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Natural logarithms functions, Exponential functions, Logarithms functions

$$y = \log_a x \Rightarrow a^y = x$$

The number *a* is called the *logarithmic base*

If $a = 10$, then it is $\log_{10}$ and it is called **Common logarithm** (available in calculator as *log*)

If $a = e$, then it is $\log_e$ or *ln* and it is called **Natural logarithm** (available in calculator as *ln)*

$\log_{10} x = \log x$ (*Common* Logarithm) $\log_e x = \ln x$ (*Natural* Logarithm)

1. Convert the following from exponential form to logarithmic form:

   a) $y = x^2$ *Answer:* $\log_x y = 2$

   b) $2^3 = 8$ *Answer:* $\log_2 8 = 3$

   c) $9^{1/2} = 3$ *Answer:* $\log_9 3 = 1/2$

   d) $e^2 = x$ *Answer:* $\log_e x = 2$ *or* $\ln x = 2$ *(base of e means ln)*

   e) $10^2 = 100$ *Answer:* $\log_{10} 100 = 2$ *or* $\log 100 = 2$ *(no base*

means base of 10)

2. Convert the following from logarithmic to exponential form:

   a) $\log_x 9 = 3$ *Answer:* $x^3 = 9$

   b) $\log x = -1$ *Answer:* $10^{-1} = x$ *(no base means base of 10)*

   c) $\ln x = -1$ *Answer:* $e^{-1} = x$ *(ln means log to the base of e)*

3. Solve for *x* (hint**:** first, convert each from logarithmic to exponential form)

   a) $\log_9 x = 1$ *Answer:* $x = 9$

   b) $\log_a x = 1$ *Answer:* $x = a$

   c) $\ln x = 1$ *Answer:* $x = e$ (why?)

4. Solve for *x* (hint**:** first, convert each from logarithmic to exponential form)

   a) $\log_9 x = 0$ *Answer:* $x = 1$

   b) $\log_x x = 0$ *Answer:* $x = 1$

   c) $\ln x = 0$ *Answer:* $x = 1$ (why?)

$log_a\, a = 1\,;\quad log_e\, e = 1$ or $ln\, e = 1$ $log_a 1 = 0\,;\quad log_e 1 = 0$ or $ln\, 1 = 0$

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Properties of logarithms

Rule	Formula	Example
I) Multiplication	$\ln(AB) = \ln A + \ln B$	$\ln 5x = \ln 5 + \ln x$
II) Division	$\ln\dfrac{A}{B} = \ln A - \ln B$	$\ln 5/x = \ln 5 - \ln x$
III) Power	$\ln A^p = p \ln A$	$\ln 5^x = x \ln 5$

Example 1 : Express in term of logarithms:

a) $\log(x^2 y^2)$
b) $\log \dfrac{x^3 y^2}{z^4}$
c) $\log \dfrac{\sqrt{x^3 y^2}}{z^3 w^5}$

Example 2: Express as a single logarithm:

a) $3\ln x + 4\ln y - 3\ln z$
b) $2\log x - 3\log y + 2\log z$

Example 3: Solve for x:

a) $5^x = 10$
b) $\ln x = 4$
c) $3^x = 5$

d) $\log_3(2x-1) - \log_3(x-4) = 2$
e) $\log_3(x-4) + \log_3(x+4) = 3$

f) $\log x + \log(x-3) = 1$
g) $\log_2 x + \log_2(x-2) = 3$

Example 4: Solve for x:

a) $\ln x = -2$
b) $\log_2 x + \log_2(x-2) = 3$
c) $e^{t(t+1)} = 1$

d) $\log_4(x+6) - \log_4 x = 2$
e) $\ln(2t+1) + \ln(2t-1) = 0$
f) $\ln(t-1) = 3$

g) $5e^{x-3} = 4$
h) $4^x = 5(2^x)$
i) $4(e^{2x}) = e^{6x}$

Answers *(not on order)*: $(2/5)$; (4) ; $(1/e^2)$; $(e^3 + 1)$; $(0, -1)$; $\left(\frac{1}{\sqrt{2}}\right)$; $(3 + ln\ 0.8)$; (0.3466) ; (2.322)