THREE PHASE SYSTEM

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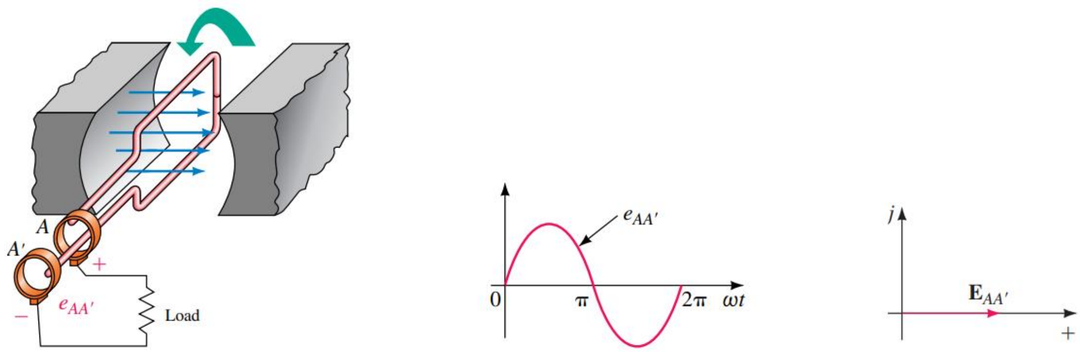
Balanced three-phase circuits

Part I/ Three-Phase Circuits (polyphase)

Figure 1. Single-phase system with two-wire type



(a) Basic ac generator (b) Voltage waveform (c) Phasor

Figure 2. A basic single-phase generator

A Polyphase system is a system in which the AC sources operate at the same frequency but different phases.

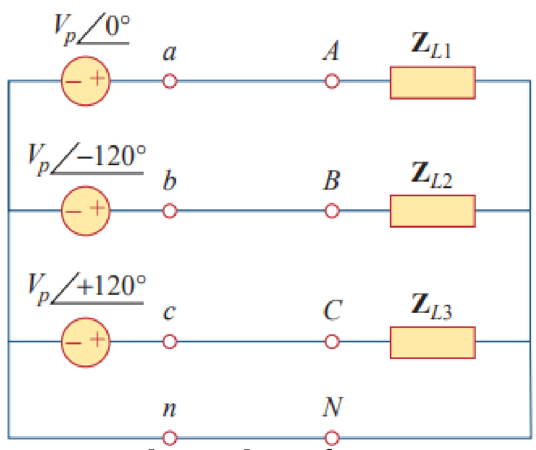
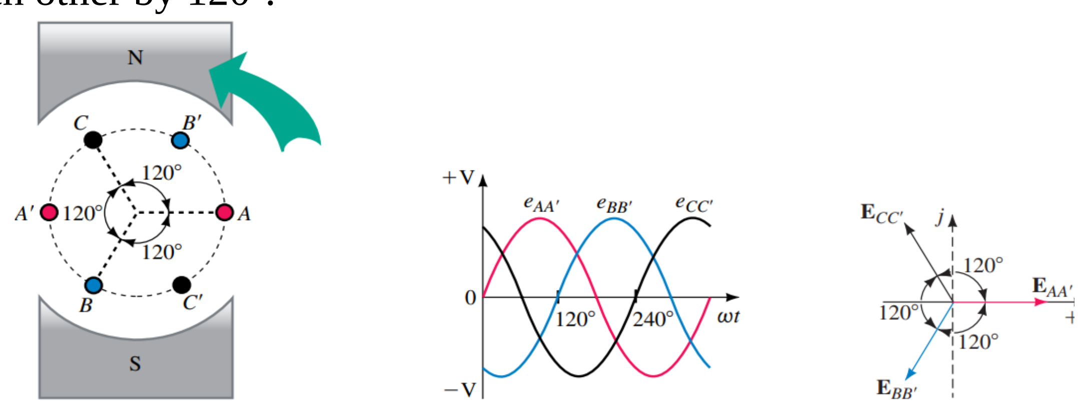


Figure 3. Three-phase four-wire system

Balanced Three-Phase Voltages

Balanced phase voltages are equal in magnitude and are out of phase with each other by 120°.



(a) Basic 3-phase generator (b) Voltage waveforms (c) Phasors

Figure 4. Three sets of coils are used to produce three balanced voltages.

- There are two phase sequences, abc and acb
- abc is similar to bca and to cab.
- The phase sequence is the time order in which the voltages pass through their respective maximum values.
- The default sequence is the abc unless otherwise stated.

Three-phase voltage sources connections

The connection could be a star (Y) or delta(Δ), but the star is more relevant one

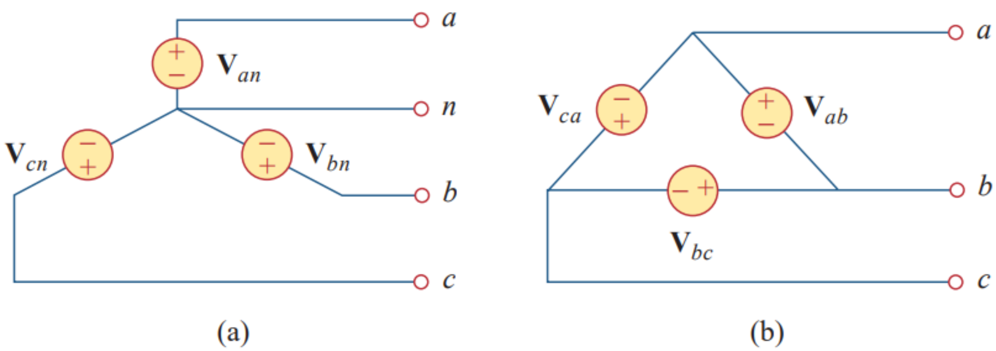
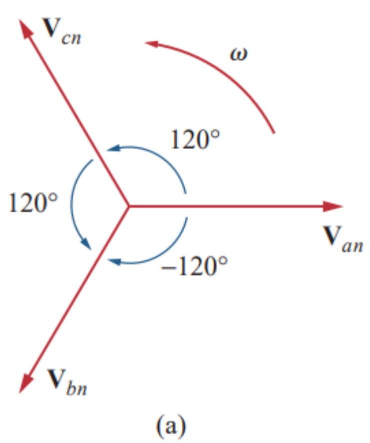
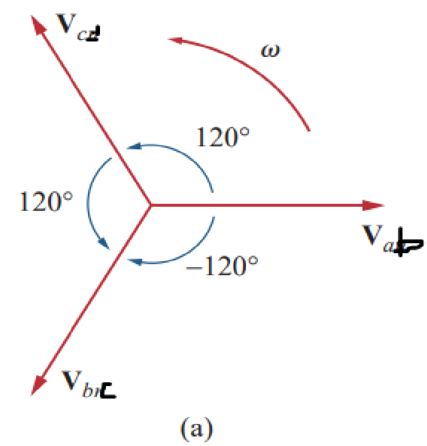


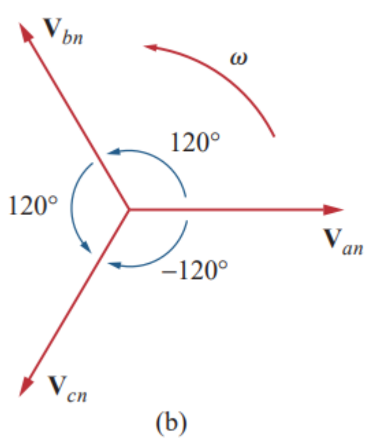
Figure 5 Three-phase voltage sources: (a) Y-connected source, (b) Δ-connected source

The phasor diagram and phase sequences



$$\mathbf{V}_{an} = V_p\angle 0^\circ$$
$$\mathbf{V}_{bn} = V_p\angle -120^\circ$$
$$\mathbf{V}_{cn} = V_p\angle -240^\circ = V_p\angle +120^\circ$$





