

4.8 Three-Dimensional Plots

1. LINE PLOTS

A three-dimensional line plot is a line that is obtained by connecting points in three-dimensional space. A basic 3-D plot is created with the `plot3` command, which is very similar to the `plot` command and has the form:

```
plot3(x,y,z, 'line specifiers', 'PropertyName', property value)
```

x , y , and z are vectors of the coordinates of the points.

(Optional) Specifiers that define the type and color of the line and markers.

(Optional) Properties with values that can be used to specify the line width, and marker's size and edge and fill colors.

For example, if the coordinates x , y , and z are given as a function of the parameter t by

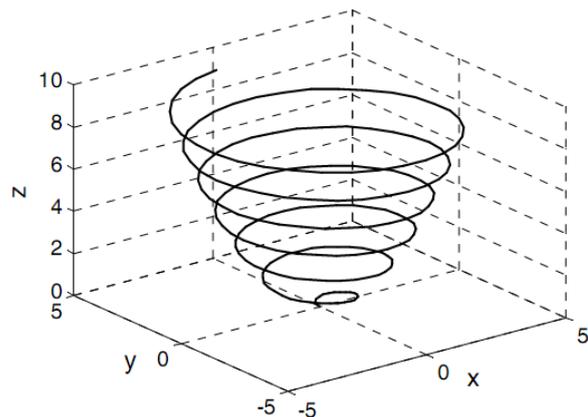
$$x = \sqrt{t} \sin(2t)$$

$$y = \sqrt{t} \cos(2t)$$

$$z = 0.5t$$

a plot of the points for $0 \leq t \leq 6\pi$ can be produced by the following script file:

```
t=0:0.1:6*pi;
x=sqrt(t).*sin(2*t);
y=sqrt(t).*cos(2*t);
z=0.5*t;
plot3(x,y,z,'k','linewidth',1)
grid on
xlabel('x'); ylabel('y'); zlabel('z')
```



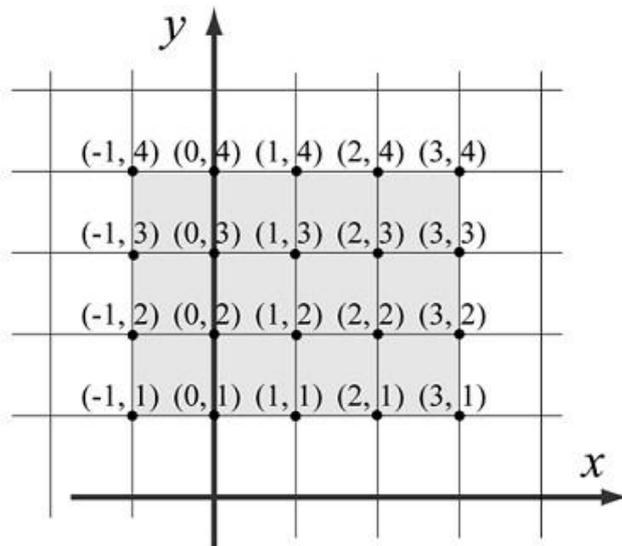
2. MESH AND SURFACE PLOTS

Mesh and surface plots are three-dimensional plots used for plotting functions of the form $z = f(x,y)$ where x and y are the independent variables and z is the dependent

variable. It means that within a given domain the value of z can be calculated for any combination of x and y . Mesh and surface plots are created in three steps. The first step is to create a grid in the $x y$ plane that covers the domain of the function. The second step is to calculate the value of z at each point of the grid. The third step is to create the plot.

4.9 Creating a grid in the $x y$ plane (Cartesian coordinates):

The grid is a set of points in the $x y$ plane in the domain of the function. The density of the grid (number of points used to define the domain) is defined by the user. Figure 10-2 shows a grid in the domain $-1 \leq x \leq 3$ and $1 \leq y \leq 4$. In this grid



A grid in the $x y$ plane for the domain $-1 \leq x \leq 3$ and $1 \leq y \leq 4$ with spacing of 1.

the distance between the points is one unit. The points of the grid can be defined by two matrices, X and Y . Matrix X has the x coordinates of all the points, and matrix Y has the y coordinates of all the points:

$$X = \begin{bmatrix} -1 & 0 & 1 & 2 & 3 \\ -1 & 0 & 1 & 2 & 3 \\ -1 & 0 & 1 & 2 & 3 \\ -1 & 0 & 1 & 2 & 3 \end{bmatrix} \quad \text{and} \quad Y = \begin{bmatrix} 4 & 4 & 4 & 4 & 4 \\ 3 & 3 & 3 & 3 & 3 \\ 2 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

