CHPTER FOUR

Fluid Dynamic

4.1 Introduction

In the process industries it is often necessary to pump fluids over long distances from storage to processing units, and there may be a substantial drop in pressure in both the pipeline and in individual units themselves. It is necessary, therefore, to consider the problems concerned with calculating the power requirements for pumping, with designing the most suitable flow system, with estimating the most economical sizes of pipes, with measuring the rate of flow, and frequently with controlling this flow at steady state rate.

It must be realized that when a fluid is flowing over a surface o through a pipe, the velocity at various points in a plane at right angles to the stream velocity is rarely uniform, and the rate change of velocity with distance from the surface will exert a vital influence on the resistance to flow and the rate of mass or heat transfer.

4.2 The Nature of Fluid Flow

When a fluid is flowing through a tube or over a surface, the pattern of flow will vary with *the velocity, the physical properties of fluid*, and *the geometry of the surface*. This problem was first examined by Reynolds in 1883. Reynolds has shown that when the velocity of the fluid is slow, the flow pattern is smooth. However, when the velocity is quite high, an unstable pattern is observed in which eddies or small packets of fluid particles are present moving in all directions and at all angles to the normal line of flow.

The first type of flow at low velocities where the layers of fluid seen to slide by one another without eddies or swirls being present is called "*laminar flow*" and Newton's law of viscosity holds.

The second type of flow at higher velocities where eddies are present giving the fluid a fluctuating nature is called "*turbulent flow*".

4.3 Reynolds Number (Re)

Studies have shown that the transition from laminar to turbulent flow in tubes is not only a function of velocity but also of density (ρ), dynamic viscosity (μ), and the diameter of tube. These variables are combining into the Reynolds number, which is dimensionless group.

$$\boxed{\mathrm{Re} = \frac{\rho u d}{\mu}}$$

where u is the average velocity of fluid, which is defined as the volumetric flow rate divided by the cross-sectional area of the pipe.

$$\boxed{u = \frac{Q}{A} = \frac{Q}{\pi/4\, d^2}}$$

$$\Rightarrow \boxed{\mathrm{Re} = \frac{4 Q\rho}{\pi\ d\ \mu} = \frac{4\dot{m}}{\pi\ d\ \mu} = \frac{G\, d}{\mu}}$$

Where, Q: volumetric flow rate m^3/s
$\dot{m}$: mass flow rate kg/s
G: mass flux or mass velocity $kg/m^2.s$

for a straight circular pipe when the value of Re is less than 2,100 the flow is always laminar. When the value is over 4,000 the flow be turbulent. In between, which is called the transition region the flow can be laminar or turbulent depending upon the apparatus details.

Example -4.1-

Water at 303 K is flowing at the rate of 10 gal/min in a pipe having an inside diameter I.D. of 2.067 in. calculate the Reynolds number using both English and S.I. units

Solution:

The volumetric flow rate (Q) = 10 gal/min (1.0 ft^3/7.481 gal) (min/60 s) = 0.0223 ft^3/s

Pipe diameter (d) = 2.067 in (ft/12 in) = 0.172 ft

Cross-sectional area (A) = $\pi/4\ d^2 = \pi/4\ (0.172)^2 = 0.0233\ ft^2$

Average velocity (u) =Q/A = (0.0223 ft^3/s) / 0.0233 ft^2 = 0.957 ft/s

At T = 303 K The density of water (ρ = 62.18 lb/ft^3),

The dynamic viscosity ($\mu = 5.38 \times 10^{-4}$) lb/ft.s

$$Re = \frac{\rho u d}{\mu} = \frac{62.18\ lb/ft^3 (0.957\ ft/s)(0.172\ ft)}{5.38\times10^{-4}\ lb/ft.s} = 1.902 \times 10^4 \text{ (turbulent)}$$

Using S.I. units

At T = 303 K The density of water (ρ = 996 kg/m^3),

The dynamic viscosity ($\mu = 8.007 \times 10^{-4}$) kg/m.s (or Pa.s)

Pipe diameter (d) = 0.172 ft (m/3.28 ft) = 0.0525m

Average velocity (u) =0.957 ft/s (m/3.28 ft) = 0.2917 m/s

$$Re = \frac{996\ kg/m^3 (0.2917 m/s)(0.0525 m)}{8.007\times10^{-4}\ kg/m.s} = 1.905 \times 10^4 \text{ (turbulent)}$$

4.4 Overall Mass Balance and Continuity Equation

In fluid dynamics, fluids are in motion. Generally, they are moved from place to place by means of mechanical devices such as pumps or blowers, by gravity head, or by pressure, and flow through systems of piping and/or process equipment.

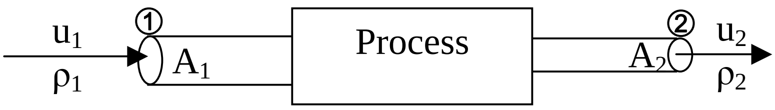
The first step in the solution of flow problems is generally to apply the principles of the conservation of mass to the whole system or any part of the system.

INPUT – OUTPUT = ACCUMULATION

At steady state, the rate of accumulation is zero

∴ INPUT = OUTPUT

In the following Figure a simple flow system is shown where fluid enters section ① with an average velocity (u_1) and density (ρ_1) through the cross-sectional area (A_1). The fluid leaves section ② with an average velocity (u_2) and density (ρ_1) through the cross-sectional area (A_2).



Thus,

At steady state $\dot{m}_1 = \dot{m}2$

$$Q_1\ \rho_1 = Q_2\ \rho_2$$

$$u_1\ A_1\ \rho_1 = u_2\ A_2\ \rho_2$$

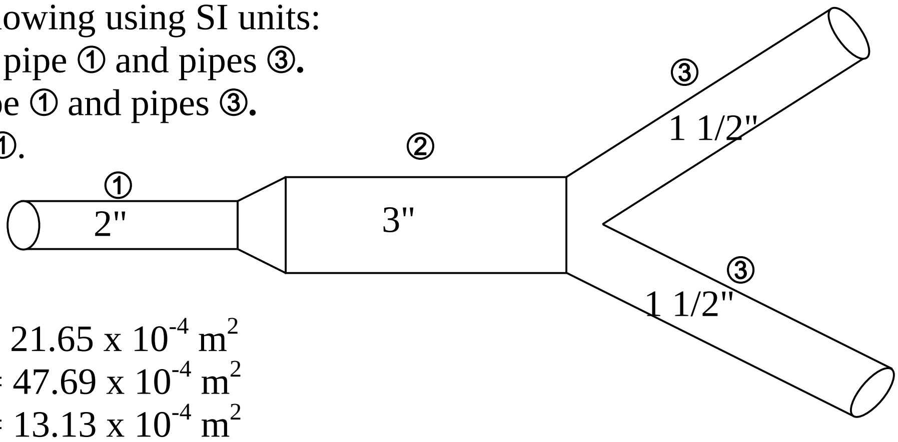
For incompressible fluids at the same temperature [$\rho_1 = \rho_2$]

$$\therefore\ u_1\ A_1 = u_2\ A_2$$

Example -4.2- **************

A petroleum crude oil having a density of 892 kg/m^3 is flowing, through the piping arrangement shown in the below Figure, at total rate of 1.388 x 10^{-3} m^3/s entering pipe ①. The flow divides equally in each of pipes ③. The steel pipes are schedule 40 pipe. **Table{{{{}}}.** Calculate the following using SI units:

a- The total mass flow rate in pipe ① and pipes ③.
b- The average velocity in pipe ① and pipes ③.
c- The mass velocity in pipe ①.



