



عنوان المحاضرة: الدوال بمتغيرين وأكثر **Functions of Several Variables**
المشتقات الجزئية والكلية **Partial & Total Derivatives**

Function of Several Variables

The equation of the surface is :-

$$F(x, y, z) = 0$$

$$z = f_1(x, y)$$

$$\frac{\partial z}{\partial x} \quad \frac{\partial z}{\partial y}$$

$$y = f_2(x, z)$$

$$\frac{\partial y}{\partial x} \quad \frac{\partial y}{\partial z}$$

$$x = f_3(y, z)$$

$$\frac{\partial x}{\partial z} \quad \frac{\partial x}{\partial y}$$

The case where $z = f(x, y)$ it regards
 x, y : is independent variable
 z : is a function (z is a dependent variable)

The first partial derivatives are write :-

$$\frac{\partial z}{\partial x} = z_x = f_x = \frac{\partial f}{\partial x} = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x, y) - f(x, y)}{\Delta x}$$

$$\frac{\partial z}{\partial y} = z_y = f_y = \frac{\partial f}{\partial y} = \lim_{\Delta y \rightarrow 0} \frac{f(x, y + \Delta y) - f(x, y)}{\Delta y}$$

And for the second partial derivatives :-

$$\frac{\partial^2 z}{\partial x^2} = z_{xx} = f_{xx} = \frac{\partial}{\partial x} (z_x)$$

$$\frac{\partial^2 z}{\partial y^2} = z_{yy} = f_{yy} = \frac{\partial}{\partial y} (z_y)$$

$$\frac{\partial^2 z}{\partial x \partial y} = z_{yx} = f_{yx} = \frac{\partial}{\partial y} (z_x)$$

$$\frac{\partial^2 z}{\partial y \partial x} = z_{xy} = f_{xy} = \frac{\partial}{\partial x} (z_y)$$



Ex:- If $Z = X^3 + Y^4 + X \sin Y + Y \cos X$ then find Z_{xy} و Z_{yx}

Solution:-

$$Z_x = 3X^2 + \sin y - y \sin X$$

$$Z_y = 4Y^3 + X \cos y + \cos X$$

$$Z_{xy} = \frac{\partial}{\partial X} (Z_y) = \cos y - \sin X$$

$$Z_{yx} = \frac{\partial}{\partial y} (Z_x) = \cos y - \sin X$$

Ex:- If $f(x,y) = y \cdot \sin(xy)$ then find f_x و f_y

Solution:-

$$f_x = y \cdot \cos(xy) \cdot y + \sin(xy) \cdot 0 = y^2 \cos(xy)$$

$$f_y = y \cdot \cos(xy) \cdot x + \sin(xy) \cdot 1 \\ = yx \cos(xy) + \sin(xy)$$

Ex:- If $Z = \tan^{-1}\left(\frac{y}{x}\right)$ then show that $x \frac{\partial Z}{\partial x} + y \frac{\partial Z}{\partial y} = 0$

Solution:-

$$\frac{\partial Z}{\partial x} = \frac{1}{1 + (y/x)^2} * \frac{x \cdot 0 - y \cdot 1}{x^2} = \frac{-y}{x^2 + y^2}$$

$$\frac{\partial Z}{\partial y} = \frac{1}{1 + (y/x)^2} * \frac{x \cdot 1 - y \cdot 0}{x^2} = \frac{x}{x^2 + y^2}$$

$$\therefore x \frac{\partial Z}{\partial x} + y \frac{\partial Z}{\partial y} = x \left(\frac{-y}{x^2 + y^2} \right) + y \left(\frac{x}{x^2 + y^2} \right) = 0$$



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Total differential :-

$$df(x,y) = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$$

Ex:- If $z = x^3 - xy^2 + 3$ then find dz .

Solution:-

$$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy$$
$$= (3x^2 - y^2) \cdot dx + (-2xy) \cdot dy = (3x^2 - y^2) \cdot dx - 2xy \cdot dy$$

----- نهاية محاضرة "الدوال بمتغيرين وأكثر" Functions of Several Variables
المشتقات الجزئية والكلية "Partial & Total Derivatives" -----