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Mathematics and Biostatistics **First Stage**

LECTURE 3

THE LIMITS AND PROPERTIES OF LIMIT

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❖ Definition of the concept of limits :

Limit is one of the basic concepts in mathematics, especially in calculus and mathematical analysis. The purpose of the function f(x) is to express the behavior of the function f when the variable of the function x approaches a certain value. Let the goal be expressed mathematically according to the following figure.

$$\lim_{x \to a} f(x) = C$$

c is a real number that represents the limit of the function or the limit of the function. The limit or limit of the function is read as f(x) is c when x leads to or approaches a.

If the limit is on the right side

$$\lim_{x \to a^+} f(x) = C_1$$

If the limit is on the left side

$$\lim_{x \to a^-} f(x) = C_2$$

2. Properties of limits :

❖ The limit of the constant function: equal to the constant itself

Find the limit of the following functions:

Ex1:

$$\lim_{x \to a} z = z \quad , \quad f(x) = z$$

$$\lim_{x \to 1} 8 = 8$$

$$\lim_{x \to 3} \frac{5}{6} = \frac{5}{6}$$

- The second : The limit is distributed over the process of addition and subtraction of functions

$$\lim_{x\to a}(f(x)\mp g(x)) = \lim_{x\to a} f(x) \mp \lim_{x\to a} g(x) = C \mp L$$

Ex2:

$$\lim_{x\to 2}(3x^2 + 6x + 8)$$

$$= \lim_{x\to 2} 3x^2 + \lim_{x\to 2} 6x + \lim_{x\to 2} 8$$

$$= 3(2)^2 + 6(2) + 8$$

$$=$$

$$= 32$$

The value 2 that the variable x denotes is substituted into the limit

- The third : The limit is distributed over the multiplication process of the two functions

$$\lim_{x\to a}(f(x) * g(x)) = \lim_{x\to a} f(x) * \lim_{x\to a} g(x) = C * L$$

Ex3:

$$\lim_{x\to 2}(x+1)(x^2-1)$$

$$= \lim_{x\to 2}(x+1)\lim_{x\to 2}(x^2-1)$$

$$= (2+1) * ((2)^2 - 1)$$

$$= 3 * (4-1)$$

$$= 3 * 3$$

$$= 9$$

- The fourth : The limit of a constant multiplied by a function is equal to the constant times the limit of the function

$$\lim_{x \to a} z\, f(x) = z \lim_{x \to a} f(x) = zC$$

Ex4:

$$\lim_{x \to 3} 9(x^2 - x)$$

$$= 9 \lim_{x \to 3} (x^2 - x)$$

$$= 9((3)^2 - 3)$$

$$= 9(9 - 3)$$

$$= 9 * 6$$

$$= 54$$

According to the fourth property, the limit is inserted into the function only and the constant is taken outside the limit

- The fifth : The limit is distributed by the quotient of two functions :

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = \frac{C}{L}$$

Ex5:

$$\lim_{x \to 6} \frac{x+2}{x-3}$$

$$= \frac{\lim_{x \to 6} x+2}{\lim_{x \to 6} x-3}$$

$$= \frac{6+2}{6-3}$$

$$= \frac{8}{3}$$

According to the fifth the limit is distributed over the numerator and denominator functions

- The Sixth: The limit of a raised function :

$$\lim_{x \to a} (f(x))^{\frac{n}{m}} = (\lim_{x \to a} f(x))^{\frac{n}{m}} = C^{\frac{n}{m}}$$

Ex6:

$$\lim_{x \to 3} (x^2 - x + 4)^{\frac{2}{3}}$$

$$= \left(\lim_{x \to 3} (x^2 - x + 4)\right)^{\frac{2}{3}}$$

$$= (\lim_{x \to 3} x^2 - \lim_{x \to 3} x + \lim_{x \to 3} 4)^{\frac{2}{3}}$$

$$= ((3)^2 - 3 + 4)^{\frac{2}{3}}$$

$$= (9 - 3 + 4)^{\frac{2}{3}}$$

$$= (10)^{\frac{2}{3}}$$

❖ The seventh : the limit of a function under the root:

$$\lim_{x \to a} \sqrt{f(x)} = \sqrt{\lim_{x \to a} f(x)}$$

$$\lim_{x \to 4} \sqrt{x^2 - 2}$$

The limit is entered on the square root

$$= \sqrt{\lim_{x \to 4} (x^2 - 2)}$$

$$= \sqrt{\lim_{x \to 4} x^2 - \lim_{x \to 4} 2}$$

$$= \sqrt{((4)^2 - 2)}$$

$$= \sqrt{16 - 2}$$

$$= \sqrt{14}$$

Notes // on finding the limit of the function f(x)