Solution:

Pipe ① I.D. = 0.0525 m, A1 = 21.65 x 10^{-4} m^2
Pipe ② I.D. = 0.07792 m, A_1 = 47.69 x 10^{-4} m^2
Pipe ③ I.D. = 0.04089 m, A_1 = 13.13 x 10^{-4} m^2

a- the total mass flow rate is the same through pipes ① and ② and is

$\dot{m}_1 = Q_1\,\rho$ = 1.388 x 10^{-3} m^3/s (892 kg/m^3) = 1.238 kg/s

Since the flow divides equally in each pipes ③'

$\Rightarrow\ \dot{m}_3 = \dot{m}_1/2 = 1.238/2 =$ 0.619 kg/s

b- $\dot{m}_1 = Q_1\,\rho = u_1 A_1\,\rho \ \Rightarrow\ u_1 = \dfrac{\dot{m}_1}{A_1\,\rho} = \dfrac{1.238\ kg/s}{(21.65\times10^{-4}\,m^2)(892\ kg/m^3)}$ = 0.641 m/s

$u_3 = \dfrac{\dot{m}_3}{A_3\,\rho} = \dfrac{0.619\ kg/s}{(13.13\times10^{-4}\,m^2)(892\ kg/m^3)}$ = 0.528 m/s

d- $G_1 = u_1\,\rho_1$ = 0.641 m/s (892 kg/m^3) = 572 kg/m^2.s

e- or $G_1 = \dfrac{\dot{m}_1}{A_1} = \dfrac{1.238\ kg/s}{21.65\times10^{-4}\,m^2}$ = 572 kg/m^2.s

4.5 Energy Relationships and Bernoulli's Equation

The total energy of a fluid in motion consists of the following components: -

Internal Energy (U)

This is the energy associated with the physical state of fluid, i.e. the energy of atoms and molecules resulting from their motion and configuration. Internal energy is a function of temperature. It can be written as (U) energy per unit mass of fluid.

Potential Energy (PE)

This is the energy that a fluid has because of its position in the earth's field of gravity. The work required to raise a unit mass of fluid to a height (z) above a datum line is (zg), where (g) is gravitational acceleration. This work is equal to the potential energy per unit mass of fluid above the datum line.

Kinetic Energy (KE)

This is the energy associated with the physical state of fluid motion. The kinetic energy of unit mass of the fluid is (u^2/2), where (u) is the linear velocity of the fluid relative to some fixed body.

Pressure Energy (Prss.E)

This is the energy or work required to introduce the fluid into the system without a change in volume. If (P) is the pressure and (V) is the volume of a mass (m) of fluid, then ($PV/m \equiv P\upsilon$) is the pressure energy per unit mass of fluid. The ratio (V/m) is the fluid density (ρ).

The total energy (E) per unit mass of fluid is given by the equation: -

$$\boxed{E = U + zg + P/\rho + u^2/2}$$

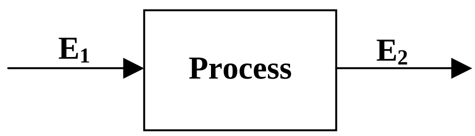
where, each term has the dimension of force times distance per unit mass. In calculation, each term in the equation must be expressed in the same units, such as J/kg, Btu/lb or lb_f.ft/lb. i.e. $(MLT^{-2})(L)(M^{-1}) = [L^2T^{-2}] \equiv \{m^2/s^2, ft^2/s^2\}$.

A flowing fluid is required to do work in order to overcome viscous frictional forces that resist the flow.

The principle of the conservation of energy will be applied to a process of input and output streams for ideal fluid of constant density and without any pump present and no change in temperature.

$E_1 = E_2$

$U_1 + z_1 g + P_1/\rho + u_1^2/2 = U_2 + z_2 g + P_2/\rho + u_2^2/2$



$U_1 = U_2$ (no change in temperature)

$P_1/\rho + u_1^2/2 + z_1 g = P_2/\rho + u_2^2/2 + z_2 g$

$\Rightarrow P/\rho + u^2/2 + z g = \text{constant}$

$\Rightarrow \Delta P/\rho + \Delta u^2/2 + \Delta z g = 0$ --------- Bernoulli's equation

4.6 Equations of Motion

According to Newton's second law of motion, *the net force* in x-direction (F_x) acting on a fluid element in x-direction is: -

F_x = (mass) x (acceleration in x-direction)

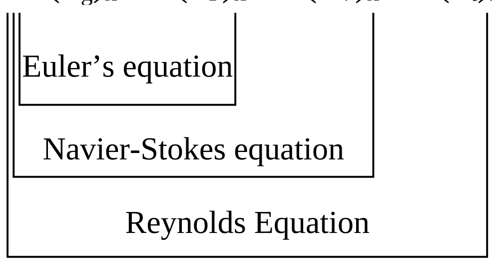
$F_x = (m)(a_x)$

In the fluid flow the following forces are present: -

1- F_g ---------force due to gravity
2- F_P ---------force due to pressure
3- F_V ---------force due to viscosity
4- F_t ---------force due to turbulence
5- F_c ---------force due to compressibility
6- F_σ ---------force due to surface tension

The net force is could be given by

$F_x = (F_g)_x + (F_P)_x + (F_V)_x + (F_t)_x + (F_c)_x + (F_\sigma)_x$



In most of the problems of fluid in motion the forces due to surface tension (F_σ), and the force due to compressibility (F_c) are neglected,

$\Rightarrow \quad F_x = (F_g)_x + (F_P)_x + (F_V)_x + (F_t)_x$

This equation is called "Reynolds equation of motion" which is useful in the analysis of turbulent flow.

In laminar (viscous) flow, the turbulent force becomes insignificant and hence the equation of motion may be written as: -

$F_x = (F_g)_x + (F_P)_x + (F_V)_x$

This equation is called "Navier-Stokes equation of motion" which is useful in the analysis of viscous flow.

If the flowing fluid is ideal and has very small viscosity, the viscous force and viscosity being almost insignificant and the equation will be: -

$F_x = (F_g)_x + (F_P)_x$

This equation is called "Euler's equation of motion".

