THREE PHASE SYSTEM

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Unbalanced three phase system

Summary of phase and line voltages/currents for balanced three-phase systems.[1]

Connection	Phase voltages/currents	Line voltages/currents
Y-Y	$\mathbf{V}_{an} = V_p\angle 0^\circ$ $\mathbf{V}_{bn} = V_p\angle -120^\circ$ $\mathbf{V}_{cn} = V_p\angle +120^\circ$ Same as line currents	$\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30^\circ$ $\mathbf{V}_{bc} = \mathbf{V}_{ab}\angle -120^\circ$ $\mathbf{V}_{ca} = \mathbf{V}_{ab}\angle +120^\circ$ $\mathbf{I}_a = \mathbf{V}_{an}/\mathbf{Z}_Y$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Y-Δ	$\mathbf{V}_{an} = V_p\angle 0^\circ$ $\mathbf{V}_{bn} = V_p\angle -120^\circ$ $\mathbf{V}_{cn} = V_p\angle +120^\circ$ $\mathbf{I}_{AB} = \mathbf{V}_{AB}/\mathbf{Z}_\Delta$ $\mathbf{I}_{BC} = \mathbf{V}_{BC}/\mathbf{Z}_\Delta$ $\mathbf{I}_{CA} = \mathbf{V}_{CA}/\mathbf{Z}_\Delta$	$\mathbf{V}_{ab} = \mathbf{V}_{AB} = \sqrt{3}V_p\angle 30^\circ$ $\mathbf{V}_{bc} = \mathbf{V}_{BC} = \mathbf{V}_{ab}\angle -120^\circ$ $\mathbf{V}_{ca} = \mathbf{V}_{CA} = \mathbf{V}_{ab}\angle +120^\circ$ $\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Δ-Δ	$\mathbf{V}_{ab} = V_p\angle 0^\circ$ $\mathbf{V}_{bc} = V_p\angle -120^\circ$ $\mathbf{V}_{ca} = V_p\angle +120^\circ$ $\mathbf{I}_{AB} = \mathbf{V}_{ab}/\mathbf{Z}_\Delta$ $\mathbf{I}_{BC} = \mathbf{V}_{bc}/\mathbf{Z}_\Delta$ $\mathbf{I}_{CA} = \mathbf{V}_{ca}/\mathbf{Z}_\Delta$	Same as phase voltages $\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Δ-Y	$\mathbf{V}_{ab} = V_p\angle 0^\circ$ $\mathbf{V}_{bc} = V_p\angle -120^\circ$ $\mathbf{V}_{ca} = V_p\angle +120^\circ$ Same as line currents	Same as phase voltages $\mathbf{I}_a = \dfrac{V_p\angle -30^\circ}{\sqrt{3}\mathbf{Z}_Y}$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$

[1] Positive or abc sequence is assumed

Power in a Balanced System

For a Y-connected load, The total instantaneous power in the load is the sum of the instantaneous powers in the three phases; that is,

$$p = p_a + p_b + p_c = v_{AN}i_a + v_{BN}i_b + v_{CN}i_c$$
$$= 3V_pI_p \cos\theta$$

Thus the total instantaneous power in a balanced three-phase system is constant.
This result is true whether the load is Y- or Δ – connected.

The average power per phase is $P_p = V_pI_p \cos\theta$

The reactive power per phase is $Q_p = V_pI_p \sin\theta$

The apparent power per phase is $S_p = V_pI_p$

The complex power per phase is $\mathbf{S}_p = P_p + jQ_p = \mathbf{V}_p\mathbf{I}_p^*$

The total average power is the sum of the average powers in the phases:

$$P = P_a + P_b + P_c = 3P_p = 3V_pI_p \cos\theta = \sqrt{3}V_LI_L \cos\theta$$

For

Y-connected load, $I_L = I_p$ but $V_L = \sqrt{3}V_p$

Δ-connected load, $I_L = \sqrt{3}I_p$ but $V_L = V_p$

The total reactive power is

$$Q = 3V_pI_p \sin\theta = 3Q_p = \sqrt{3}V_LI_L \sin\theta$$

The total complex power is

$$\mathbf{S} = P + jQ = \sqrt{3}V_LI_L\underline{/\theta}$$

Where θ is the angle between the phase voltage and the phase current.