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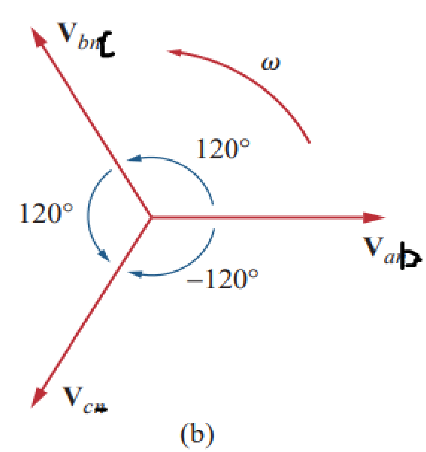


Figure 6. Phase sequences for Y-connected source
(a) abc or positive sequence,
(b) acb or negative sequence

Figure 7. Phase sequences for Δ -connected source
(a) abc or positive sequence,
(b) acb or negative sequence

For only balanced three phase systems

$$\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} = 0$$

$$|\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}|$$

$$\begin{aligned}\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} &= V_p\angle 0° + V_p\angle -120° + V_p\angle +120° \\ &= V_p(1.0 - 0.5 - j0.866 - 0.5 + j0.866) \\ &= 0\end{aligned}$$

Example 1

a. If $\mathbf{E}_{AA'} = 277\ \text{V}\angle 0°$, what are $\mathbf{E}_{BB'}$ and $\mathbf{E}_{CC'}$?

b. If $\mathbf{E}_{BB'} = 347\ \text{V}\angle -120°$, what are $\mathbf{E}_{AA'}$ and $\mathbf{E}_{CC'}$?

c. If $\mathbf{E}_{CC'} = 120\ \text{V}\angle 150°$, what are $\mathbf{E}_{AA'}$ and $\mathbf{E}_{BB'}$?

Sketch the phasors for each set.

Answers

a. $\mathbf{E}_{BB'} = 277\ \text{V}\angle -120°$; $\mathbf{E}_{CC'} = 277\ \text{V}\angle 120°$

b. $\mathbf{E}_{AA'} = 347\ \text{V}\angle 0°$; $\mathbf{E}_{CC'} = 347\ \text{V}\angle 120°$

c. $\mathbf{E}_{AA'} = 120\ \text{V}\angle 30°$; $\mathbf{E}_{BB'} = 120\ \text{V}\angle -90°$

Three-phase load

Three-phase load can be either wye-connected or delta-connected

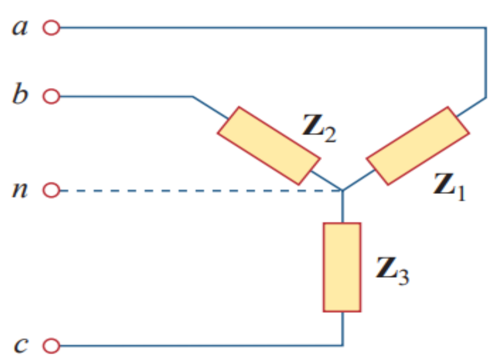


Figure 8. a Y-connected load.

For a balanced wye-connected load,

$$\mathbf{Z}_1 = \mathbf{Z}_2 = \mathbf{Z}_3 = \mathbf{Z}_Y$$

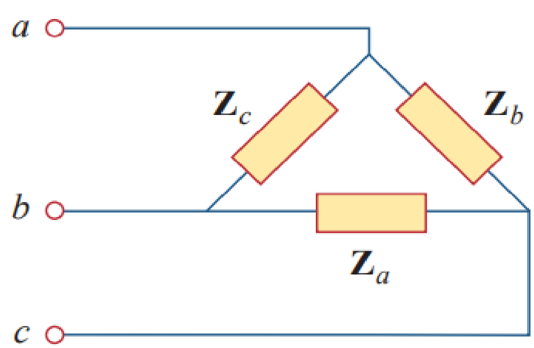


Figure 9 a Δ-connected load.

For a balanced delta-connected load,

$$\mathbf{Z}_a = \mathbf{Z}_b = \mathbf{Z}_c = \mathbf{Z}_\Delta$$

Y-to-Δ Transformation

$$\mathbf{Z}_A = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_3}$$

$$\mathbf{Z}_B = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_2}$$

$$\mathbf{Z}_C = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_1}$$

Δ-to-Y Transformation

$$\mathbf{Z}_1 = \frac{\mathbf{Z}_A\mathbf{Z}_B}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C}$$

$$\mathbf{Z}_2 = \frac{\mathbf{Z}_A\mathbf{Z}_C}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C}$$

$$\mathbf{Z}_3 = \frac{\mathbf{Z}_B\mathbf{Z}_C}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C}$$

For a balanced load only $Z_\Delta = 3Z_Y$

Example 2

Determine the phase sequence of the set of voltages

$$v_{an} = 200\cos(\omega t + 10^\circ)$$

$$v_{bn} = 200\cos(\omega t - 230^\circ), \qquad v_{cn} = 200\cos(\omega t - 110^\circ)$$

Solution:

The voltages can be expressed in phasor form as

$$\mathbf{V}_{an} = 200\underline{/10^\circ}\text{ V}, \qquad \mathbf{V}_{bn} = 200\underline{/-230^\circ}\text{ V}, \qquad \mathbf{V}_{cn} = 200\underline{/-110^\circ}\text{ V}$$

We notice that $\mathbf{V}_{an}$ leads $\mathbf{V}_{cn}$ by 120° and $\mathbf{V}_{cn}$ in turn leads $\mathbf{V}_{bn}$ by 120°. Hence, we have an *acb* sequence.

There are four possible of connections:

Source	Load
Y	Y
Y	Δ
Δ	Y
Δ	Δ

Balanced Y-Y Connection

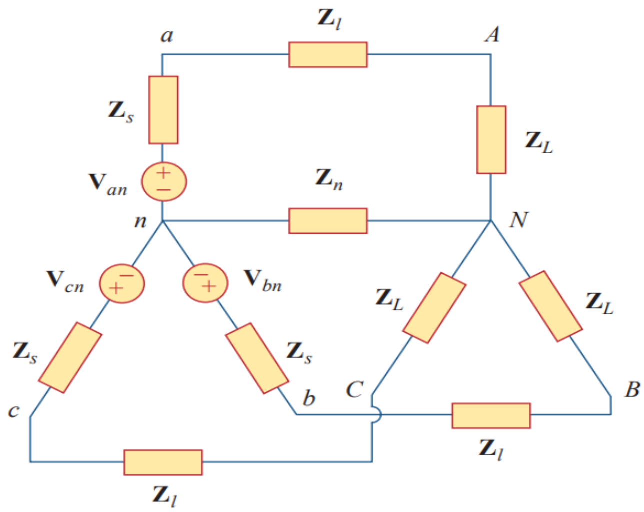


Figure 10. A balanced Y-Y system, showing the source, line, and load impedances

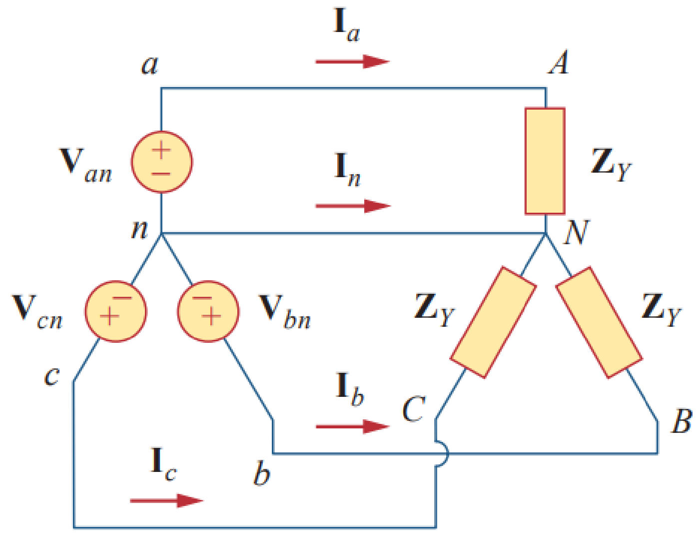


Figure 13. Balanced Y-Y connection.

