Al-Mustaqbal University
Department (Biomedical Engineering)
Class (First Stage)
Subject (Physics)
Lecturer (Asst.lec.Hiba Diaa Alrubaie)
1st/term – Lect. (Units, Trigonometry and vectors.)

SI Units

The International System of Units (SI) is the standard used worldwide:

•**Length**: meter (m)

•**Mass**: kilogram (kg)

•**Time**: second (s)

•**Electric Current**: ampere (A)

•**Temperature**: kelvin (K)

Derived Units

These are combinations of SI units for specific quantities:

•**Velocity**: meters per second (m/s)

•**Force**: newton ($N = kg \cdot m/s^2$)

•**Energy**: joule ($J = N \cdot m$)

•**Pressure**: pascal ($Pa = N/m^2$)

Unit Conversion

For example:

1 km=1000 m,1 hour=3600 seconds.

Trigonometry

A trigonometry formula is a formula that is used to represent relationships between the parts of a triangle including the side lengths, angles and the area.

The definitions of the trigonometric functions and the inverse trigonometric functions, as well as the Pythagorean theorem, can be applied to any right triangle, regardless of whether its sides correspond to x-and y coordinates. These results from trigonometry are useful in converting from rectangular coor-dinates to polar coordinates, or vice versa,

Basic Trigonometric Identities

- Quotient identities:

$$\tan(A) = \frac{\sin(A)}{\cos(A)} \qquad \cot(A) = \frac{\cos(A)}{\sin(A)}$$

Even/Odd id entities:

$$\cos(-A) = \cos(A)$$
$$\sec(-A) = \sec(A)$$

Even functions

$$\sin(-A) = -\sin(A)$$
$$\csc(-A) = -\csc(A)$$

Odd functions

$$\tan(-A) = -\tan(A)$$
$$\cot(-A) = -\cot(A)$$

Odd functions

- **Reciprocal Identities:**

$$\csc(A) = \frac{1}{\sin(A)} \qquad \sec(A) = \frac{1}{\cos(A)} \qquad \cot(A) = \frac{1}{\tan(A)}$$

$$\sin(A) = \frac{1}{\csc(A)} \qquad \cos(A) = \frac{1}{\sec(A)} \qquad \tan(A) = \frac{1}{\cot(A)}$$

- **Pythagorean Identities:**

$$\sin^2(A) + \cos^2(A) = 1$$

$$\tan^2(A) + 1 = \sec^2(A) \qquad 1 + \cot^2(A) = \csc^2(A)$$

Radius of the circle is 1.
x = cos(θ) $-1 \le \cos(\theta) \le 1$
y = sin(θ) $-1 \le \sin(\theta) \le 1$
Pythagorean Theorem: $x^2 + y^2 = 1$
This gives the identity: $\cos^2(\theta) + \sin^2(\theta) = 1$
Zeros of sin(θ) are $n\pi$ where n is an integer.
Zeros of cos(θ) are $\frac{\pi}{2} + n\pi$ where n is an integer.

- **Trigonometric Identities Summation & Difference Formulas**

$$\sin(A \pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$$

$$\cos(A \pm B) = \cos(A)\cos(B) \mp \sin(A)\sin(B)$$

$$\tan(A \pm B) = \frac{\tan(A) \pm \tan(B)}{1 \mp \tan(A)\tan(B)}$$

- **Trigonometric Identities Double Angle Formulas**

$$\sin(2A) = 2\sin(A)\cos(A)$$

$$\cos(2A) = \cos^2(A) - \sin^2(A) = 1 - 2\sin^2(A) = 2\cos^2(A) - 1$$

$$\tan(2A) = \frac{2\tan(A)}{1 - \tan^2(A)}$$

- Trigonometric Identities Half Angle Formulas

$$\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos(A)}{2}}$$

$$\cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+\cos(A)}{2}}$$

$$\tan\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos(A)}{1+\cos(A)}}$$

The quadrant of $\frac{A}{2}$ determines the sign.

Scalars and Vectors:

A quantity which is completely specified by a certain number associated with a suitable unit without any mention of direction in space is known as scalar. Examples of scalar are time, mass, length, volume, density, temperature, energy, distance, speed etc. The number describing the quantity of a particular scalar is known as its magnitude. The scalars are added subtracted, multiplied and divided by the usual arithmetical laws. A quantity which is completely described only when both their magnitude and direction are specified is known as vector. Examples of vector are force, velocity, acceleration, displacement, torque, momentum, gravitational force, electric and magnetic intensities etc. A vector is represented by a Roman letter in bold face and its magnitude, by the same letter in italics. Thus V means vector and V is magnitude.

