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Maximum shear stress

We know that maximum shear stress,

$$\begin{aligned}\tau_{max} &= \frac{1}{2} \left[\sqrt{(\sigma_t)^2 + 4 \tau^2} \right] = \frac{1}{2} \left[\sqrt{(35.8)^2 + 4 (20.9)^2} \right] \\ &= 27.5 \text{ MPa Ans.}\end{aligned}$$

5.9 Theories of Failure Under Static Load

It has already been discussed in the previous chapter that strength of machine members is based upon the mechanical properties of the materials used. Since these properties are usually determined from simple tension or compression tests, therefore, predicting failure in members subjected to uni-axial stress is both simple and straight-forward. But the problem of predicting the failure stresses for members subjected to bi-axial or tri-axial stresses is much more complicated. In fact, the problem is so complicated that a large number of different theories have been formulated. The principal theories of failure for a member subjected to bi-axial stress are as follows:

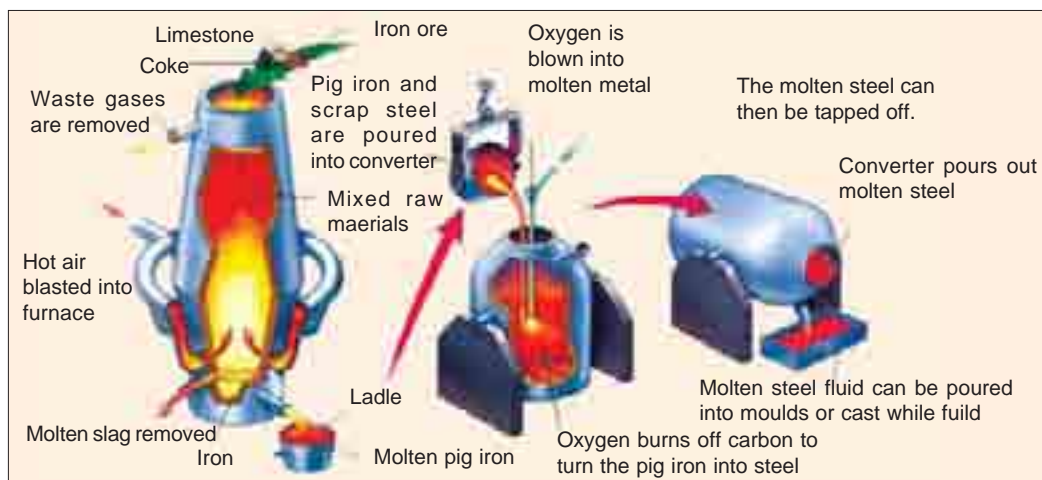
1. Maximum principal (or normal) stress theory (also known as Rankine's theory).
2. Maximum shear stress theory (also known as Guest's or Tresca's theory).
3. Maximum principal (or normal) strain theory (also known as Saint Venant theory).
4. Maximum strain energy theory (also known as Haigh's theory).
5. Maximum distortion energy theory (also known as Hencky and Von Mises theory).

Since ductile materials usually fail by yielding *i.e.* when permanent deformations occur in the material and brittle materials fail by fracture, therefore the limiting strength for these two classes of materials is normally measured by different mechanical properties. For ductile materials, the limiting strength is the stress at yield point as determined from simple tension test and it is, assumed to be equal in tension or compression. For brittle materials, the limiting strength is the ultimate stress in tension or compression.

5.10 Maximum Principal or Normal Stress Theory (Rankine's Theory)

According to this theory, the failure or yielding occurs at a point in a member when the maximum principal or normal stress in a bi-axial stress system reaches the limiting strength of the material in a simple tension test.

Since the limiting strength for ductile materials is yield point stress and for brittle materials (which do not have well defined yield point) the limiting strength is ultimate stress, therefore according



Pig iron is made from iron ore in a blast furnace. It is a brittle form of iron that contains 4-5 per cent carbon.

Note : This picture is given as additional information and is not a direct example of the current chapter.

to the above theory, taking factor of safety ($F.S.$) into consideration, the maximum principal or normal stress (σ_{t1}) in a bi-axial stress system is given by

$$\begin{aligned}\sigma_{t1} &= \frac{\sigma_{yt}}{F.S.}, \text{ for ductile materials} \\ &= \frac{\sigma_u}{F.S.}, \text{ for brittle materials}\end{aligned}$$

where

$$\begin{aligned}\sigma_{yt} &= \text{Yield point stress in tension as determined from simple tension test, and} \\ \sigma_u &= \text{Ultimate stress.}\end{aligned}$$

Since the maximum principal or normal stress theory is based on failure in tension or compression and ignores the possibility of failure due to shearing stress, therefore it is not used for ductile materials. However, for brittle materials which are relatively strong in shear but weak in tension or compression, this theory is generally used.

Note : The value of maximum principal stress (σ_{t1}) for a member subjected to bi-axial stress system may be determined as discussed in Art. 5.7.

5.11 Maximum Shear Stress Theory (Guest's or Tresca's Theory)

According to this theory, the failure or yielding occurs at a point in a member when the maximum shear stress in a bi-axial stress system reaches a value equal to the shear stress at yield point in a simple tension test. Mathematically,

$$\tau_{max} = \tau_{yt}/F.S. \quad \dots(i)$$

where

$$\begin{aligned}\tau_{max} &= \text{Maximum shear stress in a bi-axial stress system,} \\ \tau_{yt} &= \text{Shear stress at yield point as determined from simple tension test, and} \\ F.S. &= \text{Factor of safety.}\end{aligned}$$

Since the shear stress at yield point in a simple tension test is equal to one-half the yield stress in tension, therefore the equation (i) may be written as

$$\tau_{max} = \frac{\sigma_{yt}}{2 \times F.S.}$$

This theory is mostly used for designing members of ductile materials.

