Classical Mechanics

First stage

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***Lecture* Eight**

MOMENTUM AND KINETIC ENERGY IN COLLISIONS

We shall also be interested in the total kinetic energy of a system of two colliding bodies. If that total happens to be unchanged by the collision, then the kinetic energy of the system is *conserved* (it is the same before and after the collision). Such a collision is called an **elastic collision.**

In everyday collisions of common bodies, such as two cars or a ball and a bat, some energy is always transferred from kinetic energy to other forms of energy, such as thermal energy or energy of sound. Thus, the kinetic energy of the system is *not* conserved. Such a collision is called an **inelastic collision**

However, in some situations, we can *approximate* a collision of common bodies as elastic. Suppose that you drop a Superball onto a hard floor. If the collision between the ball and floor (or Earth) were elastic, the ball would lose no kinetic energy because of the collision and would rebound to its original height.

However, the actual rebound height is somewhat short, showing that at least some kinetic energy is lost in the collision and thus that the collision is somewhat inelastic. Still, we might choose to neglect that small loss of kinetic energy to approximate the collision as elastic

The inelastic collision of two bodies always involves a loss in the kinetic energy of the system. The greatest loss occurs if the bodies stick together, in which case the collision is called a **completely inelastic collision.**

One-Dimensional Inelastic Collision

Figure 1 shows two bodies just before and just after they have a onedimensional collision. The velocities before the collision (subscript i) and after the collision (subscript f) are indicated.The two bodies form our system, which is closed and isolated. We can write the law of conservation of linear momentum for this two-body system as

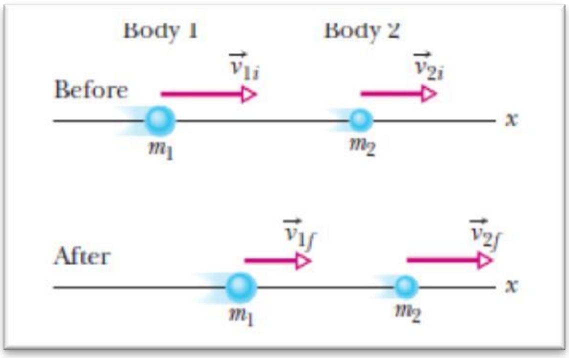

Figure 1 Bodies 1 and 2 move along an x axis, before and after they have an inelastic collision.

$$\begin{pmatrix}\text{total momentum } \vec{P}_i \\ \text{before the collision}\end{pmatrix} = \begin{pmatrix}\text{total momentum } \vec{P}_f \\ \text{after the collision}\end{pmatrix},$$

which we can symbolize as

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \quad \text{(conservation of linear momentum)}. \qquad \ldots\ldots\ldots\ldots(1)$$

Because the motion is one-dimensional, we can drop the overhead arrows for vectors and use only components along the axis, indicating direction with a sign. Thus, from p ! mv, we can rewrite Eq. 1 as

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}. \qquad \text{.............(2)}$$

If we know values for, say, the masses, the initial velocities, and one of the final velocities,
we can find the other final velocity with Eq. 2

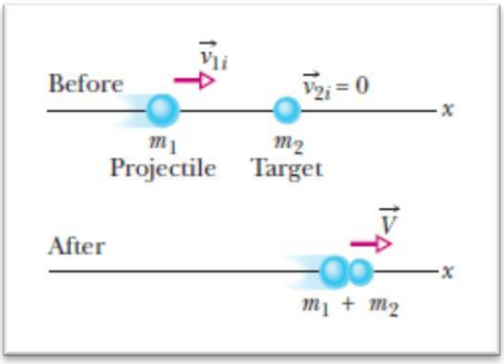

Figure 2 A completely inelastic collision between two bodies. Before the collision, the body with mass m_2 is at rest and the body with mass m_1 moves directly toward it. After the collision, the stucktogether bodies move with the same velocity *V*.

Figure 2 shows two bodies before and after they have a completely inelastic collision (meaning they stick together).The body with mass m_2 happens to be initially at rest ($v_{2i} = 0$).We can refer to that body as the *target* and to the incoming body as the *projectile*. After the collision, the stuck-together bodies move with velocity *V*. For this situation, we can rewrite Eq. 2 as

$$m_1 v_{1i} = (m_1 + m_2)V$$

$$V = \frac{m_1}{m_1 + m_2} v_{1i}.$$

............)3

If we know values for, say, the masses and the initial velocity v_{1i} of the projectile, we can find the final velocity V with Eq. 3. Note that V must be less than v_{1i} because the mass ratio $m_1/(m_1$ & $m_2)$ must be less than unity.