$$\mathbf{Z}_Y = \mathbf{Z}_s + \mathbf{Z}_\ell + \mathbf{Z}_L$$

For abc (positive sequence)

$$\mathbf{V}_{an} = V_p\underline{/0^\circ}$$

$$\mathbf{V}_{bn} = V_p\underline{/-120^\circ}, \qquad \mathbf{V}_{cn} = V_p\underline{/+120^\circ}$$

The line-to-line voltages or simply line voltages are related to the phase voltages. For example,

$$\mathbf{V}_{ab} = \mathbf{V}_{an} + \mathbf{V}_{nb} = \mathbf{V}_{an} - \mathbf{V}_{bn} = V_p\underline{/0^\circ} - V_p\underline{/-120^\circ}$$

$$= V_p\left(1 + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = \sqrt{3}V_p\underline{/30^\circ}$$

$$\mathbf{V}_{bc} = \mathbf{V}_{bn} - \mathbf{V}_{cn} = \sqrt{3}V_p\underline{/-90^\circ}$$

$$\mathbf{V}_{ca} = \mathbf{V}_{cn} - \mathbf{V}_{an} = \sqrt{3}V_p\underline{/-210^\circ}$$

$$\boxed{V_L = \sqrt{3}V_p}$$

$$V_p = |\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}|$$

$$V_L = |\mathbf{V}_{ab}| = |\mathbf{V}_{bc}| = |\mathbf{V}_{ca}|$$

$$Z_A = Z_B = Z_C = Z_y$$

$$I_N = I_A + I_B + I_C = 0$$

$$V_{AN} = |V|\measuredangle\phi_{AN}$$

$$V_{BN} = |V|\measuredangle(\phi_{AN} - 120^\circ)$$

$$V_{CN} = |V|\measuredangle(\phi_{AN} - 240^\circ)$$

$$V_{AB} = V_{AN} - V_{BN} = |V_{AN}|\sqrt{3}\measuredangle(\phi_{AN} + 30^\circ)$$

$$V_{BC} = V_{BN} - V_{CN} = |V_{AN}|\sqrt{3}\measuredangle(\phi_{AN} - 120^\circ + 30^\circ)$$

$$V_{CA} = V_{CN} - V_{AN} = |V_{AN}|\sqrt{3}\measuredangle(\phi_{AN} - 240^\circ + 30^\circ)$$

The line voltages sum up to zero as do the phase voltages.

Van+Vbn+Vcn=0 (for balanced load only)
and
Vab+Vbc+Vca=0 (for balanced load only)

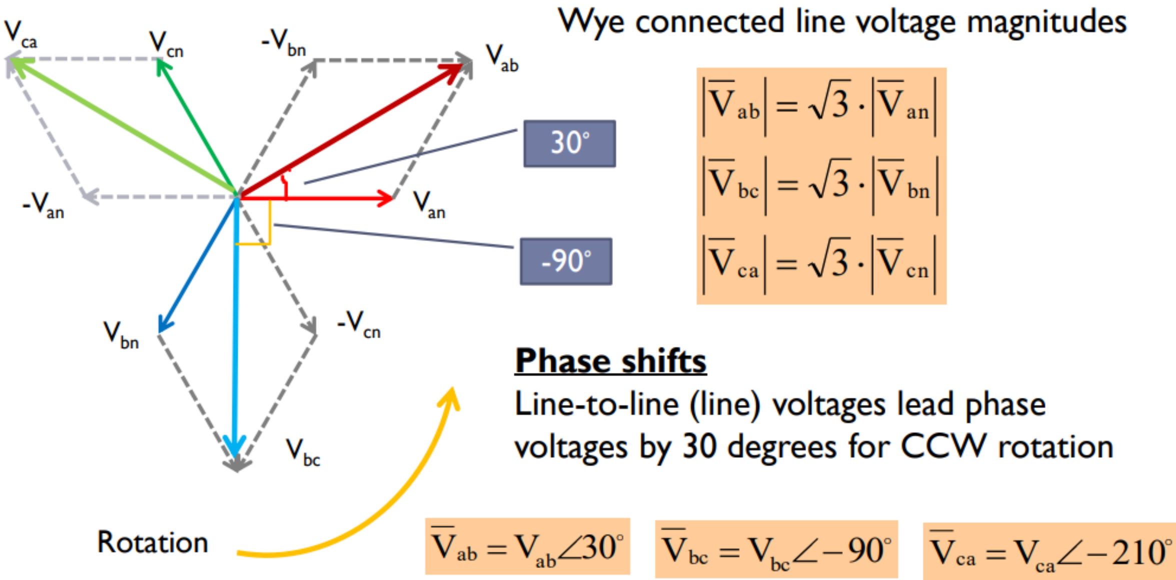


Figure 11. The relation between line and phase voltages

The line currents are equal to phase currents in Y- connection and their sum is equal to zero in a balanced system.

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}, \qquad \mathbf{I}_b = \frac{\mathbf{V}_{bn}}{\mathbf{Z}_Y} = \frac{\mathbf{V}_{an}\angle -120^\circ}{\mathbf{Z}_Y} = \mathbf{I}_a\angle -120^\circ$$

$$\mathbf{I}_c = \frac{\mathbf{V}_{cn}}{\mathbf{Z}_Y} = \frac{\mathbf{V}_{an}\angle -240^\circ}{\mathbf{Z}_Y} = \mathbf{I}_a\angle -240^\circ$$

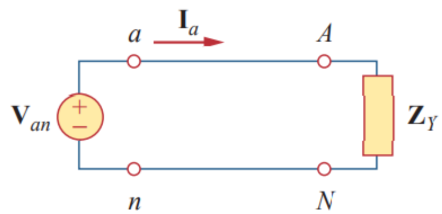
$$\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c = 0$$

$$\mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c) = 0$$

$$\mathbf{V}_{nN} = \mathbf{Z}_n \mathbf{I}_n = 0$$

The Single-Phase Equivalent

An alternative way of analyzing a balanced Y-Y system is to do so on a "per phase'



$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}$$

Figure 12. A single-phase equivalent circuit.

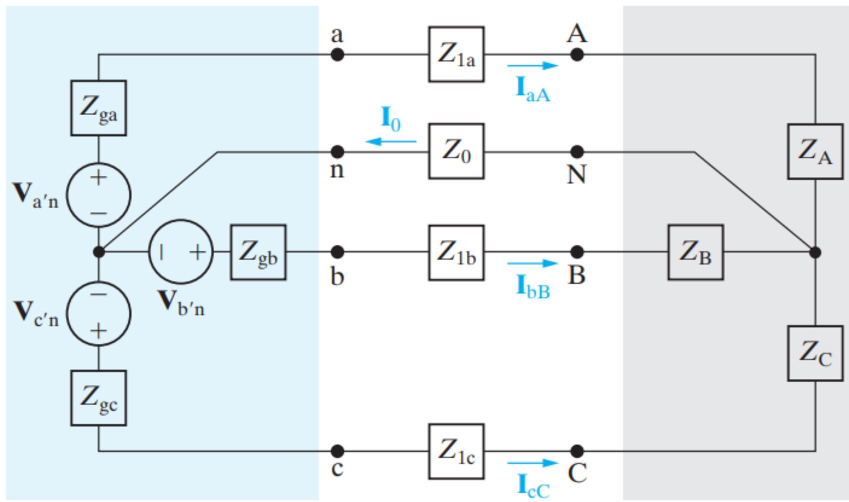


Figure 13. A three-phase Y-Y system.