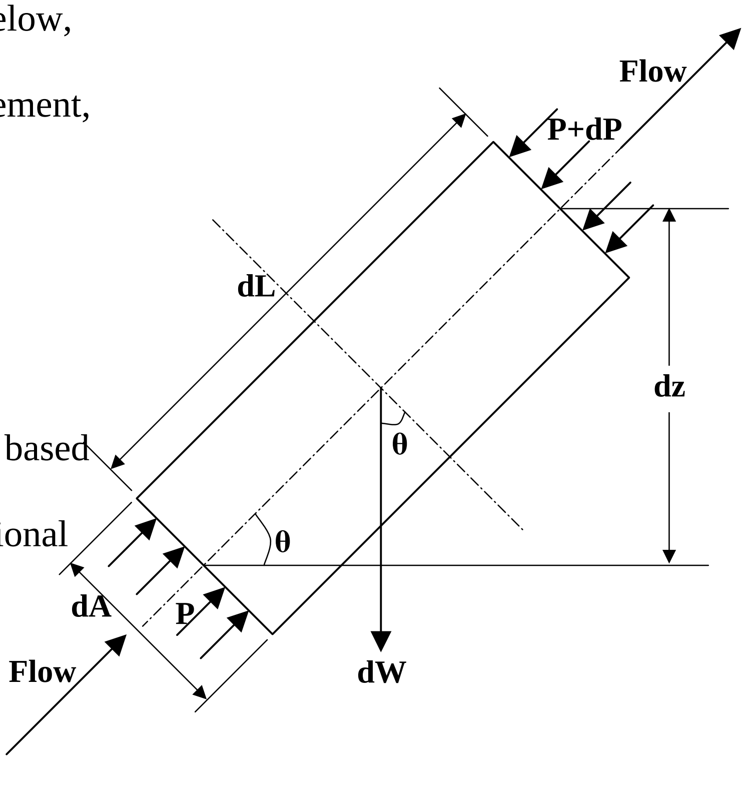
4.6.1 Euler's equation of motion

The Euler's equation for steady state flow on an ideal fluid along a streamline is based on the Newton's second law of motion. The integration of the equation gives Bernoulli's equation in the form of energy per unit mass of the flowing fluid.

Consider a steady flow of an ideal fluid along a streamline. Now consider a small element of the flowing fluid as shown below,

Let:

dA: cross-sectional area of the fluid element,
dL: Length of the fluid element'
dW: Weight of the fluid element'
u: Velocity of the fluid element'
P: Pressure of the fluid element'



The Euler's equation of motion is based on the following assumption: -

1- The fluid is non-viscous (the frictional losses are zero).
2- The fluid is homogenous and Incompressible (the density of fluid is constant).
3- The flow is continuous, steady, and along the streamline (laminar).
4- The velocity of flow is uniform over the section.
5- No energy or force except gravity and pressure forces is involved in the flow.

The forces on the cylindrical fluid element are,

1- Pressure force acting on the direction of flow (PdA)
2- Pressure force acting on the opposite direction of flow [(P+dP)dA]
3- A component of gravity force acting on the opposite direction of flow (dW sin θ)

- The pressure force in the direction of low

$F_P = PdA – (P+dP)\, dA = – dPdA$

- The gravity force in the direction of flow

$F_g = – dW \sin\theta$ $\{W=m\, g = \rho\, dA\, dL\, g\}$

$= – \rho\, g\, dA\, dL \sin\theta$ $\{\sin\theta = dz / dL\}$

$= – \rho\, g\, dA\, dz$

- The net force in the direction of flow

$F = m\, a$ $\{ m = \rho\, dA\, dL \}$

$= \rho\, dA\, dL\, a$ $\{ a = \frac{du}{dt} = \frac{du}{dL} \times \frac{dL}{dt} = u \frac{du}{dL} \}$

$= \rho\, dA\, u\, du$

We have

$F_x = (F_g)_x + (F_P)_x$

$\rho\, dA\, u\, du = – dP\, dA – \rho\, g\, dA\, dz$ $\{\div – \rho\ dA\, z\}$

$\Rightarrow$ $\boxed{dP/\rho + du^2/2 + dz\, g = 0}$ --------- Euler's equation of motion

Bernoulli's equation could be obtain by integration the Euler's equation

$\int dP/\rho + \int du^2/2 + \int dz\, g = \text{constant}$

$\Rightarrow$ $P/\rho + u^2/2 + z\, g = \text{constant}$

$\Rightarrow$ $\Delta P/\rho + \Delta u^2/2 + \Delta z\, g = 0$ --------- Bernoulli's equation

4.7 Modification of Bernoulli's Equation

1- Correction of the kinetic energy term

The velocity in kinetic energy term is the mean linear velocity in the pipe. To account the effect of the velocity distribution across the pipe [(α) dimensionless correction factor] is used.

For a circular cross sectional pipe:

- $\alpha = 0.5$ for laminar flow
- $\alpha = 1.0$ for turbulent flow

2- Modification for real fluid

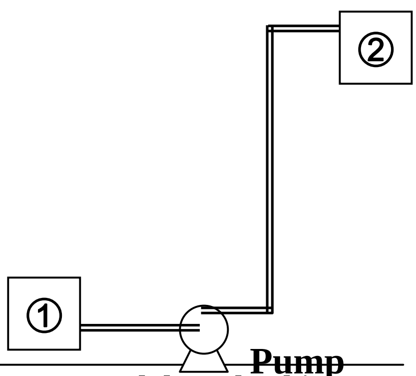
The real fluids are viscous and hence offer resistance to flow. Friction appears wherever the fluid flow is surrounding by solid boundary. Friction can be defined as the amount of mechanical energy irreversibly converted into heat in a flow in stream. As a result of that the total energy is always decrease in the flow direction i.e. ($E_2 < E_1$). Therefore $E_1 = E_2 + F$, where F is the energy losses due to friction.

Thus the modified Bernoulli's equation becomes,

$\boxed{P_1/\rho + u_1^2/2 + z_1\, g = P_2/\rho + u_2^2/2 + z_2\, g + F}$ ---------(J/kg ≡ m^2/s^2)

3- Pump work in Bernoulli's equation

A pump is used in a flow system to increase *the mechanical energy* of the fluid. The increase being used to maintain flow of the fluid. Assume a pump is installed between the stations ① and ② as shown in Figure. The work supplied to the pump is shaft work ($– W_s$), the negative sign is due to work added to fluid.



Frictions occurring within the pump are: -

a- Friction by fluid

b- Mechanical friction

Since the shaft work must be discounted by these frictional force (losses) to give net mechanical energy as actually delivered to the fluid by pump (W_p).

Thus, $W_p = \eta\, W_s$ where η, is the efficiency of the pump.