Types of Vectors:

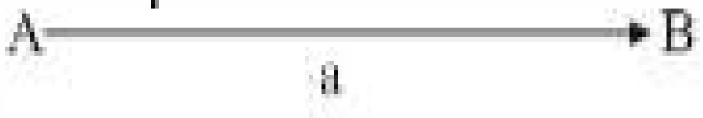
1. Unit Vector:

A vector whose magnitude is unity i.e., 1 and direction along the given vector is called a unit Vector. If a is a vector then a unit vector in the direction of a, denoted by $\hat{a}$ (read as a cap), is given as,

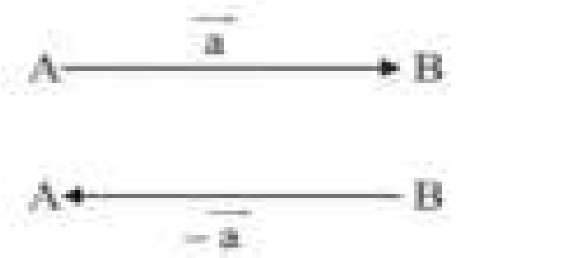
$$\hat{a} = \frac{\overline{a}}{|a|} \quad \text{or} \quad \overline{a} = |a|\,\hat{a}$$

2. Free Vector:

A vector whose position is not fixed in space. Thus, the line of action of a free vector can be shifted parallel to itself. Displacement is an example of a free vector as shown in figure 1:



- **Negative of a Vector**: The vector which has the same magnitude as the vector a but opposite in direction to a is called the negative to a . It is represented by



$-\bar{a}$. Thus of $\overline{AB} = -\bar{a}$ then $\overline{BA} = -\bar{a}$

Null or Zero Vector: It is a vector whose magnitude is zero. We denote the null vector by O. The direction of a zero vector is arbitrary. The vectors other than zero vectors are proper vectors or non-zero vectors.

Equal Vectors: Two vectors a and b are said tobe equal if they have the same magnitude and direction. If a and b are equal vectors then

a = b

Addition and Subtraction of Vectors:

1. Addition of Vectors:

Suppose $\vec{a}$ and $\vec{b}$ are any two vectors. Choose point A so that $\vec{a} = \overrightarrow{OA}$ and choose point C so that $\vec{b} = \overrightarrow{AC}$. The sum, $\vec{a} + \vec{b}$ of $\vec{a}$ and $\vec{b}$ is the vector is the vector $\overrightarrow{OC}$. Thus the sum of two vectors $\vec{a}$ and $\vec{b}$ is performed by the Triangle Law of addition.

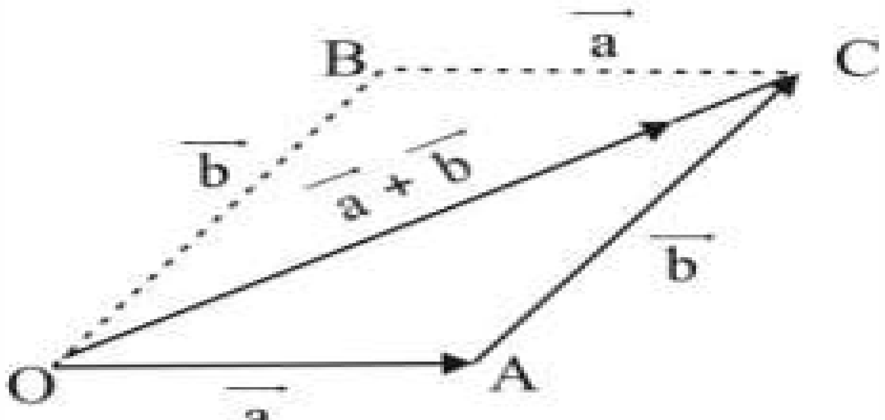


Fig. 6

2. Subtraction of Vectors:

If a vector $\vec{b}$ is to be subtracted from a vector $\vec{a}$, the difference vector $\vec{a} - \vec{b}$ can be obtained by adding vectors $\vec{a}$ and $-\vec{b}$.

The vector $-\vec{b}$ is a vector which is equal and parallel to that of vector but its arrow-head points in opposite direction. Now the vectors $\vec{a}$ and $-\vec{b}$ can be added by the head-to-tail rule. Thus the line AC represents, in magnitude and direction, the vector $\vec{a} - \vec{b}$.