Example 1

Determine the total average power, reactive power, and complex power at the source and at the load of the circuit shown in Fig. 3

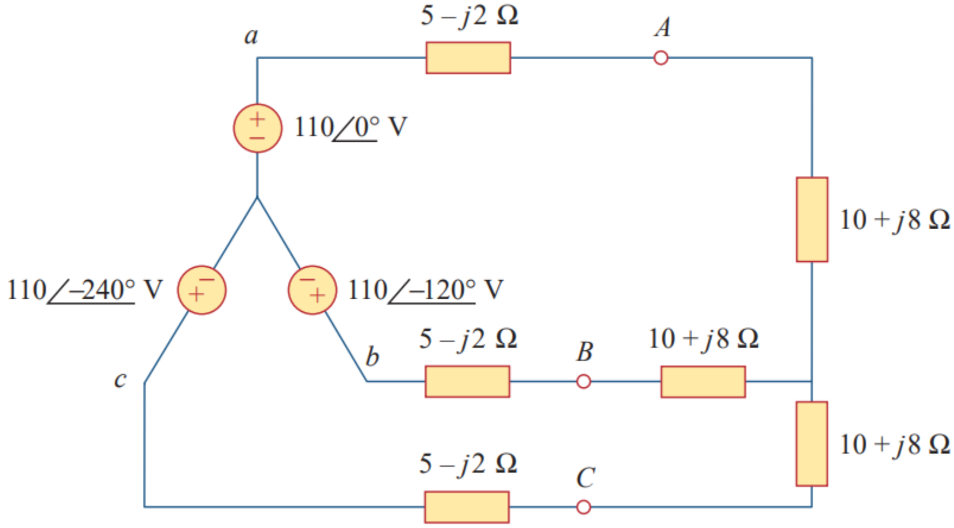

Figure 3 Y-Y connection

Solution:

$$\mathbf{V}_p = 110\angle 0^\circ \text{ V} \qquad \text{and} \qquad \mathbf{I}_p = 6.81\angle -21.8^\circ \text{ A}$$

Thus, at the source, the complex power absorbed is

$$\begin{aligned}\mathbf{S}_s &= -3\mathbf{V}_p\mathbf{I}_p^* = -3(110\angle 0^\circ)(6.81\angle 21.8^\circ)\\ &= -2247\angle 21.8^\circ = -(2087 + j834.6) \text{ VA}\end{aligned}$$

The real or average power absorbed is -2087 W and the reactive power is -834.6 VAR.

At the load, the complex power absorbed is

$$\mathbf{S}_L = 3|\mathbf{I}_p|^2\mathbf{Z}_p$$

where $\mathbf{Z}_p = 10 + j8 = 12.81\angle 38.66^\circ$ and $\mathbf{I}_p = \mathbf{I}_a = 6.81\angle -21.8^\circ$. Hence,

$$\begin{aligned}\mathbf{S}_L &= 3(6.81)^2 12.81\angle 38.66^\circ = 1782\angle 38.66\\ &= (1392 + j1113) \text{ VA}\end{aligned}$$

The real power absorbed is 1391.7 W and the reactive power absorbed is 1113.3 VAR. The difference between the two complex powers is absorbed by the line impedance $(5 - j2)\ \Omega$. To show that this is the case, we find the complex power absorbed by the line as

$$\mathbf{S}_\ell = 3|\mathbf{I}_p|^2\mathbf{Z}_\ell = 3(6.81)^2(5 - j2) = 695.6 - j278.3 \text{ VA}$$

which is the difference between $\mathbf{S}_s$ and $\mathbf{S}_L$; that is, $\mathbf{S}_s + \mathbf{S}_\ell + \mathbf{S}_L = 0$, as expected.

Unbalanced Three-Phase Systems

An unbalanced system is due to unbalanced voltage sources or an unbalanced load.

Unbalanced three-phase systems are solved by direct application of mesh and nodal analysis.

Fig. 10 shows an example of an unbalanced three-phase system that consists of balanced source voltages and an unbalanced Y-connected load. Since the load is unbalanced, and are not equal.

The line currents are determined by Ohm's law as:

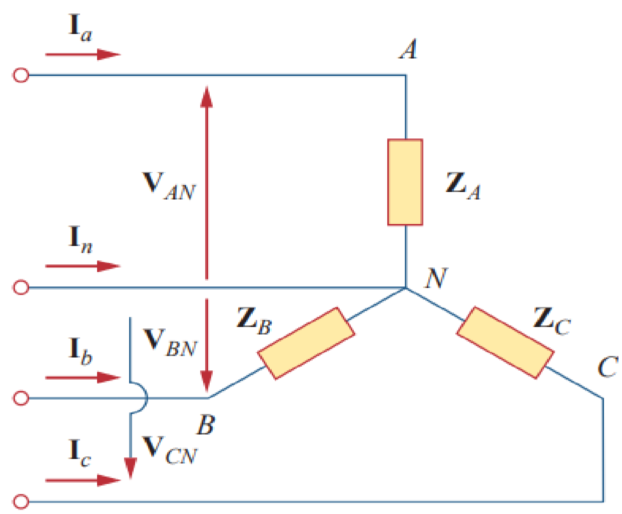

Figure 10 unbalanced three-phase Y-connected load.

