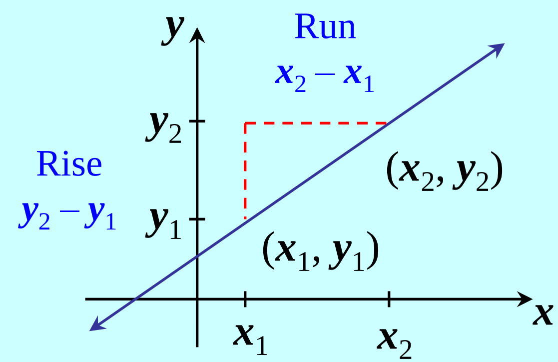
The Slope of a Line

Mathematicians have developed a useful measure of the steepness of a line, called the **slope** of the line. Slope compares the vertical change (the **rise**) to the horizontal change (the **run**) when moving from one fixed point to another along the line. A ratio comparing the change in y (the rise) with the change in x (the run) is used calculate the slope of a line.

Definition of Slope

The slope of the line through the distinct points $(\boldsymbol{x}_1, \boldsymbol{y}_1)$ and $(\boldsymbol{x}_2, \boldsymbol{y}_2)$ is

$$\frac{\text{Change in y}}{\text{Change in x}} = \frac{\text{Rise}}{\text{Run}} = \frac{y_2 - y_1}{x_2 - x_1}$$



where $x_1 - x_2 = 0.$

Sample Problems:

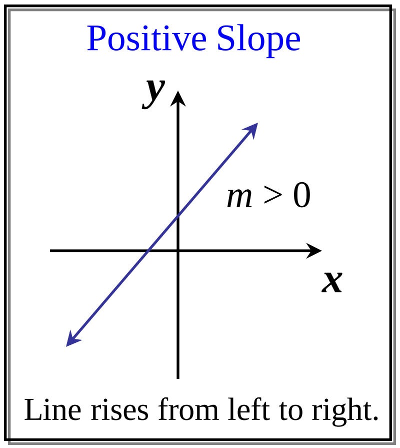
- Find the slope of the line thru the points given:
  - (-3,-1) and (-2,4)

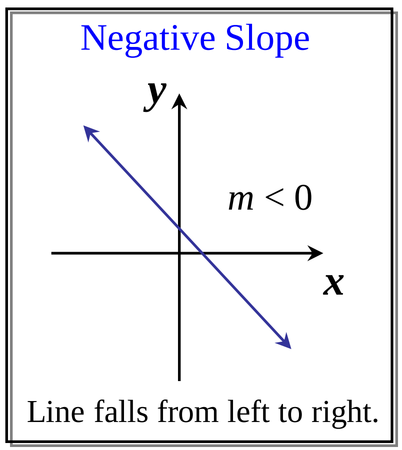
$$m = \frac{(4-(-1))}{(-2-(-3))} = \frac{5}{1} = 5$$

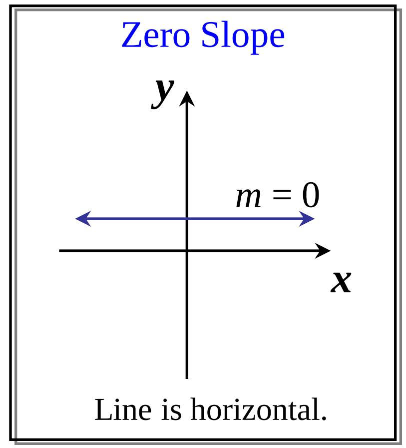
  - (-3,4) and (2,-2)

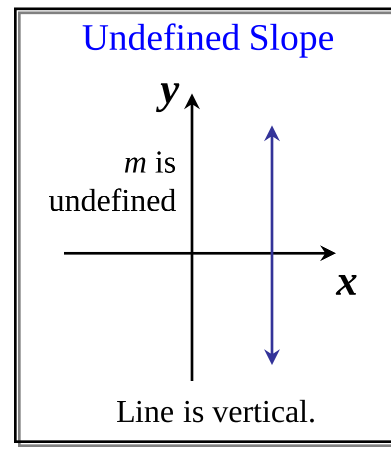
$$m = \frac{(-2-4)}{(2-(-3))} = \frac{-6}{5}$$

The Possibilities for a Line's Slope









Point-Slope Form of the Equation of a Line

The **point-slope equation** of a non-vertical line of slope *m* that passes through the point (x_1, y_1) is

$$y - y_1 = m(x - x_1).$$

Example: Writing the Point-Slope Equation of a Line

Write the point-slope form of the equation of the line passing through (-1,3) with a slope of 4. Then solve the equation for y.