Thus the modified Bernoulli's equation for present of pump between the two selected points ① and ② becomes,

$$\boxed{\frac{P_1}{\rho}+\frac{u_1^2}{2\alpha_1}+g\ z_1+\eta\ Ws=\frac{P_2}{\rho}+\frac{u_2^2}{2\alpha_2}+g\ z_2+F} \quad \text{---------}(J/kg \equiv m^2/s^2)$$

By dividing each term of this equation by (g), each term will have a length units, and the equation will be: -

$$\boxed{\frac{P_1}{\rho\ g}+\frac{u_1^2}{2\alpha_1 g}+z_1+\frac{\eta\ Ws}{g}=\frac{P_2}{\rho\ g}+\frac{u_2^2}{2\alpha_2 g}+z_2+h_F} \quad \text{---------}(m)$$

where $h_F = F/g$ ≡ head losses due to friction.

4.8 Friction in Pipes

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of energy of fluid is lost. This loss of energy is classified on: -

Energy Losses

Major energy losses
(Skin friction)
Due to surface skin of the pipe

Minor energy losses
(Form friction)

- Sudden expansion or contraction pipe
- Bends, valves and fittings
- An obstruction in pipe

4.8.1 Relation between Skin Friction and Wall Shear Stress

For the flow of a fluid in short length of pipe (dL) of diameter (d), the total frictional force at the wall is the product of shear stress (τ_{rx}) and the surface area of the pipe (π d dL). This frictional force causes a drop in pressure ($-dP_{fs}$).

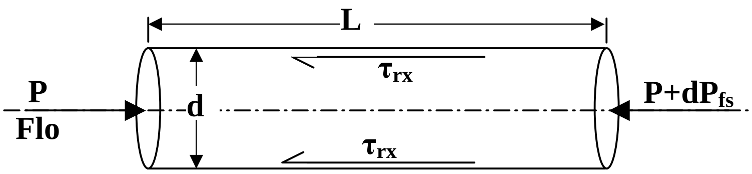
Consider a horizontal pipe as shown in Figure;

Force balance on element (dL)

$$\tau(\pi\ d\ dL) = [P-(P+dP_{fs})]\ (\pi/4\ d^2)$$

$$\Rightarrow \quad \boxed{-dP_{fs} = 4(\tau\ dL/d) = 4\ (\tau/\rho\ u_x^2)\ (dL/d)\ \rho\ u_x^2} \quad \text{------------------------}(*)$$

where, $\boxed{(\tau/\rho\ u_x^2) = \Phi = J_f = f/2 = f'/2}$



Φ(or J_f): Basic friction Factor

f: Fanning (or Darcy) friction Factor

f′: Moody friction Factor.

For incompressible fluid flowing in a pipe of constant cross-sectional area, (u) is not a function of pressure or length and equation (*) can be integrated over a length (L) to give the equation of pressure drop due to skin friction:

$-\Delta P_{fs} = 4f\,(L/d)\,(\rho u^2/2)$ ---------------------(Pa)

The energy lost per unit mass F_s is then given by:

$F_s = (-\Delta P_{fs}/\rho) = 4f\,(L/d)\,(u^2/2)$ -----------------(J/kg) or (m^2/s^2)

The head loss due to skin friction (h_{Fs}) is given by:

$h_{Fs} = F_s/g = (-\Delta P_{fs}/\rho g) = 4f\,(L/d)\,(u^2/2g)$ ---------------(m)

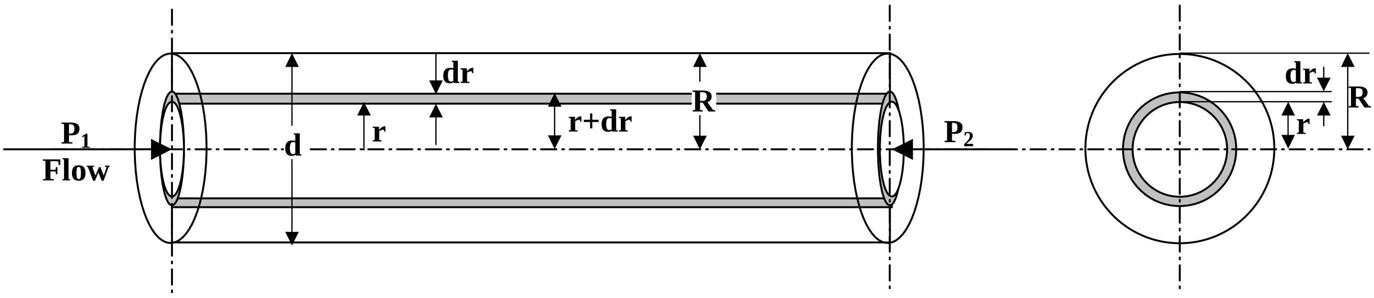
Note: -

- All the above equations could be used for laminar and turbulent flow.
- $\Delta P_{fs} = P_2 - P_1 \Rightarrow -\Delta P_{fs} = P_1 - P_2$ (+ve value)

4.8.2 Evaluation of Friction Factor in Straight Pipes

1. **Velocity distribution in laminar flow**

Consider a horizontal circular pipe of a uniform diameter in which a Newtonian, incompressible fluid flowing as shown in Figure:



Consider the cylinder of radius (r) sliding in a cylinder of radius (r+dr).
Force balance on cylinder of radius (r)

$\tau_{rx}\,(2\pi\, r\, L) = (P_1 - P_2)\,(\pi\, r^2)$

for laminar flow $\tau_{rx} = -\,\mu\,(du_x/dr)$

$\Rightarrow r\,(P_1 - P_2) = -\,\mu\,(du_x/dr)\,2L \Rightarrow [(P_2 - P_1)/(2L\,\mu)]\, r\, dr = du_x$

$\Rightarrow [\Delta P_{fs}/(2L\,\mu)]\, r^2/2 = u_x + C$

- **Boundary Condition (1)** (for evaluation of C)

at $r = R$ $u_x = 0$ $\Rightarrow C = [(\Delta P_{fs}\, R^2)/(4L\,\mu)]$

$\Rightarrow [(\Delta P_{fs}\, r^2)/(4L\,\mu)] = u_x + [(\Delta P_{fs}\, R^2)/(4L\,\mu)]$

$\Rightarrow u_x = [(-\Delta P_{fs}\, R^2)/(4L\,\mu)][1 - (r/R)^2]$ velocity distribution (profile) in laminar flow

- **Boundary Condition (2)** (for evaluation of u_{max})

at $r = o$ $u_x = u_{max}$ $\Rightarrow u_{max} = [(-\Delta P_{fs}\, R^2)/(4\,L\,\mu)]$

$\Rightarrow u_{max} = [(-\Delta P_{fs}\, d^2)/(16\,L\,\mu)]$ ----------centerline velocity in laminar flow

∴ $u_x / u_{max} = [1-(r/R)^2]$ ---------velocity distribution (profile)in laminar flow

2. Average (mean) linear velocity in laminar flow

$Q = u A$----------------------- (1)

Where, (u) is the average velocity and (A) is the cross-sectional area = (πR^2)