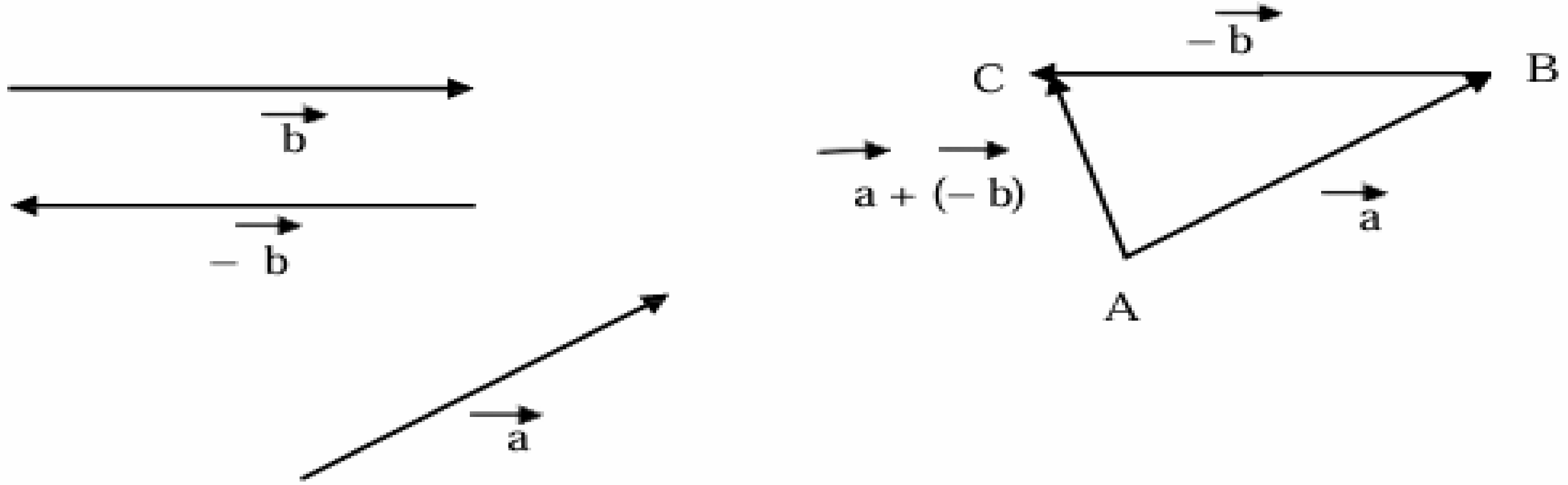


Fig . 7

Properties of Vector Addition:

i. **Vector addition is commutative**

i.e., a + b = a + b where $\vec{a}$ and b are any two vectors.

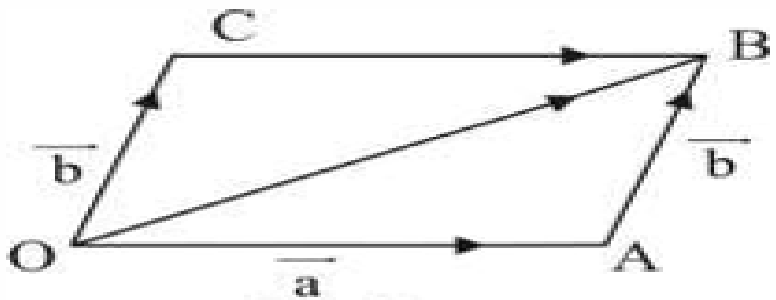


Fig. 8

(ii) Vectors Addition is Associative:

i.e. $$\left(\overline{a} + \overline{b}\right) + \overline{c} = \overline{a} + \left(\overline{b} + \overline{c}\right)$$

where $\overline{a}$, $\overline{b}$ and $\overline{c}$ are any three vectors.

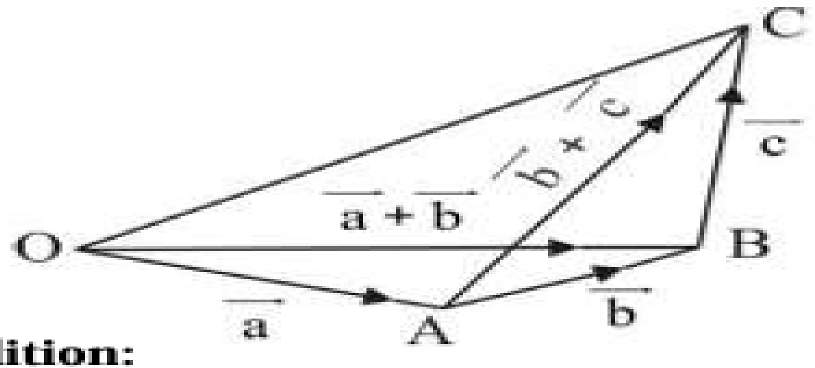


Fig.9

(iii) $\overline{O}$ is the identity in vectors addition:

For every vector $\overline{a}$

$\overline{a} + \overline{O} = \overline{a}$

Where $\overline{O}$ is the zero vector.

