4) Exact (first order first degree)

The general form

$$M(x.y)dx + N(x.y)dy = 0$$

<u>Condition</u> (test) if $\frac{\partial M(x.y)}{\partial y} = \frac{\partial N(x.y)}{\partial x} \rightarrow mean\ is\ Exact$

Assume $\frac{\partial F}{\partial x} = M(x.y)\ \&\ \frac{\partial F}{\partial y} = N(x.y)$

$$\left[F = \int M * dx + g(y)\right]$$

OR

$$\left[F = \int N\, dy + g(x)\right]$$

Example// Solve the equation

$$\sin y\ dx + (x\cos y - 2y)dy = 0$$

Solution/

$$\sin y\ dx + (x\cos y - 2y)dy = 0 \leftrightarrow M(x.y)dx + N(x.y)dy = 0$$

$$M(x.y) = \sin y \rightarrow \frac{\partial M}{\partial y} = \cos y$$

$$N(x.y) = (x\cos y - 2y) \rightarrow \frac{\partial N}{\partial x} = \cos y$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad this\ is\ Exact$$

$$F = \int M * dx + g(y)$$

$$F = \int \sin y\, dx + g(y) \rightarrow F = \cos y + g(y)$$

$$\frac{\partial F}{\partial y} = \frac{\partial}{\partial y}(x \sin y + g(y))$$

$$\frac{\partial F}{\partial y} = N(x.y) = (x\cos y - 2y)$$

$$(x\cos y - 2y) = \frac{\partial}{\partial y}\big((x \sin y) + g(y)\big) \rightarrow$$

$$(x\cos y - 2y = x \cos y + \overline{g}(y)(Integration\ of\ the\ two\ parties)$$

$$x \sin y - y^2 = x \sin y + g(y) \rightarrow g(y) = -y^2$$

$$\therefore F = \cos y - y^2 = c$$

Example// Solve $(x - xy^2)dx + (8y - yx^2)dy = 0$

Solution /

$$(x - xy^2)dx + (8y - xy^2)dy = 0$$
$$\leftrightarrow M(x.y)dx + N(x.y)dy = 0$$

$$M(x.y) = (x - xy^2) \rightarrow \frac{\partial M}{\partial y} = -2xy$$

$$N(x.y) = (8y - yx^2) \rightarrow \frac{\partial N}{\partial x} = -2xy$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \text{ this is Exact}$$

$$F = \int M * dx + g(y)$$

$$F = \int (x - xy^2)\, dx + g(y) \rightarrow F = (\frac{x^2}{2} - \frac{y^2 x^2}{2}) + g(y)$$

$$\frac{\partial F}{\partial y} = \frac{\partial}{\partial y}((\frac{x^2}{2} - \frac{y^2 x^2}{2}) + g(y))$$

$$\frac{\partial F}{\partial y} = N(x.y) = (8y - yx^2)$$

$$(8y - yx^2) = \frac{\partial}{\partial y}((\frac{x^2}{2} - \frac{y^2 x^2}{2}) + g(y)) \rightarrow$$

$$8y - yx^2 = -x^2 y + \bar{g}(y)$$

$$(\text{Integration of the two parties})$$

$$4\, y^2 - \frac{x^2 y^2}{2} = -\frac{x^2 y^2}{2} + g(y) \rightarrow g(y) = 4y^2$$

$$\therefore F = (\frac{x^2}{2} - \frac{y^2 x^2}{2}) + 4y^2 = c$$