Solution We use the point-slope equation of a line with m = 4, x_1= -1, and y_1 = 3.

$\boldsymbol{y} - \boldsymbol{y}_1 = m(\boldsymbol{x} - \boldsymbol{x}_1)$ This is the point-slope form of the equation.

$\boldsymbol{y} - 3 = 4[\boldsymbol{x} - (-1)]$ Substitute the given values. Simply.

$\boldsymbol{y} - 3 = 4(\boldsymbol{x} + 1)$ We now have the point-slope form of the equation for the given line.

We can solve the equation for y by applying the distributive property.

$\boldsymbol{y} - 3 = 4\boldsymbol{x} + 4$

$\boldsymbol{y} = 4\boldsymbol{x} + 7$ Add 3 to both sides.

Slope-Intercept Form of the Equation of a

The **slope-intercept equation** of a non-vertical line with slope m and $\boldsymbol{y}$-intercept b is

$$\boldsymbol{y} = m\boldsymbol{x} + b.$$

Equations of Horizontal and Vertical Lines

Equation of a Horizontal Line

A horizontal line is given by an equation of the form

$$\boldsymbol{y} = b$$

where b is the $\boldsymbol{y}$-intercept. Note: $m = 0$.

Equation of a Vertical Line

A vertical line is given by an equation of the form

$$\boldsymbol{x} = a$$

where a is the $\boldsymbol{x}$-intercept. Note: m is undefined.

General Form of the Equation of the a Line

Every line has an equation that can be written in the general form

$$Ax + By + C = 0$$

Where A, B, and C are three integers, and A and B are not both zero. A must be positive.

Standard Form of the Equation of the a Line

Every line has an equation that can be written in the standard form

$$Ax + By = C$$

Where A, B, and C are three integers, and A and B are not both zero. A must be positive.

In this form, m = -A/B and the intercepts are (0,C/B) and (C/A, 0).

Equations of Lines

- Point-slope form: $\boldsymbol{y} - \boldsymbol{y}_1 = m(\boldsymbol{x} - \boldsymbol{x}_1)$
- Slope-intercept form: $\boldsymbol{y} = m\,\boldsymbol{x} + b$
- Horizontal line: $\boldsymbol{y} = b$
- Vertical line: $\boldsymbol{x} = a$
- General form: $A\boldsymbol{x} + B\boldsymbol{y} + C = 0$
- Standard form: $A\boldsymbol{x} + B\boldsymbol{y} = C$

Example: Finding the Slope and the *y*-Intercept

Find the slope and the ***y***-intercept of the line whose equation is $2x - 3y + 6 = 0$.

Solution The equation is given in general form, Ax + By + C = 0. **One** method is to rewrite it in the form $y = mx + b$. We need to solve for ***y***.

$2x - 3y + 6 = 0$ This is the given equation.

$2x + 6 = 3y$ To isolate the ***y***-term, add 3 ***y*** on both sides.

$3y = 2x + 6$ Reverse the two sides. (This step is optional.)

$y = \frac{2}{3}x + 2$ Divide both sides by 3.

The coefficient of ***x***, 2/3, is the slope and the constant term, 2, is the ***y***-intercept.

Steps for Graphing $y = mx + b$

Graphing $y = mx + b$ by Using the Slope and *y*-Intercept

- Plot the ***y***-intercept on the ***y***-axis. This is the point $(0, b)$.
- Obtain a second point using the slope, *m*. Write m as a fraction, and use rise over run starting at the ***y***-intercept to plot this point.
- Use a straightedge to draw a line through the two points. Draw arrowheads at the ends of the line to show that the line continues indefinitely in both directions.

Example: Graphing by Using Slope and *y*-Intercept

Graph the line whose equation is $y = \frac{2}{3}x + 2$.

Solution The equation of the line is in the form $y = mx + b$.
We can find the slope, ***m***, by identifying the coefficient of ***x***.
We can find the ***y***-intercept, ***b***, by identifying the constant term.

$$y = \frac{2}{3}x + 2$$

The slope is 2/3.

The y-intercept is 2.