$$\mathbf{I}_a = \frac{\mathbf{V}_{AN}}{\mathbf{Z}_A}, \qquad \mathbf{I}_b = \frac{\mathbf{V}_{BN}}{\mathbf{Z}_B}, \qquad \mathbf{I}_c = \frac{\mathbf{V}_{CN}}{\mathbf{Z}_C}$$

This set of unbalanced line currents produces current in the neutral line, which is not zero as in a balanced system. Applying KCL at node N gives the neutral line current as

$$\mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c)$$

In a three-wire system

$$\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c = 0$$

To calculate power in an unbalanced three-phase system requires that we find the power in each phase using

$$P = P_a + P_b + P_c$$

The total power is not simply three times the power in one phase but the sum of the powers in the three phases.

Example 6

The unbalanced Y-load of Fig. 10 has balanced voltages of 100 V and the acb sequence. Calculate the line currents and the neutral current

$$\mathbf{Z}_A = 15\ \Omega,\ \mathbf{Z}_B = 10 + j5\ \Omega,\ \mathbf{Z}_C = 6 - j8\ \Omega.$$

Solution:

the line currents are

$$\mathbf{I}_a = \frac{100\angle 0^\circ}{15} = 6.67\angle 0^\circ \text{ A}$$

$$\mathbf{I}_b = \frac{100\angle 120^\circ}{10 + j5} = \frac{100\angle 120^\circ}{11.18\angle 26.56^\circ} = 8.94\angle 93.44^\circ \text{ A}$$

$$\mathbf{I}_c = \frac{100\angle -120^\circ}{6 - j8} = \frac{100\angle -120^\circ}{10\angle -53.13^\circ} = 10\angle -66.87^\circ \text{ A}$$

the current in the neutral line is

$$\mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c) = -(6.67 - 0.54 + j8.92 + 3.93 - j9.2)$$
$$= -10.06 + j0.28 = 10.06\angle 178.4^\circ \text{ A}$$

Example 7

For the unbalanced circuit in Fig. 11, find: (a) the line currents, (b) the total complex power absorbed by the load, and (c) the total complex power absorbed by the source

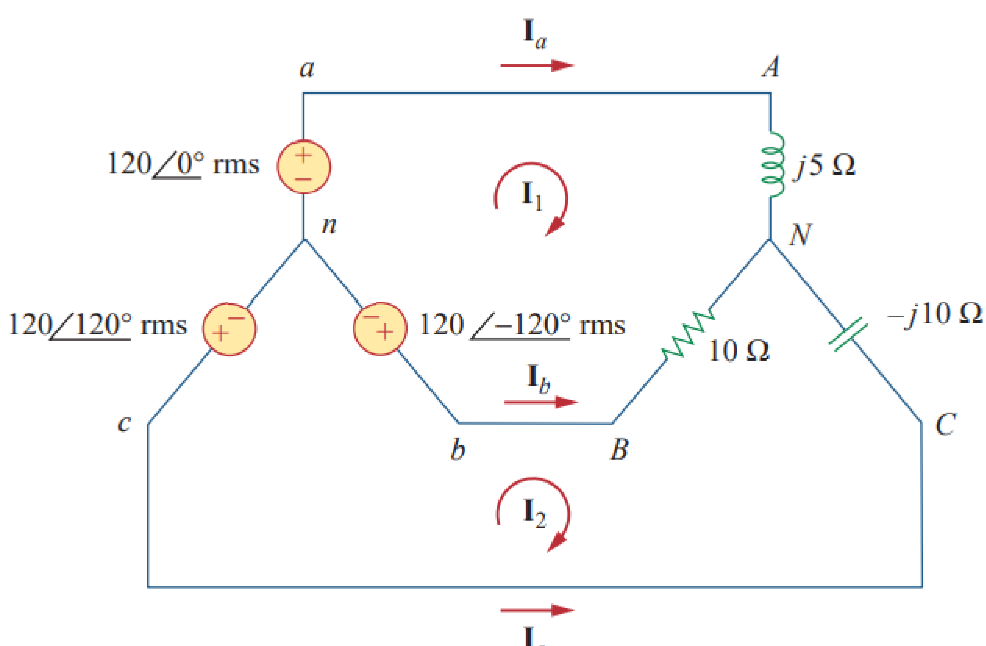

Figure 11

Solution:

(a) We use mesh analysis to find the required currents.