$dQ = u_x\, dA$ where $u_x = u_{max}[1-(r/R)^2]$, and $dA = 2\pi\, r\, dr$

$\Rightarrow dQ = u_{max}[1-(r/R)^2]\, 2\pi\, r\, dr$

$$\int_0^Q dQ = 2\pi\, u_{max} \int_0^R (r - \frac{r^3}{R^2})\, dr = 2\pi\, u_{max} \left| \frac{r^2}{2} - \frac{r^4}{4R^2} \right|_0^R$$

$\Rightarrow Q = u_{max}/2\, (\pi R^2)$ ----------------------- (2)

By equalization of equations (1) and (2)

$\Rightarrow$ $u = u_{max}/2 = [(-\Delta P_{fs}\, R^2)/(8L\, \mu)] = [(-\Delta P_{fs}\, d^2)/(32\, L\, \mu)]$ average velocity in laminar flow

∴ $-\Delta P_{fs} = (32\, L\, \mu\, u) / d^2$ Hagen–Poiseuille equation

3. Friction factor in laminar flow

We have $-\Delta P_{fs} = 4f\, (L/d)\, (\rho u^2/2)$----------------------(3)

and also $-\Delta P_{fs} = (32\, L\, \mu\, u) / d^2$ ----------------------(4)

By equalization of these equations [i.e. eqs. (3) and (4)]

$\Rightarrow (32\, L\, \mu\, u) / d^2 = 4f\, (L/d)\, (\rho u^2/2) \Rightarrow f = 16\, \mu /(\rho\, u\, d)$

∴ $f = 16 / Re$ Fanning or Darcy friction factor in laminar flow.

4. Velocity distribution in turbulent flow

The velocity, at any point in the cross-section of cylindrical pipe, in turbulent flow is proportional to the one-seventh power of the distance from the wall. This may be expressed as follows: -

$u_x / u_{max} = [1-(r/R)]^{1/7}$ Prandtl one-seventh law equation. velocity distribution (profile)in laminar flow

5. Average (mean) linear velocity in Turbulent flow

$Q = u A$---------------------- (1)

$dQ = u_x\, dA$ where $u_x = u_{max}\, [1-(r/R)]^{1/7}$, and $dA = 2\pi\, r\, dr$

$\Rightarrow dQ = u_{max}\, [1-(r/R)]^{1/7}\, 2\pi\, r\, dr$

$$\int_0^Q dQ = 2\pi\, u_{max} \int_0^R r(1-\frac{r}{R})^{1/7}\, dr$$

Let $M = (1- r/R)$ $dM = (-1/R)\, dr$

or $r = R(1 - M)$ $dr = - R\, dM$

at $r = 0$ $M=1$

at $r = R$ $M=0$

Rearranging the integration

$$Q = u_{max}\, 2\pi\, R^2 \int_1^0 (1-M) M^{1/7} (-dM) = u_{max}\, 2\pi\, R^2 \int_1^0 (M^{1/7} - M^{8/7})\, dM$$

$$Q = u_{max}\, 2\pi\, R^2 \left| \frac{M^{8/7}}{8/7} - \frac{M^{15/7}}{15/7} \right|_0^1 = u_{max}\, 2\pi\, R^2 \left[\frac{7}{8} - \frac{7}{15} \right]$$

$\Rightarrow Q = 49/60\ u_{max}\ (\pi R^2)$ ----------------------- (5)

By equalization of equations (1) and (5)

$\therefore$ $\boxed{u = 49/60\ u_{max} \approx 0.82\ u_{max}}$ ------------average velocity in turbulent flow

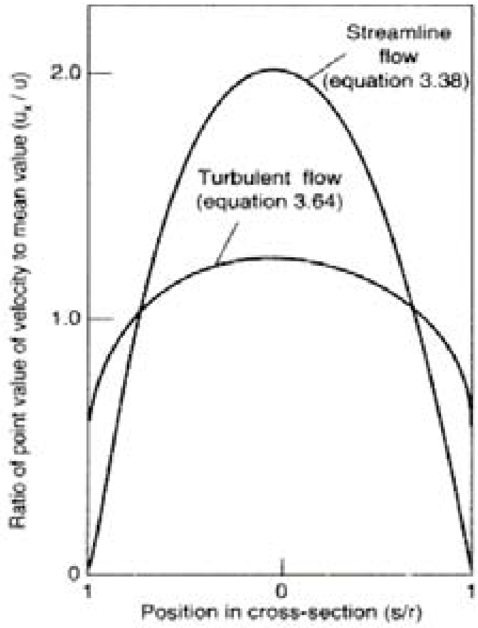


Figure of the shape of velocity profiles for streamline and turbulent flow

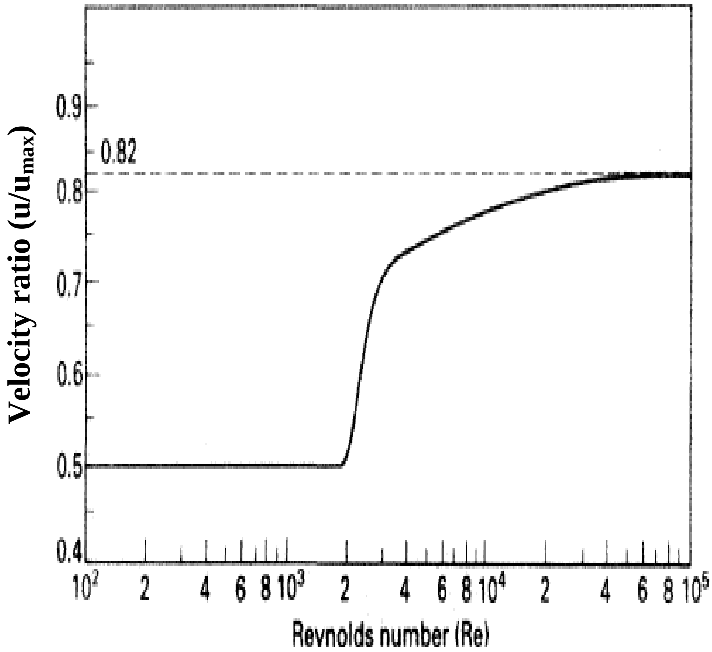


Figure of the Variation of (u/u_{max}) with Reynolds number in a pipe

6. <u>Friction factor in Turbulent flow</u>

A number of expressions have been proposed for calculating friction factor in terms of or function of (Re). Some of these expressions are given here: -

$$f = \frac{0.079}{Re^{0.25}} \qquad \text{for } 2{,}500 < Re < 100{,}000$$

and, $$f^{-0.5} = 4\log(Re\ f^{0.5}) - 0.4 \qquad \text{for } 2{,}500 < Re < 10{,}000{,}000$$

These equations are for ***<u>smooth pipes</u>*** in turbulent flow. For rough pipes, the ratio of (e/d) acts an important role in evaluating the friction factor in turbulent flow as shown in the following equation

$$(f/2)^{-0.5} = -2.5\ln\left[0.27\frac{e}{d} + 0.885\,Re^{-1}(f/2)^{-0.5}\right]$$

Table of the roughness values *e*.

