Classical Mechanics

First stage

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Lecture TWO

Vectors

3.1 Vector and Scalar Quantities

A scalar quantity is completely specified by a single value with an appropriate unit and has no direction.

Examples of scalar quantities are temperature, volume, mass, speed, and time intervals.

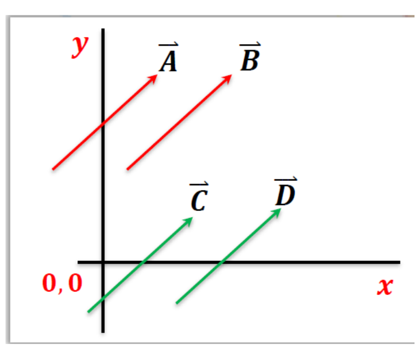
A vector quantity is completely specified by a number with an appropriate unit plus a direction. Examples of vector quantity are displacement and velocity.

3.2 Some properties of Vectors

Equality

It is said about two vector that are equal if they have equal magnitudes and same direction re- gard less of their starting point... notice figure(1) (Vectors $\vec{A}$, $\vec{B}$,$\vec{C}$,$\vec{D}$) are equal vectors and can be written as:

$$\vec{A}=\vec{B}=\vec{C}=\vec{D}$$



figure(1)

Not vector A has starting point P_1 and ending point P2, and vector B has starting point Ps and ending point P. and we can say that:

A =B Because vector A has a magnitude equal to vector (B) and has same direction.

Negative of a Vector

The negative of vector A is a vector that has the same magnitude of A and an opposite direction of it... notice figure (2) The negative of vector A is represented as A so that:

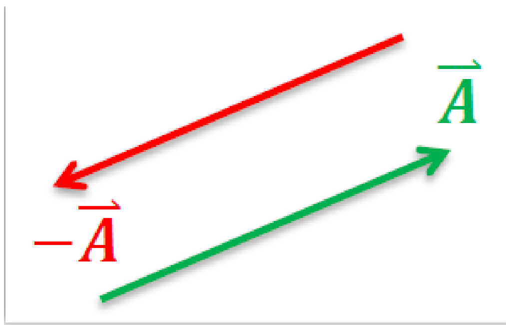
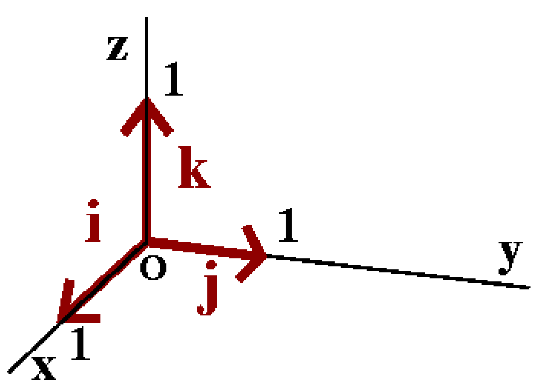


figure (2)

The vector and the negative of the same vector have equal magnitudes and opposite directions.

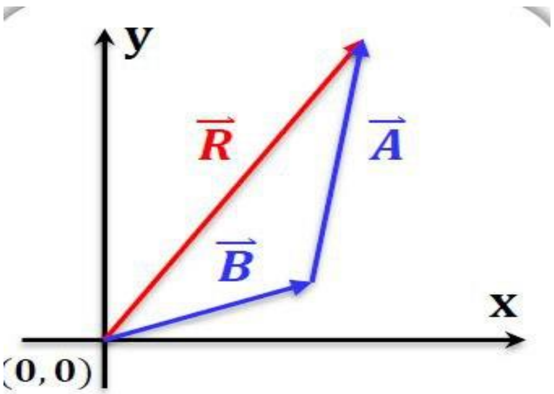
Unit Vectors

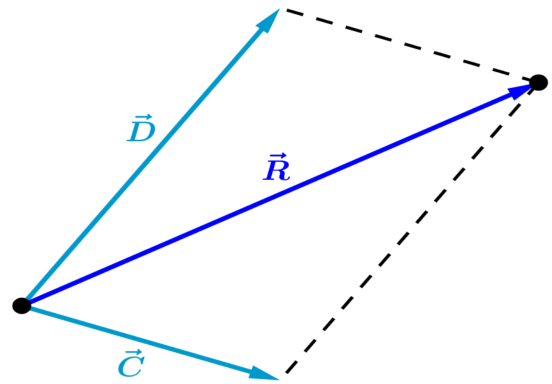
A unit vector is a vector with a magnitude (or length) of exactly 1. It's essentially an arrow pointing in a specific direction. Since its magnitude is always 1, it only provides information about irection.