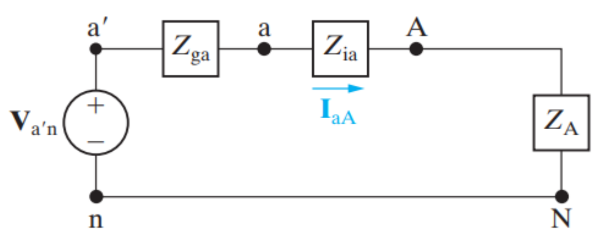


Figure 14 A single-phase equivalent circuit

Example 3

Calculate the line currents in the three-wire Y-Y system of Fig.15

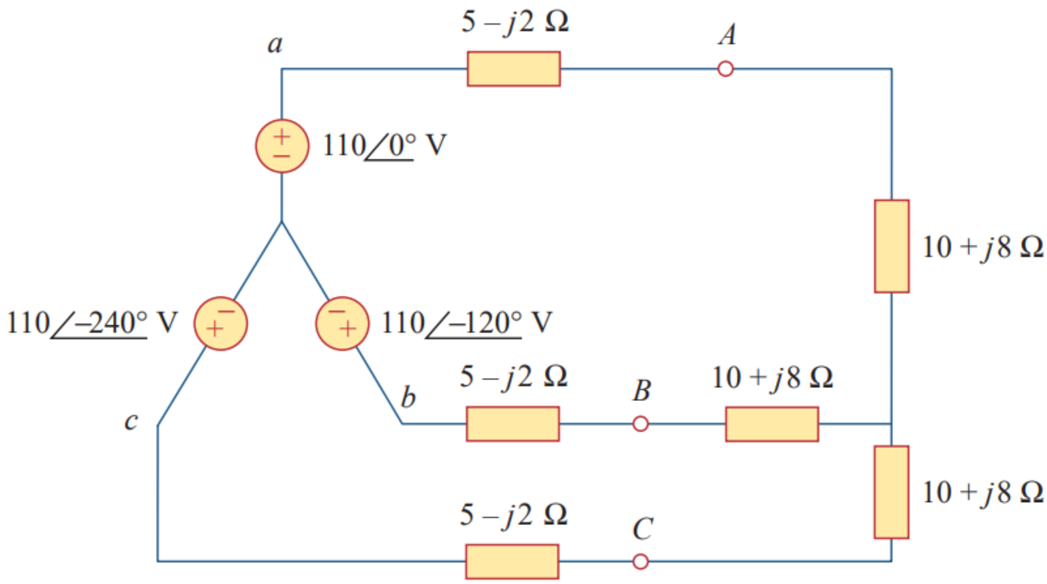


Figure 15. Three-wire Y-Y system

solution:

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}$$

where $\mathbf{Z}_Y = (5 - j2) + (10 + j8) = 15 + j6 = 16.155\underline{/21.8^\circ}$. Hence,

$$\mathbf{I}_a = \frac{110\underline{/0^\circ}}{16.155\underline{/21.8^\circ}} = 6.81\underline{/-21.8^\circ}\text{ A}$$

$$\mathbf{I}_b = \mathbf{I}_a\underline{/-120^\circ} = 6.81\underline{/-141.8^\circ}\text{ A}$$

$$\mathbf{I}_c = \mathbf{I}_a\underline{/-240^\circ} = 6.81\underline{/-261.8^\circ}\text{ A} = 6.81\underline{/98.2^\circ}\text{ A}$$

Sketch the phasor currents

Example 4

For Fig. 16, E_{AN} = 120 V∠0º.

a. Solve for the line currents.

b. Solve for the phase voltages at the load.

c. Solve for the line voltages at the load

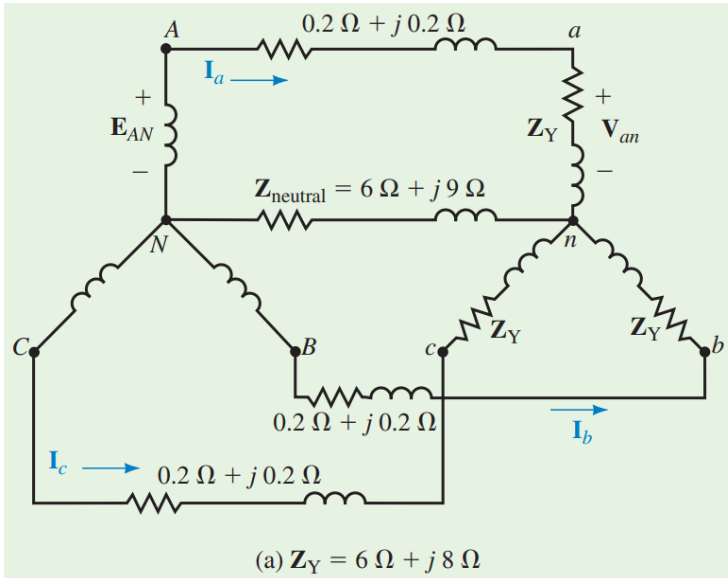


Figure 16 Y-Y connection.

Solution:

a. Reduce the circuit to its single-phase equivalent as shown in Fig. 21

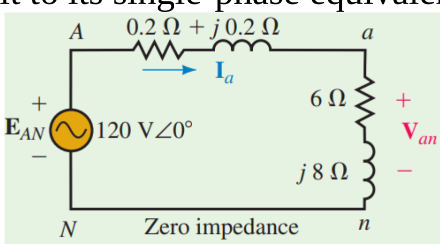


Figure 21 Single-phase equivalent.

Since the neutral conductor carries no current, its impedance has no effect on the solution

$$\mathbf{I}_a = \frac{\mathbf{E}_{AN}}{\mathbf{Z}_T} = \frac{120\angle 0^\circ}{(0.2 + j0.2) + (6 + j8)} = 11.67\ \text{A}\angle -52.91^\circ$$

Therefore,

$$\mathbf{I}_b = 11.67\ \text{A}\angle -172.91^\circ \quad \text{and} \quad \mathbf{I}_c = 11.67\ \text{A}\angle 67.09^\circ$$

b. $\mathbf{V}_{an} = \mathbf{I}_a \times \mathbf{Z}_{an} = (11.67\angle -52.91^\circ)(6 + j8) = 116.7\ \text{V}\angle 0.223^\circ$

Thus,

$$\mathbf{V}_{bn} = 116.7\ \text{V}\angle -119.777^\circ \quad \text{and} \quad \mathbf{V}_{cn} = 116.7\ \text{V}\angle 120.223^\circ$$

c. $\mathbf{V}_{ab} = \sqrt{3}\mathbf{V}_{an}\angle 30^\circ = \sqrt{3} \times 116.7\angle(0.223^\circ + 30^\circ) = 202.2\ \text{V}\angle 30.223^\circ$

Thus,

$$\mathbf{V}_{bc} = 202.2\ \text{V}\angle -89.777^\circ \quad \text{and} \quad \mathbf{V}_{ca} = 202.2\ \text{V}\angle 150.223^\circ$$