For mesh 1,

$$120\angle{-120^\circ} - 120\angle{0^\circ} + (10 + j5)\mathbf{I}_1 - 10\mathbf{I}_2 = 0$$

Or $\quad (10 + j5)\mathbf{I}_1 - 10\mathbf{I}_2 = 120\sqrt{3}\angle{30^\circ}$

For mesh 2,

$$120\angle{120^\circ} - 120\angle{-120^\circ} + (10 - j10)\mathbf{I}_2 - 10\mathbf{I}_1 = 0$$

Or $\quad -10\mathbf{I}_1 + (10 - j10)\mathbf{I}_2 = 120\sqrt{3}\angle{-90^\circ}$

$$\begin{bmatrix} 10 + j5 & -10 \\ -10 & 10 - j10 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 120\sqrt{3}\angle{30^\circ} \\ 120\sqrt{3}\angle{-90^\circ} \end{bmatrix}$$

The determinants are

$$\Delta = \begin{vmatrix} 10 + j5 & -10 \\ -10 & 10 - j10 \end{vmatrix} = 50 - j50 = 70.71\angle{-45^\circ}$$

$$\Delta_1 = \begin{vmatrix} 120\sqrt{3}\angle{30^\circ} & -10 \\ 120\sqrt{3}\angle{-90^\circ} & 10 - j10 \end{vmatrix} = 207.85(13.66 - j13.66)$$

$$= 4015\angle{-45^\circ}$$

$$\Delta_2 = \begin{vmatrix} 10 + j5 & 120\sqrt{3}\angle{30^\circ} \\ -10 & 120\sqrt{3}\angle{-90^\circ} \end{vmatrix} = 207.85(13.66 - j5)$$

$$= 3023.4\angle{-20.1^\circ}$$

The mesh currents are

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \frac{4015.23\angle{-45^\circ}}{70.71\angle{-45^\circ}} = 56.78 \text{ A}$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{3023.4\angle{-20.1^\circ}}{70.71\angle{-45^\circ}} = 42.75\angle{24.9^\circ} \text{ A}$$

The line currents are

$$\mathbf{I}_a = \mathbf{I}_1 = 56.78 \text{ A}, \qquad \mathbf{I}_c = -\mathbf{I}_2 = 42.75\angle{-155.1^\circ} \text{ A}$$

$$\mathbf{I}_b = \mathbf{I}_2 - \mathbf{I}_1 = 38.78 + j18 - 56.78 = 25.46\angle{135^\circ} \text{ A}$$

(b) We can now calculate the complex power absorbed by the load.

For phase A, $\mathbf{S}_A = |\mathbf{I}_a|^2\mathbf{Z}_A = (56.78)^2(j5) = j16{,}120 \text{ VA}$

For phase B $\mathbf{S}_B = |\mathbf{I}_b|^2\mathbf{Z}_B = (25.46)^2(10) = 6480 \text{ VA}$

For phase C $\mathbf{S}_C = |\mathbf{I}_c|^2\mathbf{Z}_C = (42.75)^2(-j10) = -j18{,}276 \text{ VA}$

The total complex power absorbed by the load is

$$\mathbf{S}_L = \mathbf{S}_A + \mathbf{S}_B + \mathbf{S}_C = 6480 - j2156 \text{ VA}$$

(c) We check the result above by finding the power absorbed by the source. For the voltage source in phase a,

$$\mathbf{S}_a = -\mathbf{V}_{an}\mathbf{I}_a^* = -(120\angle{0^\circ})(56.78) = -6813.6 \text{ VA}$$

For the source in phase b,

$$\mathbf{S}_b = -\mathbf{V}_{bn}\mathbf{I}_b^* = -(120\angle{-120^\circ})(25.46\angle{-135^\circ})$$
$$= -3055.2\angle{105^\circ} = 790 - j2951.1 \text{ VA}$$

For the source in phase c,

$$\mathbf{S}_c = -\mathbf{V}_{bn}\mathbf{I}_c^* = -(120\angle{120^\circ})(42.75\angle{155.1^\circ})$$
$$= -5130\angle{275.1^\circ} = -456.03 + j5109.7 \text{ VA}$$

The total complex power absorbed by the three-phase source is

$$\mathbf{S}_s = \mathbf{S}_a + \mathbf{S}_b + \mathbf{S}_c = -6480 + j2156 \text{ VA}$$

showing that and confirming the conservation principle of ac power. Ss + SL=0