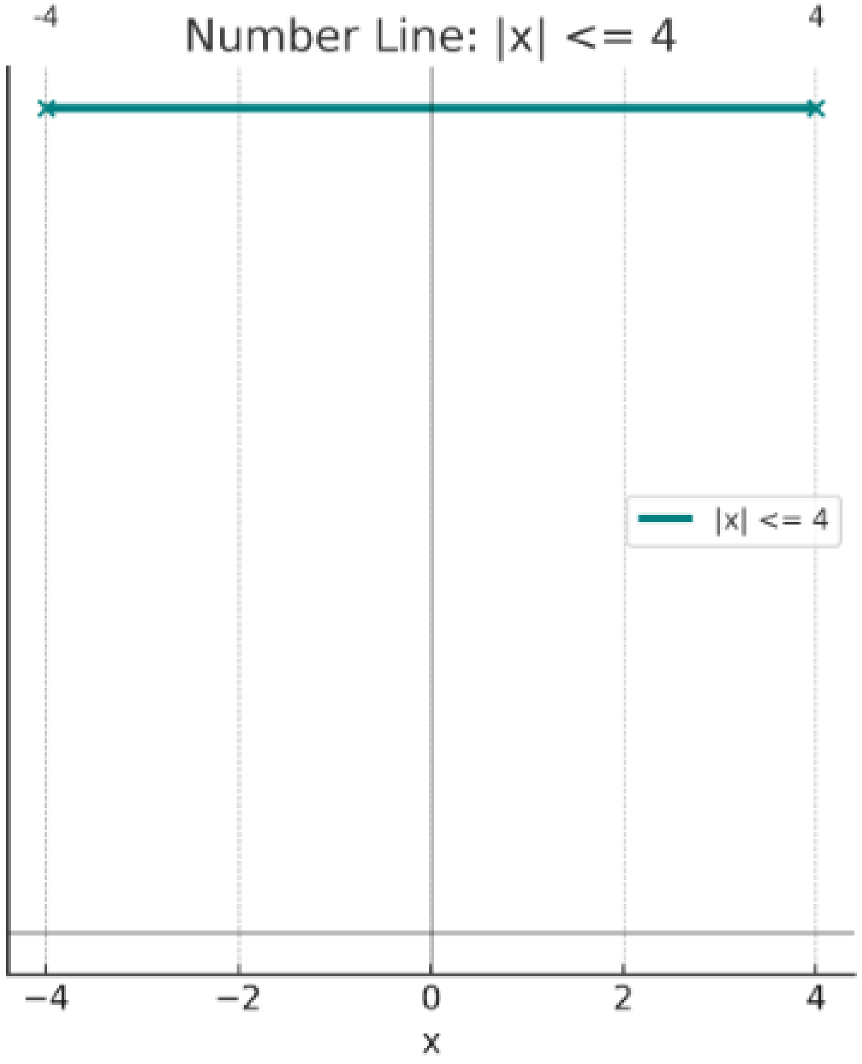
Inequality: y > 2x + 1
y > 2x + 1
y = 2x + 1
20
15
10
5
0
−5
−10
y
−4
−2
0
2
4
x



Graph of Absolute Value: y = |x|
y = |x|
5
4
3
2
1
0
y
−4
−2
0
2
4
x



Displacement Function: s(t) = 10t + 5
s (position)
t (time)
0
20
40
60
80
100
0
2
4
6
8
10
s(t) = 10t + 5



Number Line: |x| <= 4
-4
4
|x| <= 4
−4
−2
0
2
4
x

- Thanks for lessening ..

Any questions?