Star- Delta Connection

Consider the Y- Δ connection in Fig. 17

$$V_{an} = V_p\angle 0^\circ$$

$$V_{bn} = V_p\angle -120^\circ, \qquad V_{cn} = V_p\angle +120^\circ$$

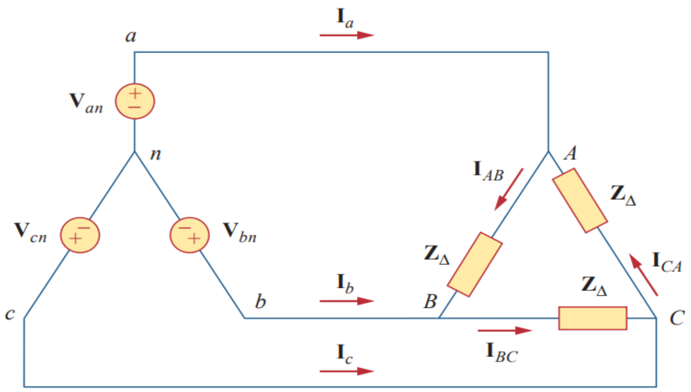


Figure 17 Y-Δ connection

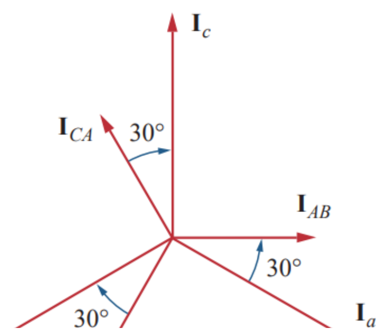
$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta}, \qquad \mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_\Delta}, \qquad \mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_\Delta}$$

Or

$$\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30^\circ = \mathbf{V}_{AB}, \qquad \mathbf{V}_{bc} = \sqrt{3}V_p\angle -90^\circ = \mathbf{V}_{BC}$$

$$\mathbf{V}_{ca} = \sqrt{3}V_p\angle +150^\circ = \mathbf{V}_{CA}$$

$$-\mathbf{V}_{an} + \mathbf{Z}_\Delta \mathbf{I}_{AB} + \mathbf{V}_{bn} = 0$$



$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{an} - \mathbf{V}_{bn}}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta}$$

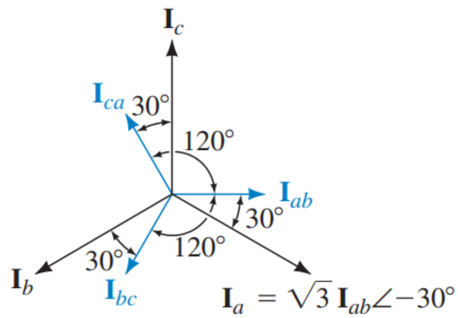
$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA}, \qquad \mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB}, \qquad \mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC}$$

Since $\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle -240^\circ$,

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA} = \mathbf{I}_{AB}(1 - 1\angle -240^\circ)$$

$$= \mathbf{I}_{AB}(1 + 0.5 - j0.866) = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$$

$$I_L = \sqrt{3} I_p$$



$$I_L = |\mathbf{I}_a| = |\mathbf{I}_b| = |\mathbf{I}_c|$$

Figure 23 phasor diagram of the line and phase currents in a Δ connected load

$$I_p = |\mathbf{I}_{AB}| = |\mathbf{I}_{BC}| = |\mathbf{I}_{CA}|$$

The set of line currents lags the set of phase currents by 30°.

$$\mathbf{I}_{aA} = \mathbf{I}_{AB} - \mathbf{I}_{CA}$$

$$= I_\phi \angle 0^\circ - I_\phi \angle 120^\circ$$

$$= \sqrt{3} I_\phi \angle -30^\circ,$$

$$\mathbf{I}_{bB} = \mathbf{I}_{BC} - \mathbf{I}_{AB}$$

$$= I_\phi \angle -120^\circ - I_\phi \angle 0^\circ$$

$$= \sqrt{3} I_\phi \angle -150^\circ,$$

$$\mathbf{I}_{cC} = \mathbf{I}_{CA} - \mathbf{I}_{BC}$$

$$= I_\phi \angle 120^\circ - I_\phi \angle -120^\circ$$

$$= \sqrt{3} I_\phi \angle 90^\circ.$$

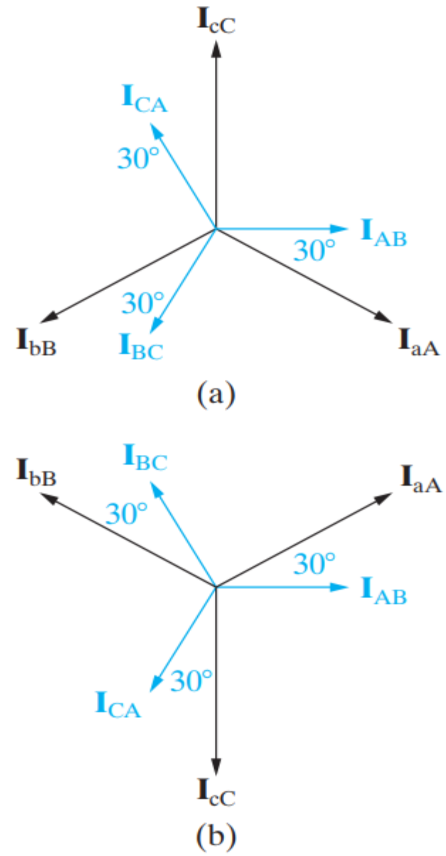
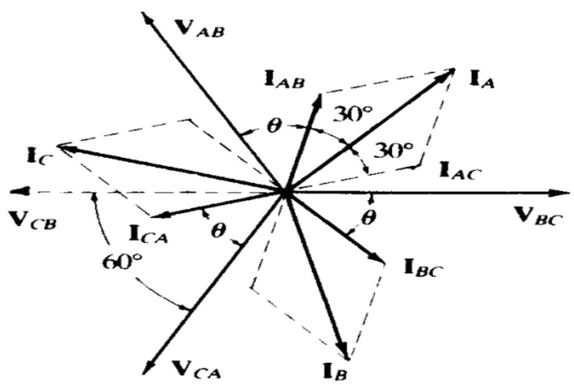


Figure 24 Phasor diagrams of the line and phase currents in a Δ connected load.

(a) The positive sequence.

(b) The negative sequence



$$Z_{AB} = Z_{BC} = Z_{CA} = Z_\Delta$$

$$I_{AB} = |I_\phi| \measuredangle \phi_{AB}$$

$$I_{BC} = |I_\phi| \measuredangle (\phi_{AB} - 120^\circ)$$

$$I_{CA} = |I_\phi| \measuredangle (\phi_{AB} - 240^\circ)$$

$$I_A = I_{AB} - I_{CA} = |I_\phi| \sqrt{3} \measuredangle (\phi_{AB} - 30^\circ)$$

$$I_B = I_{BC} - I_{AB} = |I_\phi| \sqrt{3} \measuredangle (\phi_{AB} - 120^\circ - 30^\circ)$$

$$I_C = I_{CA} - I_{BC} = |I_\phi| \sqrt{3} \measuredangle (\phi_{AB} - 240^\circ - 30^\circ)$$

Example 6

A balanced *abc*-sequence Y-connected source with $\mathbf{V}_{an} = 100\angle 10^\circ$ V is connected to a Δ-connected balanced load $(8 + j4)\ \Omega$ per phase. Calculate the phase and line currents.

Solution:

This can be solved in two ways.

METHOD 1 The load impedance is

$$\mathbf{Z}_\Delta = 8 + j4 = 8.944\angle 26.57^\circ\ \Omega$$

If the phase voltage $\mathbf{V}_{an} = 100\angle 10^\circ$, then the line voltage is

$$\mathbf{V}_{ab} = \mathbf{V}_{an}\sqrt{3}\angle 30^\circ = 100\sqrt{3}\angle 10^\circ + 30^\circ = \mathbf{V}_{AB}$$

or

$$\mathbf{V}_{AB} = 173.2\angle 40^\circ\ \text{V}$$

The phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = \frac{173.2\angle 40^\circ}{8.944\angle 26.57^\circ} = 19.36\angle 13.43^\circ\ \text{A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120^\circ = 19.36\angle -106.57^\circ\ \text{A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle +120^\circ = 19.36\angle 133.43^\circ\ \text{A}$$

The line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ = \sqrt{3}(19.36)\angle 13.43^\circ - 30^\circ$$
$$= 33.53\angle -16.57^\circ\ \text{A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ = 33.53\angle -136.57^\circ\ \text{A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ = 33.53\angle 103.43^\circ\ \text{A}$$

METHOD 2 Alternatively, using single-phase analysis,

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_\Delta/3} = \frac{100\angle 10^\circ}{2.981\angle 26.57^\circ} = 33.54\angle -16.57^\circ\ \text{A}$$

as above. Other line currents are obtained using the *abc* phase sequence.

