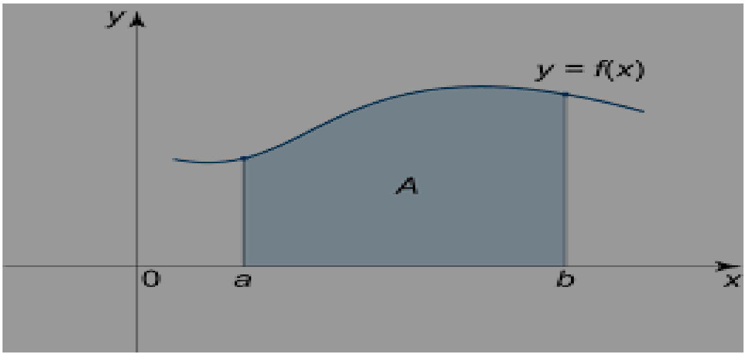
2.13 Integration application/area under the curve

1.The area under the curve

Define: Let F be continuing function over the closed value [a, b], then the area under the curve define: -



$$A=\int_a^b f(x)\,dx \quad with\ x-axis$$

Or $$A=\int_a^b f(y)\,dy \quad with\ y-axis$$

Example 1: Find the area under the curve bounded by the curve $y=\sqrt{x}$ $\quad and\ 0 \le x \le 1\ \ with\ the\ x-axis$

Solution //

$$A=\int_a^b f(x)\,dx \quad \rightarrow A=\int_0^1 \sqrt{x}\,dx = \frac{x^{\frac{3}{2}}}{3/2}\Bigg|_0^1 = \frac{2}{3}unit^2$$

Example 1: Find the area bounded by the curve $y = x - x^2$. with $x - axis$.

Solution /

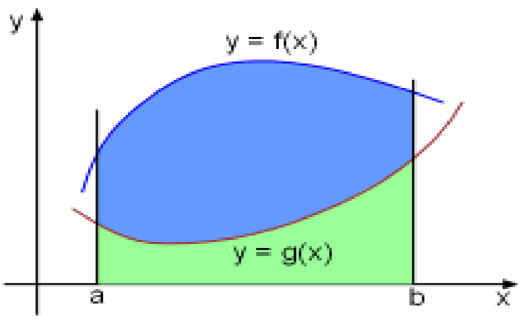
$$y = x - x^2.\text{ with } x - \text{axis} \rightarrow 0 = x - x^2 \rightarrow x(1 - x^2) = 0$$

x=0 & x=1

$$A = \int_0^1 (x - x^2)\, dx = \frac{x^2}{2} - \frac{x^3}{3}\Big|_0^1 = \frac{1}{6} unit^2$$

2. The area between two curve

Define: Let F1&F2 are two functions over the closed value [a,b],then between two curves define as follows:-



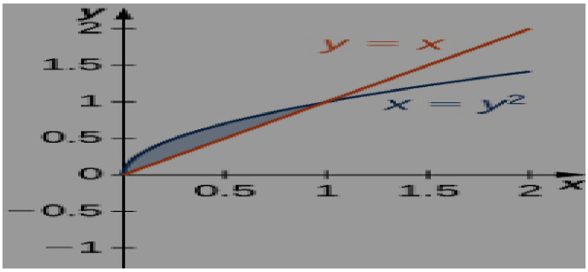
$$A=\int_a^b |f1(x) - f2(x)|\, dx \quad with\ x - axis$$

Or $$A=\int_a^b |f1(y) - f2(y)| dx \quad with\ y - axis$$

Example 1: Find the area of region bonded by the curves

$$y=\sqrt{x} \quad \& \quad y=x$$

Solution //



$$\sqrt{x}=\mathrm{x}\rightarrow \mathrm{x}-x^2=0\rightarrow x(1-\mathrm{x})=0$$

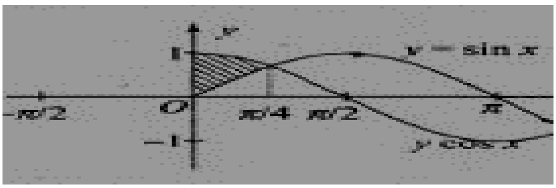
$$x=0 \;\&\; x=1$$

$$A=\int_0^1(\sqrt{x}-x)\,dx=\frac{x^{\frac{3}{2}}}{3/2}-\frac{x^2}{2}\Bigg|_0^1=\frac{1}{6}unit^2$$

Example 2: Find the area of region bonded by the curves

$$y=\sin x \;\;\&\;\; y=\cos x \;\; bounded\ by\ the\ lines\ x=0 \;\&\; x=\frac{\pi}{2}$$

solution //



$$\sin x=\cos x\rightarrow\frac{\sin x}{\cos x}=1\;\rightarrow\tan x=1$$

$$\mathbf{x}=\frac{\pi}{4} \in (\mathbf{0}.\frac{\pi}{2})$$

$$A = \int_{0}^{\frac{\pi}{4}} (\cos x - \sin x)dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin x - \cos x)dx$$