Surface type	ft	mm
Planed wood or finished concrete	0.00015	0.046
Unplaned wood	0.00024	0.073
Unfinished concrete	0.00037	0.11
Cast iron	0.00056	0.17
Brick	0.00082	0.25
Riveted steel	0.0017	0.51
Corrugated metal	0.0055	1.68
Rubble	0.012	3.66

7. Graphical evaluation of friction factor

As with the results of Reynolds number the curves are in three regions (Figure 3.7 vol.I). At low values of Re (Re < 2,000), the friction factor is independent of the surface roughness, but at high values of Re (Re > 2,500) the friction factor vary with the surface roughness. At very high Re, the friction factor become independent of Re and a function of the surface roughness only. Over the transition region of Re from 2,000 to 2,500 the friction factor increased rapidly showing the great increase in friction factor as soon as turbulent motion established.

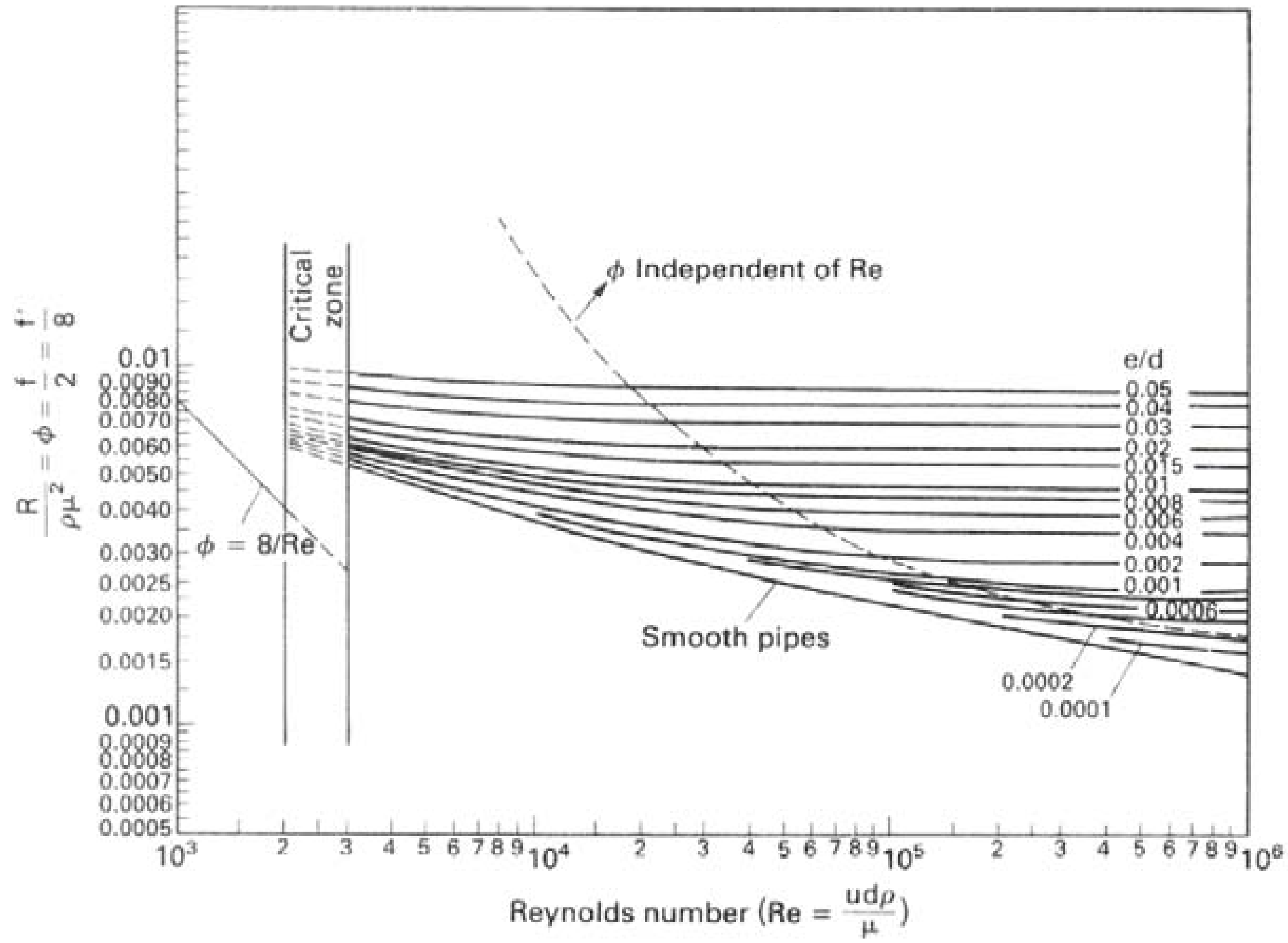
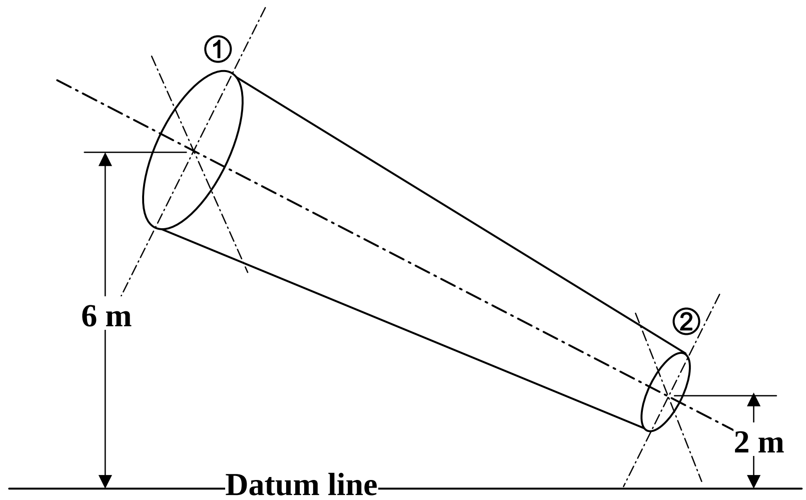


Figure (3.7) Pipe friction chart *Φ* versus *Re*

Example -4.3-

Water flowing through a pipe of 20 cm I.D. at section ① and 10 cm at section ②. The discharge through the pipe is 35 lit/s. The section ① is 6 m above the datum line and section ② is 2 m above it. If the pressure at section ① is 245 kPa, find the intensity of pressure at section ②. Given that ρ = 1000 kg/m^3, μ= 1.0 mPa.s.