Example 7

Suppose Vab=240 V∠15°for the circuit of Fig. 25

a. Determine the phase currents.

b. Determine the line currents.

c. Sketch the phasor diagram.

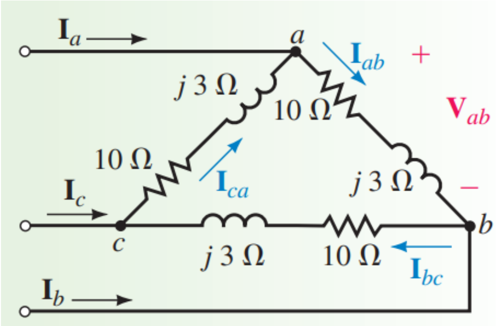


Figure 25 balanced delta load

Solution:

a. $$\mathbf{I}_{ab} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_{ab}} = \frac{240\angle 15°}{10 + j3} = 23.0\text{ A}\angle -1.70°$$

Thus,

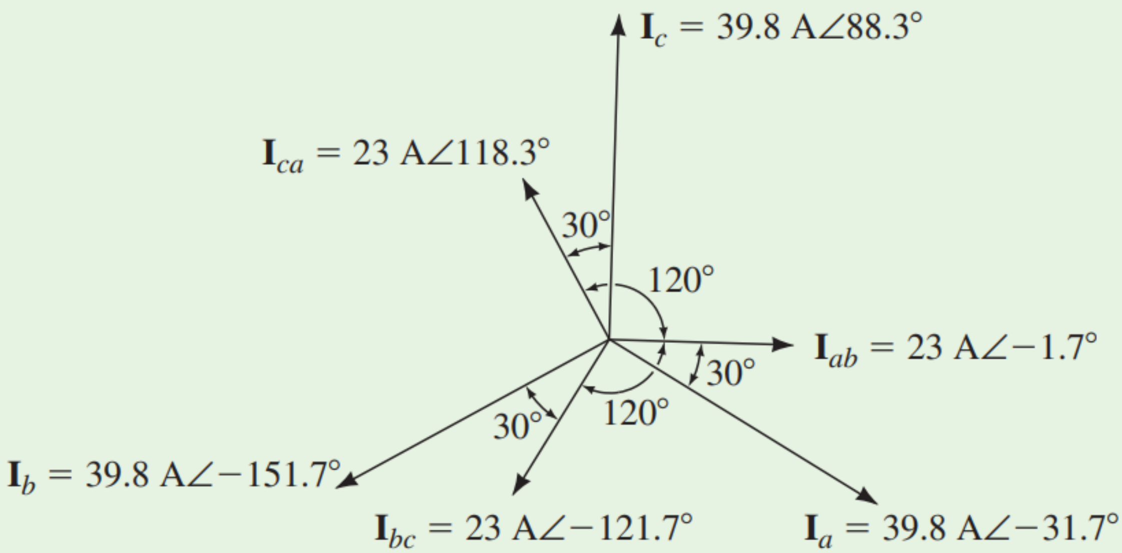
$$\mathbf{I}_{bc} = 23.0\text{ A}\angle -121.7° \text{ and } \mathbf{I}_{ca} = 23.0\text{ A}\angle 118.3°$$

b. $$\mathbf{I}_a = \sqrt{3}\mathbf{I}_{ab}\angle -30° = 39.8\text{ A}\angle -31.7°$$

Thus,

$$\mathbf{I}_b = 39.8\text{ A}\angle -151.7° \text{ and } \mathbf{I}_c = 39.8\text{ A}\angle 88.3°$$

c. Phasors are shown in Figure 24–15.



Example 8

For the circuit of Fig. 26, $E_{AB} = 208\ V\angle 30^o$.

a. Determine the phase currents.

b. Determine the line currents.

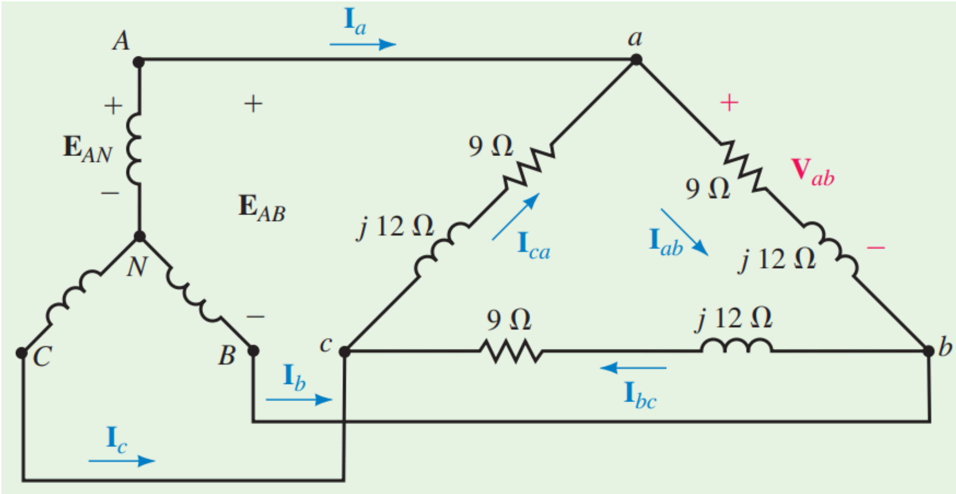


Figure 26 A Y-Δ connection

Solution:

a. Since this circuit has no line impedance, the load connects directly to the source and $\mathbf{V}_{ab} = \mathbf{E}_{AB} = 208\ V\angle 30°$. Current $\mathbf{I}_{ab}$ can be found as

$$\mathbf{I}_{ab} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_{ab}} = \frac{208\angle 30°}{9 + j12} = \frac{208\angle 30°}{15\angle 53.13°} = 13.9\ A\angle -23.13°$$

Thus,

$$\mathbf{I}_{bc} = 13.9\ A\angle -143.13° \quad \text{and} \quad \mathbf{I}_{ca} = 13.9\ A\angle 96.87°$$

b. $\mathbf{I}_a = \sqrt{3}\mathbf{I}_{ab}\angle -30° = \sqrt{3}(13.9)\angle(-30° - 23.13°) = 24\ A\angle -53.13°$

Thus,

$$\mathbf{I}_b = 24\ A\angle -173.13° \quad \text{and} \quad \mathbf{I}_c = 24\ A\angle 66.87°$$

Example 9

For the circuit of Fig. 27, the magnitude of the line voltage at the generator is 207.8 volts. Solve for the line voltage V_{ab} at the load.