The purpose of reaching the goal of the function (x)) is to substitute the value of a to which the variable x leads in the function / as in the previous examples and the result of the substitution is either a specific (known) quantity at which point the solution ends or the result of the substitution is an unspecified (unknown) quantity such as or This case usually appears in rational functions in which both the numerator and denominator are polynomial functions and in order to find a solution to these cases the following is applied

1. If the exposure result is equal to “, do one of the following:

- Analyze the numerator and denominator using one of the methods of analysis (difference of squares, experiment, difference of cubes, conjugate of the root).

- Use the following law if it fits the question.

$$\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$$

2. If the result of the substitution is equal to - and there is no more than or equal to x in the numerator, divide both the numerator and denominator by x raised to the largest power in the denominator. However, if there is no more than x in the denominator, divide both the numerator and denominator by the largest power in the row.

Example (11) Find the limit of the following function:

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$$

According to the fifth , the limit is distributed over the numerator and denominator

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = \frac{\lim_{x \to 1} x^2 - 1}{\lim_{x \to 1} x - 1}$$

$$= \frac{(1)^2 - 1}{1 - 1}$$

$$= \frac{0}{0}$$

Since the result of the substitution is 0/0, we must analyze the function to reach the limit of the function. The analysis is done using the law of the difference between two squares.

$$(x^2 - a^2) = (x - a)(x + a)$$

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = \lim_{x \to 1} \frac{(x - 1)(x + 1)}{x - 1}$$

$$= \lim_{x \to 1} (x + 1)$$

$$= 1 + 1 = 2$$

12. Find the limit of the following function

$$\lim_{x\to 3}\frac{x^3-27}{x-3}$$

$$\lim_{x\to 3}\frac{x^3-27}{x-3}=\frac{\lim_{x\to 3} x^3-27}{\lim_{x\to 3} x-3}$$

$$=\frac{(3)^3-27}{3-3}$$

$$=\frac{27-27}{0}=\frac{0}{0}$$

It is solved by using the law of the difference between two cubes

$$(x^3-a^3)=(x-a)(x^2+ax+a^2)$$

$$\lim_{x\to 3}\frac{x^3-27}{x-3}=\lim_{x\to 3}\left(\frac{(x-3)(x^2+3x+9)}{(x-3)}\right)$$

$$=\lim_{x\to 3}(x^2+3x+9)$$

According to the second , it is divided into addition and subtraction

$$=(3)^2+3*3+9$$

$$=9+9+9$$

$$=27$$

12. Find the limit of the following function

$$\lim_{x\to1}\frac{x^2-1}{\sqrt{x}-1}$$

$$\lim_{x\to1}\left(\frac{x^2-1}{\sqrt{x}-1}\right)=\frac{\lim_{x\to1}(x^2-1)}{\lim_{x\to1}(\sqrt{x}-1)}$$

$$=\frac{(1)^2-1}{\sqrt{1}-1}$$

$$=\frac{0}{0}$$

- To reach the limit of the function, we use the rule of the limit of the root, that is, multiplying the numerator and denominator by the conjugate of the root, which represents the same amount in the opposite direction.

$$\lim_{x\to1}\frac{x^2-1}{\sqrt{x}-1}=\lim_{x\to1}\frac{x^2-1}{\sqrt{x}-1}*\frac{\sqrt{x}+1}{\sqrt{x}+1}$$

$$=\lim_{x\to1}\frac{(x^2-1)(\sqrt{x}+1)}{(x-1)}$$

$$=\lim_{x\to1}\frac{(x-1)(x+1)(\sqrt{x}+1)}{(x-1)}$$

$$=\lim_{x\to1}(x+1)(\sqrt{x}+1)$$

$$=(1+1)*(\sqrt{1}+1)$$

$$=2*2$$

$$=4$$

- Thanks for lessening ..

Any questions?