Remarks: Non-parallel vectors are not added or subtracted by the ordinary algebraic Laws because their resultant depends upon their directions as well.

Multiplication of a Vector by a Scalar:

If $\overline{a}$ is any vectors and K is a scalar, then $K\overline{a} = \overline{a}K$ is a vector with magnitude $|K|.|a|$ i.e., $|K|$ times the magnitude of $\overline{a}$ and whose direction is that of vector $\overline{a}$ or opposite to vector $\overline{a}$ according as K is positive or negative resp. In particular $\overline{a}$ and $-\overline{a}$ are opposite vectors.

Properties of Multiplication of Vectors by Scalars:

1. The scalar multiplication of a vectors satisfies

$$m(n\overline{a}) = (mn)\overline{a} = n(m\overline{a})$$

2. The scalar multiplication of a vector satisfies the distributive laws

i.e., $$(m+n)\overline{a} = m\overline{a} + n\overline{a}$$

and $$m(\overline{a} + \overline{b}) = m\overline{a} + m\overline{b}$$

Where m and n are scalars and $\overline{a}$ and $\overline{b}$ are vectors.

- The Unit Vectors i, j, k (orthogonal system of unit Vectors): Let us consider three mutually perpendicular straight lines OX, OY and OZ. These three mutually perpendicular lines determine uniquely the position of a point. Hence these lines may be taken as the co-ordinates axes with O as the origin. We shall use i, j and k to denote the Unit Vectors along OX, OY and OX respectively.

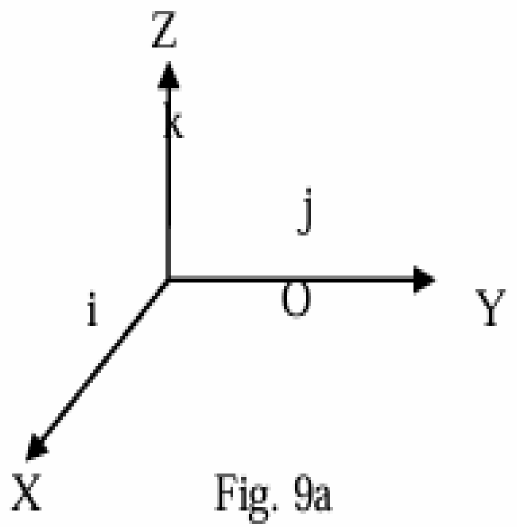


Fig. 9a

Representation of a Vector in the Form of Unit Vectors i, j and k.

Let us consider a vector $\vec{r} = \overrightarrow{OP}$ as shown in fig. 11. Then x i, y j and z k are vectors directed along the axes,

$$\overline{OQ} = \overline{OA} + \overline{AQ} = \overline{OA} + \overline{OB} \qquad \text{because}$$

and $\overline{OQ} = xi + yi$

Because $\overline{QP} = zk$

$\overline{OP} = \overline{OQ} + \overline{QP}$

and $\overline{r} = \overline{OP} = xi + yj + zk$

Here the real numbers x, y and z are the components of Vector r or the co-ordinates of point P in the direction of OX, OY and OZ respectively. The vectors xi, yj and zk are called the resolved parts of the vector r in the direction of the Unit vectors i, j and k respectively.

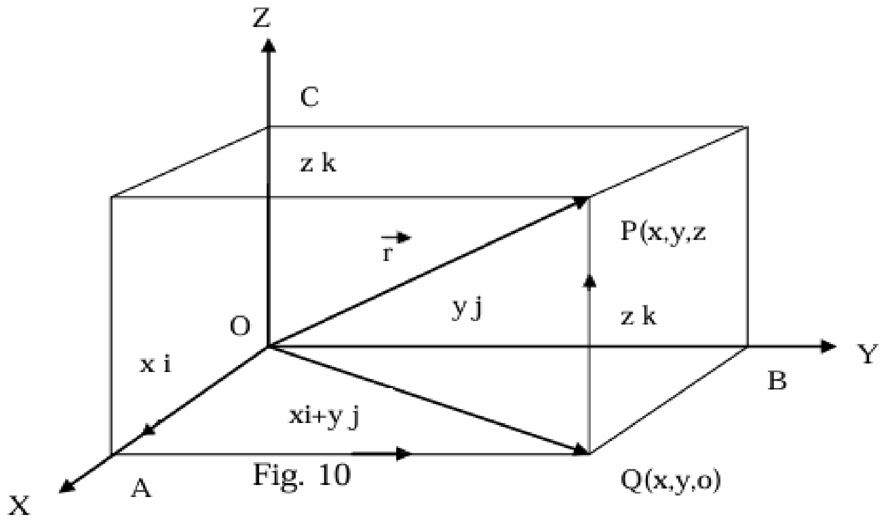


Fig. 10

Components of a Vector when the Tail is not at the Origin:

Consider a vector $\vec{r} = \vec{PQ}$ whose tail is at the point $P(x_1, y_1, z_1)$ and the head at the point $Q(x_2, y_2, z_2)$. Draw perpendiculars PP′ and QQ′ on x-aixs.

$P'Q' = x_2 - x_1 = $ x-component of $\vec{r}$

Now draw perpendiculars PP^o and QQ^o on y-axis.

Then $P^oQ^o = y_2 - y_1 = $ y-component of $\vec{r}$

Similarly $z_2 - z_1 = $ z-component of $\vec{r}$

Hence the vector $\vec{r}$ can be written as,

$\vec{r} = \vec{PQ} = (x_2 - x_1)i + (y_2 - y_1)j + (z_2 - z_1)k$

Or, $\vec{r} = \vec{PQ} = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$

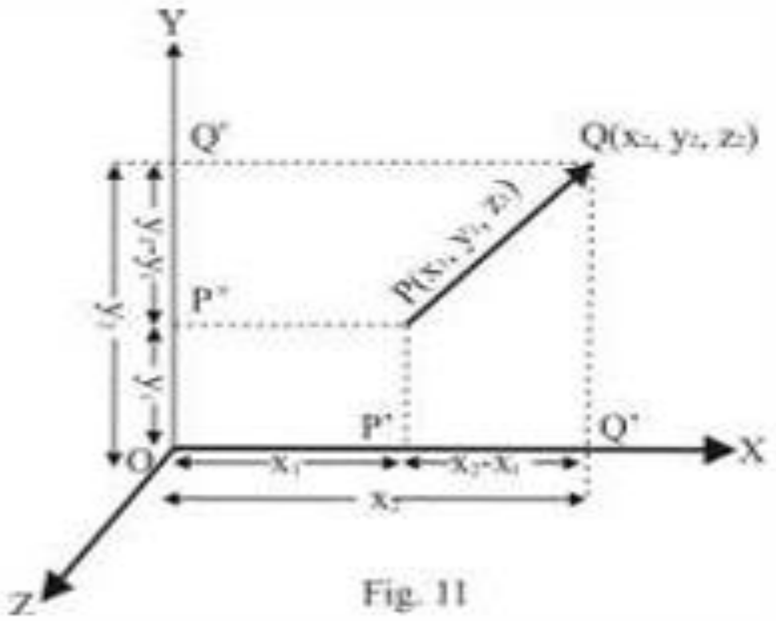


Fig. 11

Magnitude or Modulus of a Vector:

Suppose x, y and z are the magnitude of the vectors OA , OBand OC as shown in fig. 10.

In the right triangle OAQ, by Pythagorean Theorem

$$OQ^2 = x^2 + y^2$$

Also in the right triangle OQP, we have

$$OP^2 = OQ^2 + QP^2$$

$$OP^2 = x^2 + y^2 + z^2$$

Or $$|\bar{r}| = |OP| = \sqrt{x^2 + y^2 + z^2}$$

Thus if $\dot{r} = P\dot{Q} = xi + yj + zk$

Then , its magnitude is

$$|\bar{r}| = \sqrt{x^2 + y^2 + z^2}$$

If $r = (x_2 - x_1)i + (y_2 - y_1)j + (z_2 - z_1)k$

Then $|\bar{r}| = \sqrt{(x_2 - x_1)2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Example 1

Find the magnitude of the vector

$$|\bar{u}| = \frac{3}{5}i - \frac{2}{5}j + \frac{2\sqrt{3}}{5}k$$

Solution:

$$|\bar{u}| = \sqrt{\left(\frac{3}{5}\right)^2 + \left(-\frac{2}{5}\right)^2 + \left(\frac{2\sqrt{3}}{5}\right)^2}$$

$$= \sqrt{\frac{9}{25} + \frac{4}{25} + \frac{12}{25}} = \sqrt{\frac{25}{25}}$$

$$|\bar{u}| = 1$$