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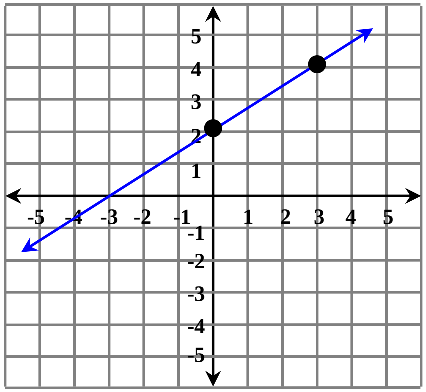
Example: Graphing by Using Slope and *y*-Intercept

Graph the line whose equation is $y = \frac{2}{3}x + 2$.

Solution

We need two points in order to graph the line. We can use the y-intercept, 2, to obtain the first point (0, 2). Plot this point on the y-axis.

$$m = \frac{2}{3} = \frac{\text{Rise}}{\text{Run}}.$$



We plot the second point on the line by starting at (0, 2), the first point.

Then move 2 units up (the rise) and 3 units to the right (the run). This gives us a second point at (3, 4).

Sample Problems

Give the slope and y-intercept of the given line then graph.

$$y = 3x + 2$$

$$y = -\frac{2}{5}x + 6$$

Example: Finding the slope and the *x*-&*y*-intercepts.

Find the slope and the intercepts of the line whose equation is $2x - 3y = -6$.

Solution When an equation is given in standard form, Ax + By = C, the slope can be determine by using the coefficients A and B, so that ***m*** = -A/B.

$2x - 3y = -6$ For the given equation, A = 2 and B = -3. So *m* = 2/3.

To find the intercepts, recall that the ***x***-intercept has the form (***x***,0) and the y-intercept has the form (0,***y***).

$2x - 3(0) = -6$ Let **y** = 0 and solve for x.

$2x = -6$

$x = -3$ So the ***x***-intercept is (-3,0).

$2(0) - 3y = -6$ Likewise, let **x** = 0 and solve for y.

$-3y = -6$

$y = 2$ So the ***y***-intercept is (0,2).

Problems

For the given equations,

1. Rewrite the equation in slope-intercept form and in standard form.
2. Graph the lines using both methods – using slope and y-intercept and using the x- & y-intercepts.

- $4x + y - 6 = 0$
- $4x + 6y + 12 = 0$
- $6x - 5y - 20 = 0$
- $4y + 28 = 0$

Exercises page 138, numbers 1-60.

Section 1.2 (cont'd)

<u>Review</u>

- Defintion of a slope : $m = \frac{y_2 - y_1}{x_2 - x_1}$

- 6 Forms for the Equation of a Line

  - Point-slope form: $\boldsymbol{y} - \boldsymbol{y}_1 = m(\boldsymbol{x} - \boldsymbol{x}_1)$
  - Slope-intercept form: $\boldsymbol{y} = m\,\boldsymbol{x} + b$
  - Horizontal line: $\boldsymbol{y} = b$
  - Vertical line: $\boldsymbol{x} = a$
  - General form: $A\boldsymbol{x} + B\boldsymbol{y} + C = 0$
  - Standard form: $A\boldsymbol{x} + B\boldsymbol{y} = C$

- Graphing Techniques
  - Using slope and y-intercept
  - Using x- & y-intercepts

Slope and Parallel Lines

- If two non-vertical lines are parallel, then they have the **same** slope.
- If two distinct non-vertical lines have the **same** slope, then they are parallel.
- Two distinct vertical lines, both with undefined slopes, are parallel.

Example: Writing Equations of a Line Parallel to a Given Line

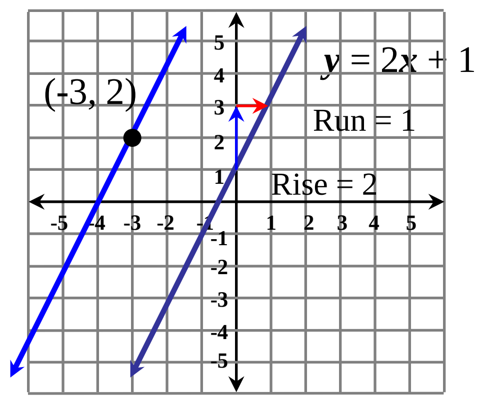
Write an equation of the line passing through (-3, 2) and parallel to the line whose equation is $y = 2x + 1$. Express the equation in point-slope form and y-intercept form.