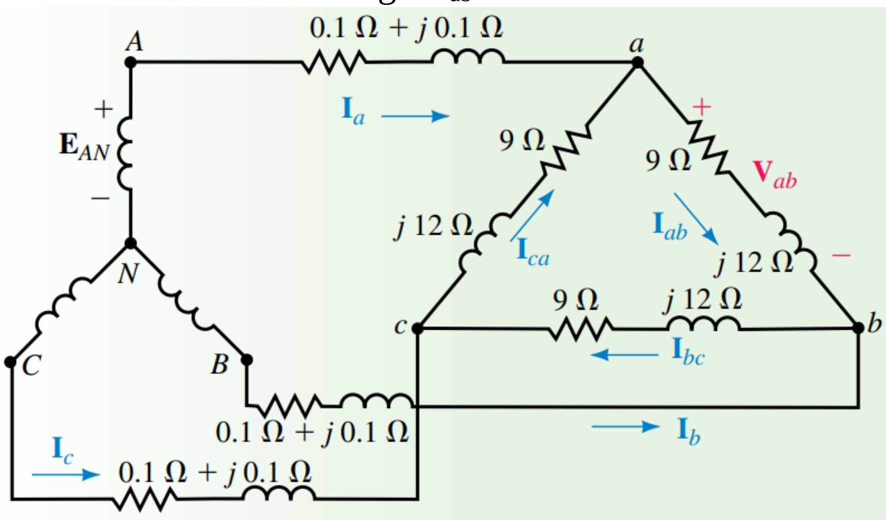


Figure 27 A Y- Δ circuit with line impedances.

Solution:

Since points A-a and B-b are not directly joined, $V_{ab} \neq E_{AB}$.

Use the single-phase equivalent as in Fig. 28.

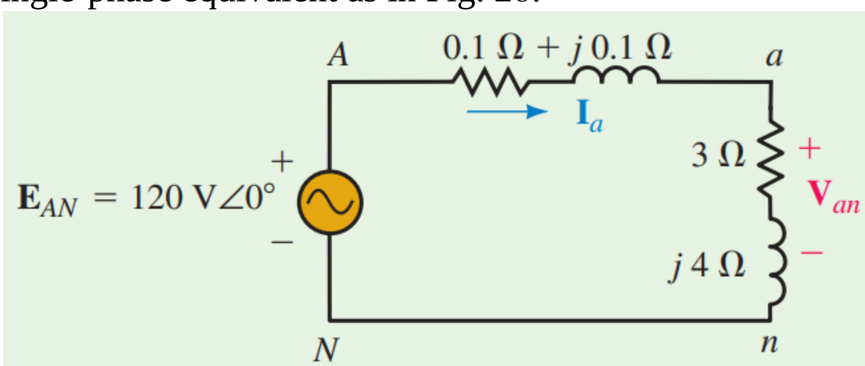


Figure 28 Single-phase equivalent

Phase voltage at the source is $207.8/\sqrt{3} = 120$ V. Choose $\mathbf{E}_{AN}$ as reference: $\mathbf{E}_{AN} = 120\text{ V}\angle 0°$.

$$\mathbf{Z}_Y = \mathbf{Z}_\Delta/3 = (9 + j12)/3 = 3\ \Omega + j4\ \Omega.$$

Now use the voltage divider rule to find Van:

$$\mathbf{V}_{an} = \left(\frac{3 + j4}{3.1 + j4.1}\right) \times 120\angle 0° = 116.7\text{ V}\angle 0.223°$$

Thus,

$$\mathbf{V}_{ab} = \sqrt{3}\mathbf{V}_{an}\angle 30° = \sqrt{3}(116.7\text{ V})\angle 30.223° = 202.2\text{ V}\angle 30.223°$$

Balanced Delta-Wye Connection

A balanced -Y system consists of a balanced Δ -connected source feeding a balanced Y-connected load. Consider the circuit in Fig. 31

$$\mathbf{V}_{ab} = V_p\angle 0^\circ, \qquad \mathbf{V}_{bc} = V_p\angle -120^\circ$$
$$\mathbf{V}_{ca} = V_p\angle +120^\circ$$

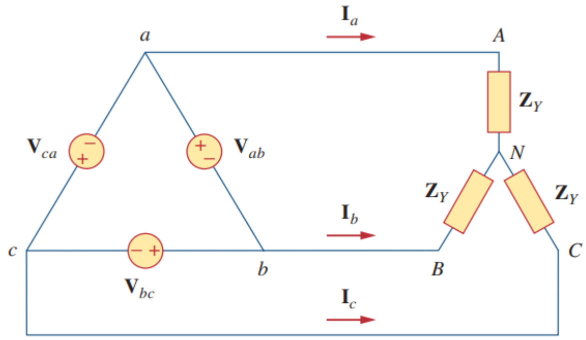


Figure 31 A balanced Δ-Y connection

To obtain the line currents, apply KVL to loop aANBba in Fig. 31, writing

$$-\mathbf{V}_{ab} + \mathbf{Z}_Y\mathbf{I}_a - \mathbf{Z}_Y\mathbf{I}_b = 0$$

or

$$\mathbf{Z}_Y(\mathbf{I}_a - \mathbf{I}_b) = \mathbf{V}_{ab} = V_p\angle 0^\circ$$

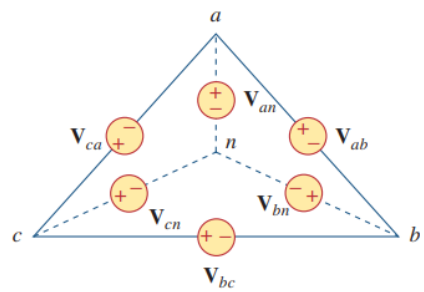
Thus,

$$\mathbf{I}_a - \mathbf{I}_b = \frac{V_p\angle 0^\circ}{\mathbf{Z}_Y}$$

But I_b lags I_a by ∠120° since we assumed the abc sequence; that is, $I_b = I_a\angle -120^\circ$ Then,

$$\mathbf{I}_a - \mathbf{I}_b = \mathbf{I}_a(1 - 1\angle -120^\circ)$$
$$= \mathbf{I}_a\left(1 + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = \mathbf{I}_a\sqrt{3}\angle 30^\circ$$

$$\mathbf{I}_a = \frac{V_p/\sqrt{3}\angle -30^\circ}{\mathbf{Z}_Y}$$
$$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ, \mathbf{I}_c = \mathbf{I}_a\angle +120^\circ.$$



Another way to obtain the line currents is to replace the delta-connected source with its equivalent wye-connected source, as shown in Fig. 32

Figure 32 Transforming a Δ -connected source to an equivalent Y-connected source

Thus, the equivalent wye-connected source has the phase voltages

$$\mathbf{V}_{an} = \frac{V_p}{\sqrt{3}}\angle -30^\circ$$

$$\mathbf{V}_{bn} = \frac{V_p}{\sqrt{3}}\angle -150^\circ, \qquad \mathbf{V}_{cn} = \frac{V_p}{\sqrt{3}}\angle +90^\circ$$

Therefore, we can use the equivalent single-phase circuit shown in Fig. 33

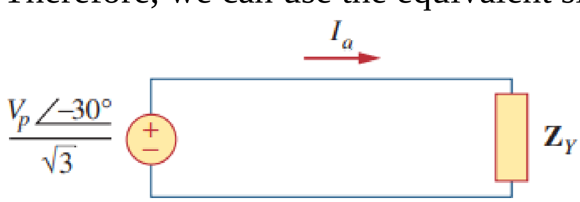


Figure 33 The single-phase equivalent circuit.

$$\mathbf{I}_a = \frac{V_p/\sqrt{3}\angle -30^\circ}{\mathbf{Z}_Y}$$

$$\mathbf{V}_{AN} = \mathbf{I}_a\mathbf{Z}_Y = \frac{V_p}{\sqrt{3}}\angle -30^\circ$$
$$\mathbf{V}_{BN} = \mathbf{V}_{AN}\angle -120^\circ, \qquad \mathbf{V}_{CN} = \mathbf{V}_{AN}\angle +120^\circ$$

Example 11

A balanced Y-connected load with a phase impedance of $40 + j25\ \Omega$ is supplied by a balanced, positive sequence Δ-connected source with a line voltage of 210 V. Calculate the phase currents. Use $\mathbf{V}_{ab}$ as a reference.