Solution:

$Q = 35$ lit/s $= 0.035\ m^3/s$

$u = Q/A \quad \Rightarrow u_1 = (0.035\ m^3/s) / (0.2^2\ \pi/4)\ m^2 = 1.114$ m/s

$\Rightarrow u_2 = (0.035\ m^3/s) / (0.1^2\ \pi/4)\ m^2 = 4.456$ m/s

$Re = \rho u d / \mu \quad \Rightarrow Re_1 = (1000\ kg/m^3 \times 1.114\ m/s \times 0.2\ m) / (0.001 Pa.s) = 222{,}800$

$Re = \rho u d / \mu \quad \Rightarrow Re_2 = (1000\ kg/m^3 \times 4.456\ m/s \times 0.1\ m) / (0.001 Pa.s) = 445{,}600$

The flow is turbulent along the tube (i.e. $\alpha_1 = \alpha_2 = 1.0$)

$$\frac{P_1}{\rho} + \frac{u_1^2}{2\alpha_1} + g\ z_1 + \eta W_s = \frac{P_2}{\rho} + \frac{u_2^2}{2\alpha_2} + g\ z_2 + F$$

$$\Rightarrow P_2 = \rho\left[\frac{P_1}{\rho} + g(z_1 - z_2) + \left(\frac{u_1^2}{2\alpha_1} - \frac{u_2^2}{2\alpha_2}\right)\right] = 253.3 \text{ kPa}$$

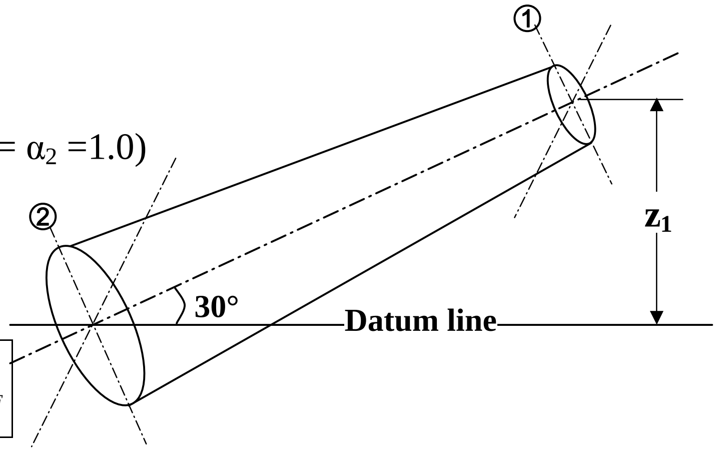
H.W.

If the pipe is smooth and its length is 20 m, find P_2. Ans. $P_2 = 246.06$ kPa

Example -4.4-

A conical tube of 4 m length is fixed at an inclined angle of 30° with the horizontal-line and its small diameter upwards. The velocity at smaller end is ($u_1 = 5$ m/s), while ($u_2 = 2$ m/s) at other end. The head losses in the tub is $[0.35\ (u_1 - u_2)^2/2g]$. Determine the pressure head at lower end if the flow takes place in down direction and the pressure head at smaller end is 2 m of liquid.

Solution:



No information of the fluid properties.

Then assume the flow is turbulent, (i.e. $\alpha_1 = \alpha_2 = 1.0$)

$z_1 = L \sin\theta$

$= 4 \sin 30$

$= 2$ m

$$\frac{P_1}{\rho\ g} + \frac{u_1^2}{2\alpha_1 g} + z_1 + \frac{\eta W s}{g} = \frac{P_2}{\rho\ g} + \frac{u_2^2}{2\alpha_2 g} + z_2 + h_F$$

$$\frac{P_2}{\rho\ g} = \frac{P_1}{\rho\ g} + z_1 + \frac{u_1^2 - u_2^2}{2g} - 0.35\frac{(u_1 - u_2)}{2g}$$

$= 2.0 + 2.0 + (25 - 4)/(2 \times 9.81) - 0.35(5-2)^2 / (2 \times 9.81) = 4.9$ m

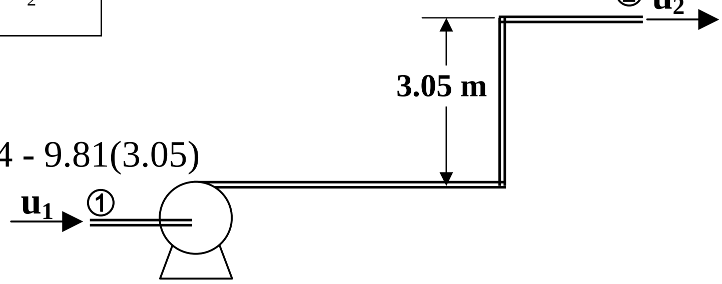
Example -4.5-

Water with density $\rho = 998\ kg/m^3$, is flowing at steady mass flow rate through a uniform-diameter pipe. The entrance pressure of the fluid is 68.9 kPa in the pipe, which connects to a pump, which actually supplies 155.4 J/kg of fluid flowing in the pipe. The exit pipe from the pump is the same diameter as the inlet pipe. The exit section of the pipe is 3.05 m higher than the entrance, and the exit pressure is 137.8 kPa. The Reynolds number in the pipe is above 4,000 in this system. Calculate the frictional loss (F) in the pipe system.

Solution:

Setting the datum line at z_1 thus, $z_1 = 0$, $z_2 = 3.05$ m

$$\frac{P_1}{\rho}+\cancel{\frac{u_1^2}{2\alpha_1}}+g\,z_1+\eta\,W_s=\frac{P_2}{\rho}+\cancel{\frac{u_2^2}{2\alpha_2}}+g\,z_2+F$$



$$\Rightarrow F=\left[\frac{P_1-P_2}{\rho}+\eta\,Ws-g\,z_2\right]$$

= (68.9 - 137.8) x 1000/998 +155.4 - 9.81(3.05)

= 56.5 J/kg or m^2/s^2

Example -4.6

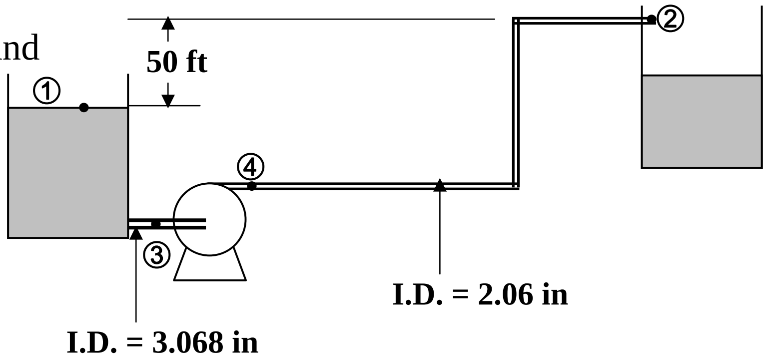
A pump draws 69.1 gal/min of liquid solution having a density of 114.8 lb/ft^3 from an open storage feed tank of large cross-sectional area through a 3.068″I.D. suction pipe. The pump discharges its flow through a 2.067″I.D. line to an open over head tank. The end of the discharge line is 50′ above the level of the liquid in the feed tank. The friction losses in the piping system are F = 10 ft lb_f/lb. what pressure must the pump develop and what is the horsepower of the pump if its efficiency is η=0.65.