Elastic Collisions in One Dimension

An elastic collision is a special type of collision in which the kinetic energy of a system of colliding bodies is conserved. If the system is closed and isolated, its linear momentum is also conserved. For a onedimensional collision in which body 2 is a target and body 1 is an incoming projectile, conservation of kinetic energy and linear momentum and yield the following expressions for the velocities immediately after the collision:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}.$$

everyday collisions are inelastic but we can approximate some of them as being elastic; that is, we can approximate that the total kinetic energy of the colliding bodies is conserved and is not transferred to other forms of energy.

Collisions in Two Dimensions

If two bodies collide and their motion is not along a single axis (the collision is not head-on), the collision is two-dimensional. If the twobody system is closed and isolated, the law of conservation of momentum applies to the collision and can be written as

$$\vec{P}_{1i} + \vec{P}_{2i} = \vec{P}_{1f} + \vec{P}_{2f}.$$

In component form, the law gives two equations that describe the collision (one equation for each of the two dimensions). If the collision is also elastic (a special case), the conservation of kinetic energy during the collision gives a third equation

$$K_{1i} + K_{2i} = K_{1f} + K_{2f}.$$

Example 1

A 1850 kg luxury sedan stopped at a traffic light is struck from the rear by a compact car with a mass of 975 kg. The two cars become entangled as a result of the collision. If the compact car was moving at a velocity of 22.0 m/s to the north before the collision, what is the velocity of the entangled mass after the colliston?

Given: $m_1 = 1850\ \text{kg}$

$m_2 = 975\ \text{kg}$

$\mathbf{v}_{1,i} = 0\ \text{m/s}$

$\mathbf{v}_{2,i} = 22.0\ \text{m/s to the north}$

Unknown: $\mathbf{v}_f = ?$

Use the equation for a perfectly inelastic collision.

$$m_1\mathbf{v}_{1,i} + m_2\mathbf{v}_{2,i} = (m_1 + m_2)\,\mathbf{v}_f$$

$$\mathbf{v}_f = \frac{m_1\mathbf{v}_{1,i} + m_2\mathbf{v}_{2,i}}{m_1 + m_2}$$

$$\mathbf{v}_f = \frac{(1850\ \text{kg})(0\ \text{m/s}) + (975\ \text{kg})(22.0\ \text{m/s north})}{1850\ \text{kg} + 975\ \text{kg}}$$

$$\boxed{\mathbf{v}_f = 7.59\ \text{m/s to the north}}$$

Example 2

A 0.015 kg marble moving to the right at 0.225 m/s makes an elastic head-on collision with a 0.030 kg shooter marble moving to the left at 0.180 m/s. After the collision, the smaller marble moves to the left at 0.315 m/s. Assume that neither marble rotates before or after the collision and that both marbles are moving on a frictionless surface. What is the velocity of the 0.030 kg marble after the collision?

Given: $m_1 = 0.015$ kg $\quad m_2 = 0.030$ kg

$\mathbf{v_{1,i}} = 0.225$ m/s to the right, $v_{1,i} = +0.225$ m/s

$\mathbf{v_{2,i}} = 0.180$ m/s to the left, $v_{2,i} = -0.180$ m/s

$\mathbf{v_{1,f}} = 0.315$ m/s to the left, $v_{1,f} = -0.315$ m/s

Unknown: $\mathbf{v_{2,f}} = ?$

Diagram:

Choose an equation or situation:
Use the equation for the conservation of momentum to find the final velocity of m_2, the 0.030 kg marble.

$$m_1\mathbf{v_{1,i}} + m_2\mathbf{v_{2,i}} = m_1\mathbf{v_{1,f}} + m_2\mathbf{v_{2,f}}$$

Rearrange the equation to isolate the final velocity of m_2.

$$m_2\mathbf{v_{2,f}} = m_1\mathbf{v_{1,i}} + m_2\mathbf{v_{2,i}} - m_1\mathbf{v_{1,f}}$$

$$\mathbf{v_{2,f}} = \frac{m_1\mathbf{v_{1,i}} + m_2\mathbf{v_{2,i}} -t\, m_1\mathbf{v_{1,f}}}{m_2}$$

$$v_{2,f} = \frac{(0.015\ \text{kg})(0.225\ \text{m/s}) + (0.030\ \text{kg})(-0.180\ \text{m/s}) - (0.015\ \text{kg})(-0.315\ \text{m/s})}{0.030\ \text{kg}}$$

$$v_{2,f} = \frac{(3.4 \times 10{-}3\ \text{kg}\bullet\text{m/s}) + (-5.4 \times 10{-}3\ \text{kg}\bullet\text{m/s}) - (-4.7 \times 10{-}3\ \text{kg}\bullet\text{m/s})}{0.030\ \text{kg}}$$

$$v_{2,f} = \frac{2.7 \times 10^{-3}\ \text{kg}\bullet\text{m/s}}{3.0 \times 10^{-2}\ \text{kg}}$$

$$\mathbf{v_{2,f}} = 9.0 \times 10^{-2}\ \text{m/s to the right}$$