3.3Vectors Addition

Two or more vectors can be added together to produce a third vector that represents the resultant of these vectors. We draw the first vector and then use a special operation equal to . When representing this operation by drawing the vectors, we first move the second vector, maintaining its magnitude, direction, and position at the end of the first vector, then the second and the third as well. After that, we perform the mathematical calculation of the resultant polygonal shape as shown in the following examples.





Exp1/ Find the vectors sum A=(3i+4j) ; B=(-2i+5j)

A+B =(3i+4j)+(-2+5j)= (3-2)i+(4=5)j= i+9j

Exp2/ Find the vectors sum A=(2i-j) ; B=(-i+3j) ; c=4k

A+B+C= (2-1)i +(-1+3)j +4k= -i+2j+4k

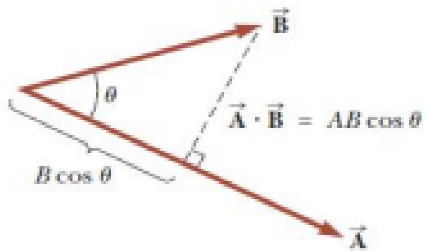
3.4 Scalar product

The scalar product of any two vectors $\vec{A}$ and $\vec{B}$ is defined as (a scalar quantity equal to the product of the magnitudes of the two vectors and the cosine of the angle θ between them):

$$\vec{A} \bullet \vec{B} = AB\cos\theta$$

We write scalar product of vectors $\vec{A}$ and $\vec{B}$ as $\vec{A} \bullet \vec{B}$ (Because of the dot symbol, the scalar product is often called the **dot product**).

- The scalar product ($\vec{A} \bullet \vec{B}$) equals the magnitude of $\vec{A}$ multiplied by the projection of $\vec{B}$ onto $\vec{A}$:($B\ cos\theta$) as shown in the figure below.



Properties of the scalar product:

1. Scalar product is **commutative**:
$$\vec{A} \bullet \vec{B} = \vec{B} \bullet \vec{A}$$
2. Scalar product obeys the **distributive law of multiplication**:
$$\vec{A} \bullet (\vec{B} + \vec{C}) = \vec{A} \bullet \vec{B} + \vec{A} \bullet \vec{C}$$
3. If $\vec{A}$ is perpendicular to $\vec{B}$ ($\theta = 90°$), then $\vec{A} \bullet \vec{B} = 0$.
4. If $\vec{A}$ is parallel to $\vec{B}$ ($\theta = 0°$), then $\vec{A} \bullet \vec{B} = AB$.
5. If $\theta = 180°$, then $\vec{A} \bullet \vec{B} = -AB$.
6. The scalar product is negative when ($90° < \theta \leq 180°$).

$$\hat{i} \bullet \hat{i} = \hat{j} \bullet \hat{j} = \hat{k} \bullet \hat{k} = 1$$

$$\hat{i} \bullet \hat{j} = \hat{i} \bullet \hat{k} = \hat{j} \bullet \hat{k} = 0$$

Two vectors $\vec{A}$ and $\vec{B}$ can be expressed in unit vector form as:

$$\vec{A} = \hat{i}A_x + \hat{j}A_y + \hat{k}A_z$$

$$\vec{B} = \hat{i}B_x + \hat{j}B_y + \hat{k}B_z$$

so $\vec{A} \bullet \vec{B} = A_xB_x + A_yB_y + A_zB_z$ and $\vec{A} \bullet \vec{A} = A^2$.

Exp1/

The vectors $\vec{A}$ and $\vec{B}$ are given by: $\vec{A} = 2\hat{\imath} + 3\hat{\jmath}$ and $\vec{B} = -\hat{\imath} + 2\hat{\jmath}$

(A) Determine the scalar product $\vec{A} \bullet \vec{B}$

(B) Find the angle (θ) between $\vec{A}$ and $\vec{B}$

Solution:

(A) $\vec{A} \bullet \vec{B} = (2\hat{\imath} + 3\hat{\jmath}) \bullet (-\hat{\imath} + 2\hat{\jmath})$

$$= -2\hat{\imath} \bullet \hat{\imath} + 2\,\hat{\imath} \bullet 2\hat{\jmath} - 3\hat{\jmath} \bullet \hat{\imath} + 3\hat{\jmath} \bullet 2\hat{\jmath}$$

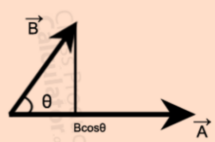
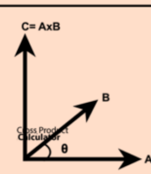
$$= -2 + 0 - 0 + 6 = 4$$

(B) The magnitude of $\vec{A}$: $A = \sqrt{A_x^2 + A_y^2} = \sqrt{(2)^2 + (3)^2} = \sqrt{13}$