MATLAB has a built-in function, called **meshgrid**, that can be used for creating the X and Y matrices. The form of the **meshgrid** function is:

$$[X, Y] = \text{meshgrid}(x, y)$$

X is the matrix of the x coordinates of the grid points.
 Y is the matrix of the y coordinates of the grid points.

x is a vector that divides the domain of x .
 y is a vector that divides the domain of y .

```
x=-3:3  
y=-3:1  
[X,Y]=meshgrid(x,y)
```

Ans:

X =

```
-3 -2 -1 0 1 2 3
```

```
-3 -2 -1 0 1 2 3
```

```
-3 -2 -1 0 1 2 3
```

```
-3 -2 -1 0 1 2 3
```

```
-3 -2 -1 0 1 2 3
```

Y =

```
-3 -3 -3 -3 -3 -3 -3
```

```
-2 -2 -2 -2 -2 -2 -2
```

```
-1 -1 -1 -1 -1 -1 -1
```

```
0 0 0 0 0 0 0
```

```
1 1 1 1 1 1 1
```

4.10 Making mesh and surface plots:

A mesh or surface plot is created with the `mesh` or `surf` command, which has the form:

```
mesh(X,Y,Z)
```

```
surf(X,Y,Z)
```

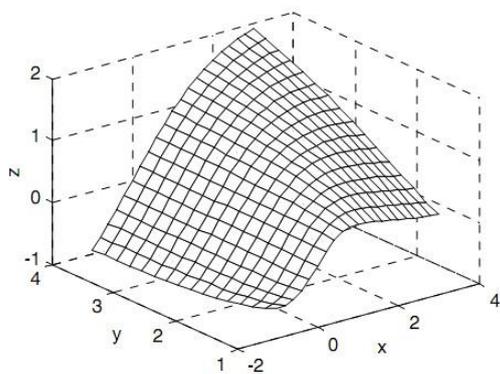
where `X` and `Y` are matrices with the coordinates of the grid and `Z` is a matrix with the value of `z` at the grid points. The mesh plot is made of lines that connect the points. In the surface plot, areas within the mesh lines are colored.

As an example, the following script file contains a complete program that creates the grid and then makes a mesh (or surface) plot of the function

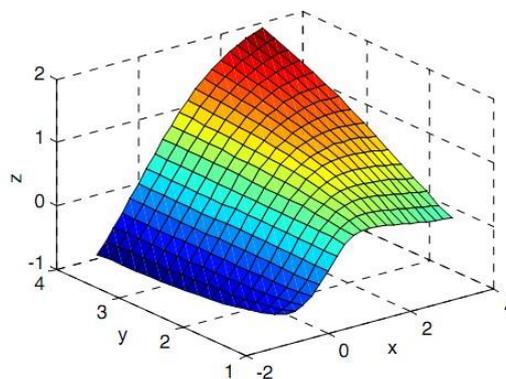
$$z = \frac{xy^2}{x^2 + y^2} \text{ over the domain } -1 \leq x \leq 3 \text{ and } 1 \leq y \leq 4.$$

```
x=-1:0.1:3;  
y=1:0.1:4;  
[X,Y]=meshgrid(x,y);  
Z=X.*Y.^2./(X.^2+Y.^2);
```

Note that in the program above the vectors x and y have a much smaller spacing than the spacing earlier in the section. The smaller spacing creates a denser grid. The figures created by the program are:



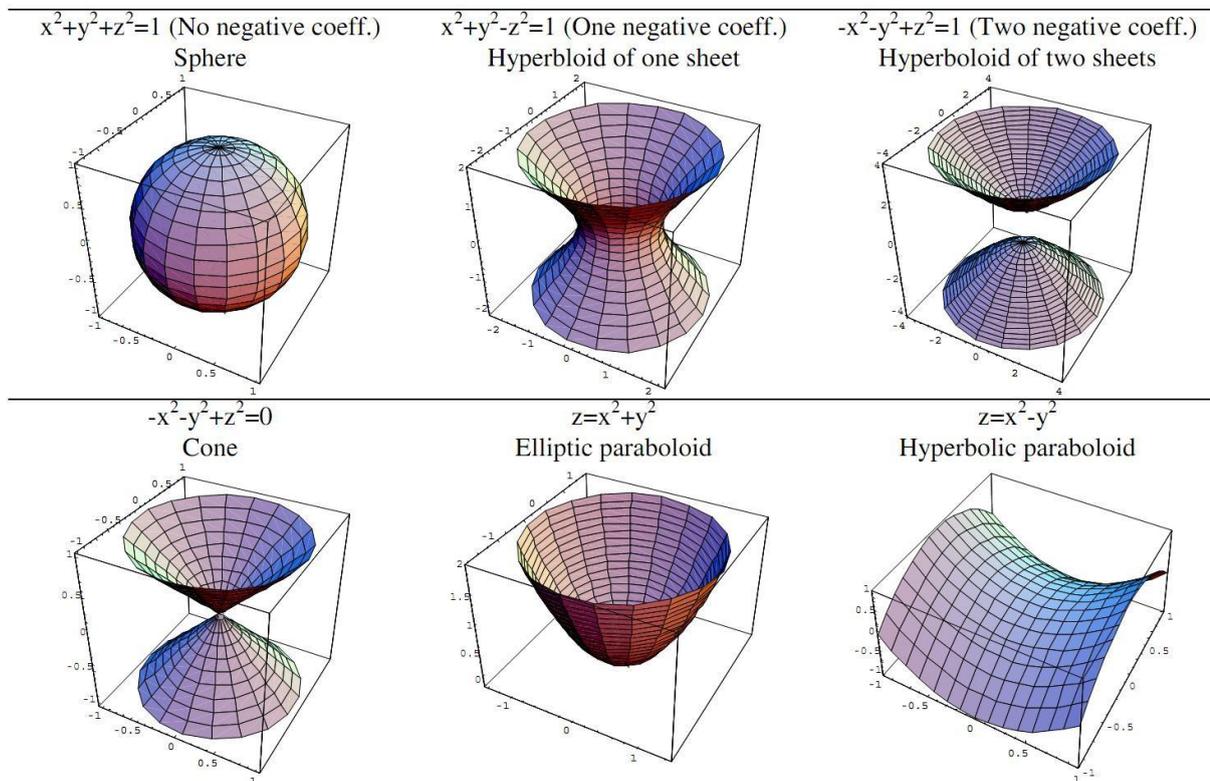
Mesh plot



Surface plot

4.11 Important surfaces

The eq. of important surfaces as in the figure:

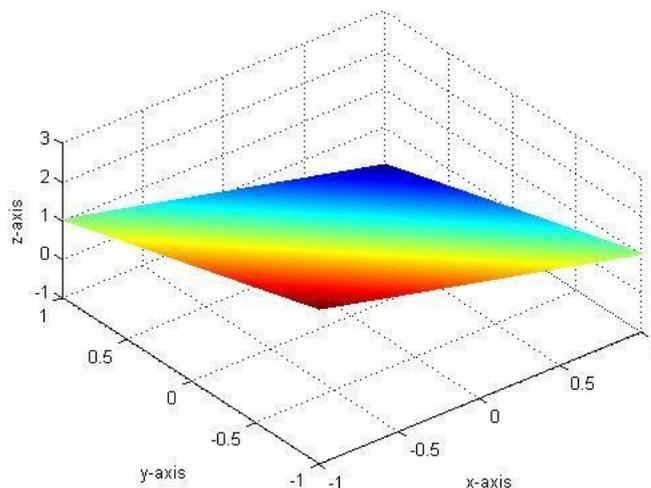


Ex.

1. plot $x+y+z=1$ in 3D?

Solution:

```
x=-1:.01:1;  
y=x;  
[x y]=meshgrid(x,y);  
z=1-x-y;  
mesh(x,y,z)  
xlabel('x-axis')  
ylabel('y-axis')  
zlabel('z-axis')
```



2. plot $x^2+y^2=z^2$ (Cone) in 3D

```
x=-1:.01:1;  
y=x;  
[x y]=meshgrid(x,y);  
z1=sqrt(x.^2+y.^2);  
z2=-sqrt(x.^2+y.^2);  
    mesh(z1)  
    hold on  
    mesh(z2)  
  
xlabel('x-axis')  
ylabel('y-axis')  
zlabel('z-axis')
```