Solution We are looking for the equation of the line shown on the left on the graph. Notice that the line passes through the point (-3, 2). Using the point-slope form of the line's equation, we have $x_1 = -3$ and $y_1 = 2$.

$$y - y_1 = m(x - x_1)$$

$y_1 = 2$

$x_1 = -3$



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Example continued:

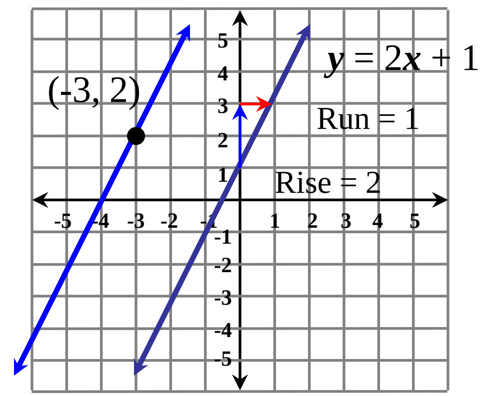
Since parallel lines have the same slope and the slope of the given line is 2, $m = 2$ for the new equation. So we know that $m = 2$ and the point (-3, 2) lies on the line that will be parallel. Plug all that into the point-slope equation for a line to give us the line parallel we are looking for.

$$\boldsymbol{y} - \boldsymbol{y}_1 = m(\boldsymbol{x} - \boldsymbol{x}_1)$$

$\boldsymbol{y}_1 = 2$

$m = 2$

$\boldsymbol{x}_1 = -3$



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Example continued:

Solution The point-slope form of the line's equation is

$$y - 2 = 2[x - (-3)]$$

$$y - 2 = 2(x + 3)$$

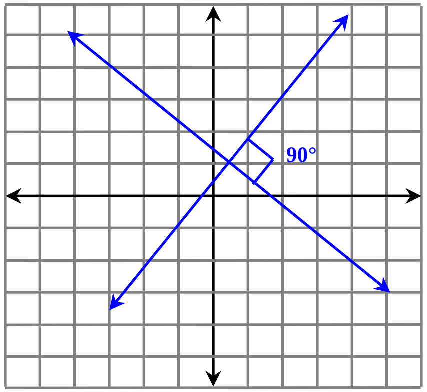
Solving for y, we obtain the slope-intercept form of the equation.

$$y - 2 = 2x + 6$$ Apply the distributive property.

$$y = 2x + 8$$ Add 2 to both sides. This is the slope-intercept form of the equation.

Slope and Perpendicular Lines

Two lines that intersect at a right angle (90°) are said to be perpendicular. There is a relationship between the slopes of perpendicular lines.



Slope and Perpendicular Lines

- If two non-vertical lines are perpendicular, then the product of their slopes is –1.
- If the product of the slopes of two lines is –1, then the lines are perpendicular.
- A horizontal line having zero slope is perpendicular to a vertical line having undefined slope.

Example: Finding the Slope of a Line Perpendicular to a Given Line

Find the slope of any line that is perpendicular to the line whose equation is $x + 4y - 8 = 0$.

Solution We begin by writing the equation of the given line in slope-intercept form. Solve for y.

$x + 4y - 8 = 0$ — This is the given equation.

$4y = -x + 8$ — To isolate the y-term, subtract x and add 8 on both sides.

$y = -1/4x + 2$ — Divide both sides by 4.

Slope is –1/4.

The given line has slope –1/4. Any line perpendicular to this line has a slope that is the negative reciprocal, 4.

Example: Writing the Equation of a Line Perpendicular to a Given Line

Write the equation of the line perpendicular to $x + 4y - 8 = 0$ that passes thru the point (2,8) in **standard form**.

Solution: The given line has slope –1/4. Any line perpendicular to this line has a slope that is the negative reciprocal, 4.

So now we need know the perpendicular slope and are given a point (2,8). Plug this into the point-slope form and rearrange into the standard form.

$$y - y_1 = m(x - x_1)$$

$y_1 = 8$ $m = 4$ $x_1 = 2$

$$y - 8 = 4[x - (2)]$$

$$y - 8 = 4x - 8$$

$$-4x + y = 0$$

$$4x - y = 0$$ Standard form

Problems

1. Find the slope of the line that is
   a) parallel
   b) perpendicular to the given lines.

- $y = 3x$
- $8x + y = 11$
- $3x - 4y + 7 = 0$
- $y = 9$

2. Write the equation for each line in slope-intercept form.

- Passes thru (-2,-7) and parallel to $y = -5x+4$
- Passes thru (-4, 2) and perpendicular to
  $y = x/3 + 7$

Exercises pg 138, numbers 61-68