The magnitude of $\vec{B}$: $B = \sqrt{B_x^2 + B_y^2} = \sqrt{(-1)^2 + (2)^2} = \sqrt{5}$

$$\cos\theta = \frac{\vec{A} \bullet \vec{B}}{AB} = \frac{4}{\sqrt{13}\sqrt{5}} = \frac{4}{\sqrt{65}}$$

$$\theta = \cos^{-1}\frac{4}{\sqrt{65}} = 60.3°$$

Dot Product	Cross Product
Product of magnitude of vectors and cos of the angle between them.	Product of magnitude of vectors and sine of the angle between them.
In terms of vectors A and B **A · B = |A| |B| cos θ**	In terms of vectors A and B **A · B = |A| |B| sin θ n**
The final product is a scalar quantity.	The final product is a vector quantity.
Follows a commutative law: A.B=B.A	Does not follow a commutative law: AxB is not equal to BxA
If the vectors are perpendicular to each other, their dot result is 0. As in, A.B=0	If the vectors are parallel to each other, their cross result is 0. As in, AxB=0
	

3.5 Vector Product

(Given any two vectors $\vec{A}$ *and* $\vec{B}$, the vector product $(\vec{A}\times\vec{B})$ is defined as a third vector $\overline{C}$, which has a magnitude of $(AB\sin\theta)$).

$$\vec{C} = \vec{A}\times\vec{B} \qquad \text{Vector product}$$

$$C = AB\sin\theta \qquad \text{magnitude of vector product}$$

- The vector product$(\vec{A}\times\vec{B})$ is also called (**cross product**).

  Properties of the vector product:

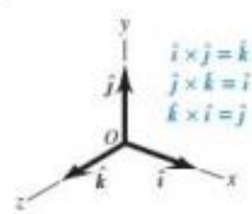
1. It is not commutative $(\vec{A}\times\vec{B} = -\vec{B}\times\vec{A})$ Therefore, if you change the order of the vectors in a vector product, you must change the sign.
2. If $\vec{A}$ is parallel to $\vec{B}$ (θ = 0 or 180°), then

   $$\vec{A}\times\vec{B}=0 \quad \text{and} \quad \vec{A}\times\vec{A}=0$$
3. If $\vec{A}$ is perpendicular to $\vec{B}$ (θ = 90°), then

   $$|\vec{A}\times\vec{B}| = AB$$
4. The vector product obeys the distributive law:

   $$\vec{A}\times(\vec{B}+\vec{C}) = \vec{A}\times\vec{B}+\vec{A}\times\vec{C}$$
5. The derivative of the vector product with respect to some variable such as (t) is: $\frac{d}{dt}(\vec{A}\times\vec{B}) = \frac{d\vec{A}}{dt}\times\vec{B}+\vec{A}\times\frac{d\vec{B}}{dt}$

- The cross products of the unit vectors (î , ĵ , and k̂) obey the following rules: $\hat{\imath}\times\hat{\imath} = \hat{\jmath}\times\hat{\jmath} = \hat{k}\times\hat{k} = 0$
- $\hat{\imath}\times\hat{\jmath} = \hat{k}$, $\hat{\jmath}\times\hat{\imath} = -\hat{k}$
- $\hat{\jmath}\times\hat{k} = \hat{\imath}$, $\hat{k}\times\hat{\jmath} = -\hat{\imath}$
- $\hat{k}\times\hat{\imath} = \hat{\jmath}$, $\hat{\imath}\times\hat{k} = -\hat{\jmath}$



The cross product of any two vectors $\overline{A}$ *and* $\vec{B}$ can be expressed in the following determinant form:

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{\imath}\begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} + \hat{\jmath}\begin{vmatrix} A_z & A_x \\ B_z & B_x \end{vmatrix} + \hat{k}\begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

$$= \hat{\imath}\,(A_yB_z - A_zB_y) + \hat{\jmath}\,(A_zB_x - A_xB_z) + \hat{k}\,(A_xB_y - A_yB_x)$$

Example):

Vector A has a magnitude of 6 units and it is in the direction of positive x-axis. Vector has a magnitude of 4 units and lies in x y plane making an angle 30° with x- axis. Find A* B ?

Solution:

$$\vec{A} = 6\,\hat{\imath} + 0\,\hat{\jmath} + 0\,\hat{k}$$

$$\vec{B} = 4\,\hat{\imath}\cos 30 + 4\,\hat{\jmath}\,sin 30 + 0\,\hat{k} = 2\sqrt{3}\,\hat{\imath} + 2\,\hat{\jmath}$$

$$\vec{C} = \vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 6 & 0 & 0 \\ 2\sqrt{3} & 2 & 0 \end{vmatrix} = 12\,\hat{k}$$

Discussion questions

1. **What is the negative of a vector and how is it represented?**
2. **What is the dot product of two vectors, and how is it calculated?**
3. **How can we compare two vectors to determine if they are equal?**
4. **What are some properties of the dot product?**