Solution:

The load impedance is

$$\mathbf{Z}_Y = 40 + j25 = 47.17\angle 32^\circ\ \Omega$$

and the source voltage is

$$\mathbf{V}_{ab} = 210\angle 0^\circ\ \text{V}$$

When the Δ-connected source is transformed to a Y-connected source,

$$\mathbf{V}_{an} = \frac{\mathbf{V}_{ab}}{\sqrt{3}}\angle -30^\circ = 121.2\angle -30^\circ\ \text{V}$$

The line currents are

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y} = \frac{121.2\angle -30^\circ}{47.12\angle 32^\circ} = 2.57\angle -62^\circ\ \text{A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ = 2.57\angle -178^\circ\ \text{A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120^\circ = 2.57\angle 58^\circ\ \text{A}$$

which are the same as the phase currents.

Balanced Delta-Delta Connection

Consider the Δ-Δ connection in the circuit shown in Fig. 34

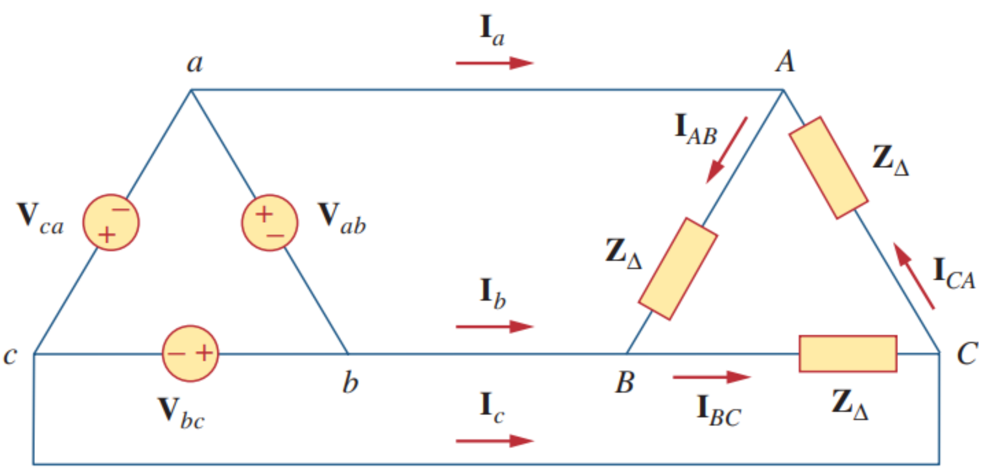


Figure 34 A balanced Δ-Δ connection.

For abc sequence:

$$\mathbf{V}_{ab} = V_p\underline{/0^\circ}$$

$$\mathbf{V}_{bc} = V_p\underline{/-120^\circ}, \qquad \mathbf{V}_{ca} = V_p\underline{/+120^\circ}$$

The line voltages are the same as the phase voltages.
Assuming there is no line impedances,

$$\mathbf{V}_{ab} = \mathbf{V}_{AB}, \qquad \mathbf{V}_{bc} = \mathbf{V}_{BC}, \qquad \mathbf{V}_{ca} = \mathbf{V}_{CA}$$

The phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_\Delta}, \qquad \mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{bc}}{\mathbf{Z}_\Delta}$$

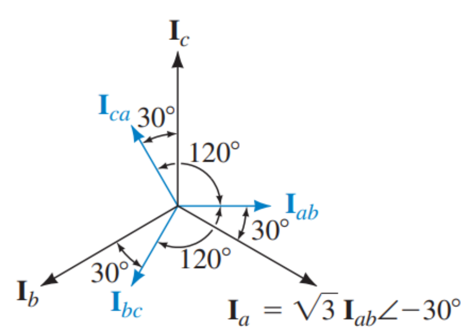
$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{ca}}{\mathbf{Z}_\Delta}$$

The line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA}, \qquad \mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB}, \qquad \mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC}$$

Each line current lags the corresponding phase current by 30° and the magnitude of

$$I_L = \sqrt{3} I_p$$



An alternative way of analyzing the circuit is to convert both the source and the load to their Y equivalents

Example 12

A balanced Δ-connected load having an impedance $20 - j15\ \Omega$ is connected to a Δ-connected, positive-sequence generator having $\mathbf{V}_{ab} = 330\angle 0^\circ$ V. Calculate the phase currents of the load and the line currents.

Solution:

The load impedance per phase is

$$\mathbf{Z}_\Delta = 20 - j15 = 25\angle -36.87^\circ\ \Omega$$

Since $\mathbf{V}_{AB} = \mathbf{V}_{ab}$, the phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = \frac{330\angle 0^\circ}{25\angle -36.87} = 13.2\angle 36.87^\circ \text{ A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120^\circ = 13.2\angle -83.13^\circ \text{ A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle +120^\circ = 13.2\angle 156.87^\circ \text{ A}$$

For a delta load, the line current always lags the corresponding phase current by 30° and has a magnitude $\sqrt{3}$ times that of the phase current. Hence, the line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ = (13.2\angle 36.87^\circ)(\sqrt{3}\angle -30^\circ) = 22.86\angle 6.87^\circ \text{ A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ = 22.86\angle -113.13^\circ \text{ A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ = 22.86\angle 126.87^\circ \text{ A}$$

Summary of phase and line voltages/currents for balanced three-phase systems.[1]

Connection	Phase voltages/currents	Line voltages/currents
Y-Y	$\mathbf{V}_{an} = V_p\angle 0^\circ$ $\mathbf{V}_{bn} = V_p\angle -120^\circ$ $\mathbf{V}_{cn} = V_p\angle +120^\circ$ Same as line currents	$\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30^\circ$ $\mathbf{V}_{bc} = \mathbf{V}_{ab}\angle -120^\circ$ $\mathbf{V}_{ca} = \mathbf{V}_{ab}\angle +120^\circ$ $\mathbf{I}_a = \mathbf{V}_{an}/\mathbf{Z}_Y$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Y-Δ	$\mathbf{V}_{an} = V_p\angle 0^\circ$ $\mathbf{V}_{bn} = V_p\angle -120^\circ$ $\mathbf{V}_{cn} = V_p\angle +120^\circ$ $\mathbf{I}_{AB} = \mathbf{V}_{AB}/\mathbf{Z}_\Delta$ $\mathbf{I}_{BC} = \mathbf{V}_{BC}/\mathbf{Z}_\Delta$ $\mathbf{I}_{CA} = \mathbf{V}_{CA}/\mathbf{Z}_\Delta$	$\mathbf{V}_{ab} = \mathbf{V}_{AB} = \sqrt{3}V_p\angle 30^\circ$ $\mathbf{V}_{bc} = \mathbf{V}_{BC} = \mathbf{V}_{ab}\angle -120^\circ$ $\mathbf{V}_{ca} = \mathbf{V}_{CA} = \mathbf{V}_{ab}\angle +120^\circ$ $\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Δ-Δ	$\mathbf{V}_{ab} = V_p\angle 0^\circ$ $\mathbf{V}_{bc} = V_p\angle -120^\circ$ $\mathbf{V}_{ca} = V_p\angle +120^\circ$ $\mathbf{I}_{AB} = \mathbf{V}_{ab}/\mathbf{Z}_\Delta$ $\mathbf{I}_{BC} = \mathbf{V}_{bc}/\mathbf{Z}_\Delta$ $\mathbf{I}_{CA} = \mathbf{V}_{ca}/\mathbf{Z}_\Delta$	Same as phase voltages $\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Δ-Y	$\mathbf{V}_{ab} = V_p\angle 0^\circ$ $\mathbf{V}_{bc} = V_p\angle -120^\circ$ $\mathbf{V}_{ca} = V_p\angle +120^\circ$ Same as line currents	Same as phase voltages $\mathbf{I}_a = \dfrac{V_p\angle -30^\circ}{\sqrt{3}\mathbf{Z}_Y}$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$

[1] Positive or abc sequence is assumed