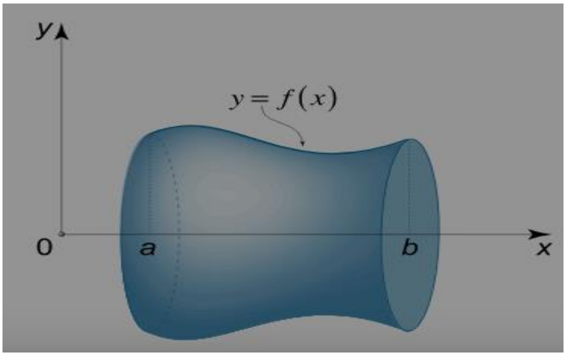
$$= \sin x + \cos x \Big|_{0}^{\frac{\pi}{4}} + (-\cos x - \sin x)\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} =$$

$$(\frac{2}{\sqrt{2}} - 1) - (1 - \frac{2}{\sqrt{2}}) = \frac{4 - 2\sqrt{2}}{\sqrt{2}} unit^2$$

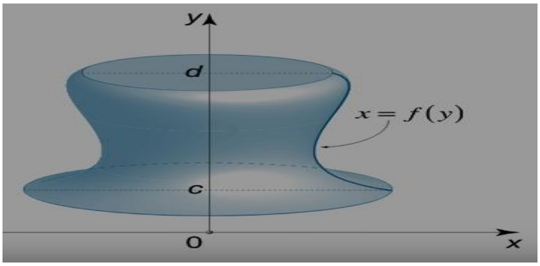
2.14 Integration application/volume

A) by disk

1. Rotation with x-axis [V=$\pi \int_a^b [f(x)]^2 \, dx$]



2. Rotation with y-axis [V=$\pi\int_a^b [f(y)]^2\,dy$]



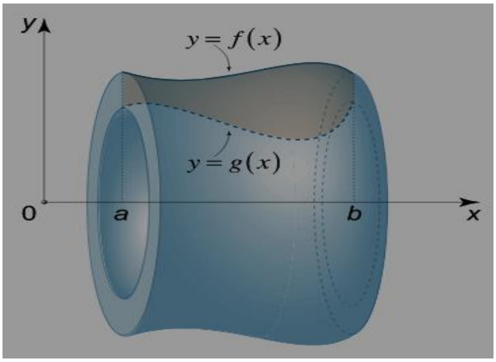
Example: Find the volume of the solid generated by revolving curve $y = \sqrt{x}$ $with\ x - axis\ from\ x = 0\ to\ x = 1$

Solution/

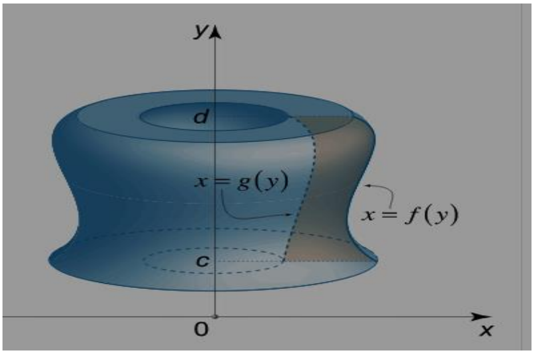
V=$\pi\int_0^1 [\sqrt{x}]^2\,dx = \frac{\pi}{2} unit^3$

B) by washer

1. Rotation with x-axis [V=$\pi\int_a^b [f1(x)^2 - f2(x)^2]dx$



2. Rotation with y-axis $[V=\pi \int_a^b [f1(y)^2 - f2(y)^2]dy]$



Example: Find the volume of the solid bounded by revolving curve $y = \sqrt{x}$ &$y = x$. *is revolving with*

a). $x - axis$

b). $y - axis$

Solution//

a). $x - axis$ $\sqrt{x} = x \rightarrow x - x^2 = 0 \rightarrow x(1 - x) = 0$

x=0 & x=1

$$V = \pi \int_0^1 ((\sqrt{x})^2 - x^2)dx = \pi \left(\frac{x^2}{2} - \frac{x^3}{3}\bigg|_0^1\right) = \frac{\pi}{6} unit^3$$

b). $y - axis$ $x1 = x2 \rightarrow y = y^2 \rightarrow y - y^2 = 0$

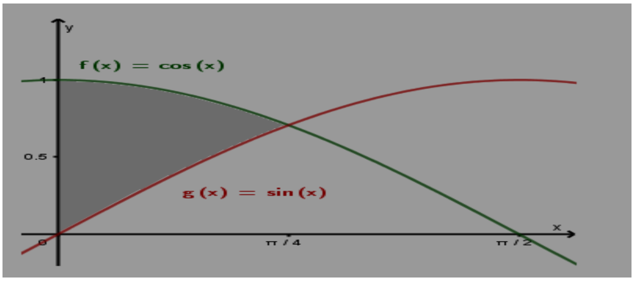
y=0 & y=1

$$V=\pi\int_{0}^{1}((y)^2-y^4)\,dx=\pi(\frac{1}{3}-\frac{1}{5})=\frac{2\pi}{15}unit^3$$

Example: Find the volume of the solid bonded by the curves

$y=\sin x$ & $y=\cos x\,.\,0\le x\le\frac{\pi}{2}$ $\;is\ revolved\ about\ the\ x-axis$

Solution//



$$\sin x=\cos x\rightarrow\frac{\sin x}{\cos x}=1\ \rightarrow\tan x=1$$

$$x=\frac{\pi}{4}\in(0.\frac{\pi}{2})$$

$$V=\pi[\int_{0}^{\frac{\pi}{4}}((\cos x)^2-\sin x)^2)dx+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}((\sin x)^2-\cos x)^2)dx]$$

$$V=\pi\quad unit^3$$