Solution:

No information of the type of fluid and then its viscosity, therefore assume the flow is turbulent.

$P_1 = P_2$ = atmospheric press.

$u_1 \approx 0$ large area of the tank



$$\cancel{\frac{P_1}{\rho}}+\cancel{\frac{u_1^2}{2\,\alpha_1\,g_c}}+\cancel{\frac{z_1 g}{g_c}}+\eta\,W_s=\cancel{\frac{P_2}{\rho}}+\frac{u_2^2}{2\,\alpha_2\,g_c}+\frac{z_2 g}{g_c}+F$$

$$\Rightarrow \eta\,Ws=\left[\frac{g\,z_2}{g_c}+\frac{u_2^2}{2\,g_c}+F\right]$$

Q = 69.1 gal/min (ft^3/7.48 gal)(min/60 s) = 0.154 ft^3/s

A_3 (area of suction line) = $\pi/4$ (3.068 in)^2 (ft/12 in)^2 = 0.0513 ft^2

$A_4 = A_2$ (area of discharge line) = $\pi/4$ (2.067 in)^2 (ft/12 in)^2 = 0.0235 ft^2

$u_2 = Q / A_2$ = (0.154 ft^3/s) / (0.0235 ft^2) = 6.55 ft/s

$u_3 = Q / A_3$ = (0.154 ft^3/s) / (0.0513 ft^2) = 3.0 ft/s

$$\Rightarrow\ \eta\,Ws=\frac{32.174\,ft/s^2\times 50\ \ ft}{32.174\,lb\ \ ft/lb_f\ \ s^2}+\frac{(6.55\ \ ft/s)^2}{2\times 32.174 lb\ \ ft/lb_f\ \ s^2}+10\,ft\ lb_f\ /lb = 60.655 \text{ ft } lb_f/lb$$

Ws = ηWs/η = 60.655/0.65 = 93.3 lb_f ft/lb

Mass flow rate $\dot{m}=Q\rho$ = 0.1539 ft^3/s (114.8 lb/ft^3) = 17.65 lb/s

Power required for pump = $\dot{m}\,Ws$ = 17.65 lb/s (93.3 ft lb_f/lb)(hp/550 ft lb_f/s)

= 3.0 hP

To calculate the pressure that must be developed by the pump, Energy Balance equation must be applied over the pump itself (points ③ and ④)

$u_4 = u_2$ = 6.55 ft / s and u_3 = 3 ft / s

$$\frac{P_3}{\rho}+\frac{u_3^2}{2\,\alpha_1\,g_c}+\frac{z_3 g}{g_c}+\eta\,W_s=\frac{P_4}{\rho}+\frac{u_4^2}{2\,\alpha_2\,g_c}+\frac{z_4 g}{g_c}+F$$

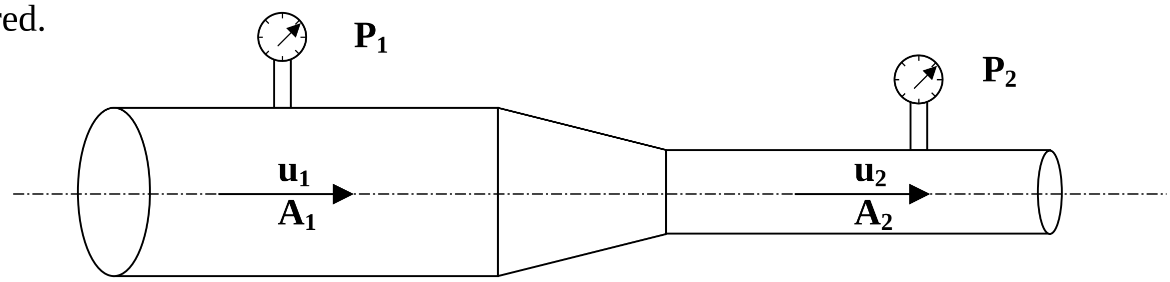
$$\Rightarrow \frac{P_4-P_3}{\rho}=\eta\;Ws+\frac{u_3^2-u_4^2}{2\,g_c}\;=\;60.655+(-\,0.527)=60.13\text{ ft lb}_f\text{ / lb}$$

$\Rightarrow$ ΔP = 60.13 ft lb_f/lb (114.8) lb/ft^3 = 69.03 lb_f/ft^2
= 47.94 psi
= 3.26 bar

Example -4.7-

A liquid with a constant density (ρ) is flowing at an unknown velocity (u_1) through a horizontal pipe of cross-sectional area (A_1) at a pressure (P_i), and then it passes to a section of the pipe in which the area is reduced gradually to (A_2) and the pressure (P_2). Assume no friction losses, find the velocities (u_1) and (u_2) if the pressure difference (P_1-P_2) is measured.

Solution:



From continuity equation $\dot{m}=\dot{m}_1=\dot{m}_2$ $\Rightarrow$ ρ Q = ρ_1 Q_1 = ρ_2 Q_2

And for constant density $\Rightarrow$Q = Q_1 = Q_2 $\Rightarrow$ u A = u_1 A_1 = u_2 A_2

$$\Rightarrow u_2 = u_1\,A_1/A_2$$

$$\frac{P_1}{\rho}+\frac{u_1^2}{2\alpha_1}+g\,z_1+\eta\,W_s=\frac{P_2}{\rho}+\frac{u_2^2}{2\alpha_2}+g\,z_2+F$$

Assume the flow is turbulent ($\alpha_1 = \alpha_2$)

$$\Rightarrow \frac{P_1-P_2}{\rho}=\frac{u_2^2-u_1^2}{2}=\frac{u_1^2\left[\left(\frac{A_1}{A_2}\right)^2-1\right]}{2}\Rightarrow\quad u_1=\sqrt{\left(\frac{P_1-P_2}{\rho}\right)\frac{2}{\left(\frac{A_1}{A_2}\right)^2-1}}\;,\;u_2=\sqrt{\left(\frac{P_1-P_2}{\rho}\right)\frac{2}{1-\left(\frac{A_1}{A_2}\right)^2}}$$

Example -4.8-

A nozzle of cross-sectional area (A_2) is discharging to the atmosphere and is located in the side of a large tank, in which the open surface of liquid in the tank is (H) above the centerline of the nozzle. Calculate the velocity (u_2) in the nozzle and the volumetric rate of discharge if no friction losses are assumed and the flow is turbulent.

Solution:

Since A_1 is very large compared to A_2 ($\Rightarrow u_1 \approx 0$).
The pressure P_1 is greater than atmosphere pressure by the head of fluid H.
The pressure P_2 which is at nozzle exit, is at atmospheric pressure .