Note: The value of maximum shear stress in a bi-axial stress system (τ_{max}) may be determined as discussed in Art. 5.7.

5.12 Maximum Principal Strain Theory (Saint Venant's Theory)

According to this theory, the failure or yielding occurs at a point in a member when the maximum principal (or normal) strain in a bi-axial stress system reaches the limiting value of strain (*i.e.* strain at yield point) as determined from a simple tensile test. The maximum principal (or normal) strain in a bi-axial stress system is given by

$$\epsilon_{max} = \frac{\sigma_{t1}}{E} - \frac{\sigma_{t2}}{m \cdot E}$$

∴ According to the above theory,

$$\epsilon_{max} = \frac{\sigma_{t1}}{E} - \frac{\sigma_{t2}}{m \cdot E} = \epsilon = \frac{\sigma_{yt}}{E \times F.S.} \quad \dots(i)$$

where

$$\begin{aligned}\sigma_{t1} \text{ and } \sigma_{t2} &= \text{Maximum and minimum principal stresses in a bi-axial stress system,} \\ \epsilon &= \text{Strain at yield point as determined from simple tension test,} \\ 1/m &= \text{Poisson's ratio,} \\ E &= \text{Young's modulus, and} \\ F.S. &= \text{Factor of safety.}\end{aligned}$$

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From equation (i), we may write that

$$\sigma_{t1} - \frac{\sigma_{t2}}{m} = \frac{\sigma_{yt}}{F.S.}$$

This theory is not used, in general, because it only gives reliable results in particular cases.

5.13 Maximum Strain Energy Theory (Haigh's Theory)

According to this theory, the failure or yielding occurs at a point in a member when the strain energy per unit volume in a bi-axial stress system reaches the limiting strain energy (*i.e.* strain energy at the yield point) per unit volume as determined from simple tension test.



This double-decker A 380 has a passenger capacity of 555. Its engines and parts should be robust which can bear high torsional and variable stresses.

We know that strain energy per unit volume in a bi-axial stress system,

$$U_1 = \frac{1}{2E} \left[(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2 \sigma_{t1} \times \sigma_{t2}}{m} \right]$$

and limiting strain energy per unit volume for yielding as determined from simple tension test,

$$U_2 = \frac{1}{2E} \left(\frac{\sigma_{yt}}{F.S.} \right)^2$$

According to the above theory, $U_1 = U_2$.

$$\therefore \frac{1}{2E} \left[(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2 \sigma_{t1} \times \sigma_{t2}}{m} \right] = \frac{1}{2E} \left(\frac{\sigma_{yt}}{F.S.} \right)^2$$

or
$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2 \sigma_{t1} \times \sigma_{t2}}{m} = \left(\frac{\sigma_{yt}}{F.S.} \right)^2$$

This theory may be used for ductile materials.

5.14 Maximum Distortion Energy Theory (Hencky and Von Mises Theory)

According to this theory, the failure or yielding occurs at a point in a member when the distortion strain energy (also called shear strain energy) per unit volume in a bi-axial stress system reaches the limiting distortion energy (*i.e.* distortion energy at yield point) per unit volume as determined from a simple tension test. Mathematically, the maximum distortion energy theory for yielding is expressed as

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - 2\sigma_{t1} \times \sigma_{t2} = \left(\frac{\sigma_{yt}}{F.S.} \right)^2$$

This theory is mostly used for ductile materials in place of maximum strain energy theory.

Note: The maximum distortion energy is the difference between the total strain energy and the strain energy due to uniform stress.

Example 5.16. The load on a bolt consists of an axial pull of 10 kN together with a transverse shear force of 5 kN. Find the diameter of bolt required according to

1. Maximum principal stress theory; 2. Maximum shear stress theory; 3. Maximum principal strain theory; 4. Maximum strain energy theory; and 5. Maximum distortion energy theory.

Take permissible tensile stress at elastic limit = 100 MPa and poisson's ratio = 0.3.

Solution. Given : $P_{t1} = 10 \text{ kN}$; $P_s = 5 \text{ kN}$; $\sigma_{t(el)} = 100 \text{ MPa} = 100 \text{ N/mm}^2$; $1/m = 0.3$

Let $d =$ Diameter of the bolt in mm.

∴ Cross-sectional area of the bolt,

$$A = \frac{\pi}{4} \times d^2 = 0.7854 d^2 \text{ mm}^2$$

We know that axial tensile stress,

$$\sigma_1 = \frac{P_{t1}}{A} = \frac{10}{0.7854 d^2} = \frac{12.73}{d^2} \text{ kN/mm}^2$$

and transverse shear stress,

$$\tau = \frac{P_s}{A} = \frac{5}{0.7854 d^2} = \frac{6.365}{d^2} \text{ kN/mm}^2$$

1. According to maximum principal stress theory

We know that maximum principal stress,

$$\begin{aligned} \sigma_{r1} &= \frac{\sigma_1 + \sigma_2}{2} + \frac{1}{2} \left[\sqrt{(\sigma_1 - \sigma_2)^2 + 4 \tau^2} \right] \\ &= \frac{\sigma_1}{2} + \frac{1}{2} \left[\sqrt{(\sigma_1)^2 + 4 \tau^2} \right] \quad \dots(\because \sigma_2 = 0) \\ &= \frac{12.73}{2 d^2} + \frac{1}{2} \left[\sqrt{\left(\frac{12.73}{d^2}\right)^2 + 4 \left(\frac{6.365}{d^2}\right)^2} \right] \\ &= \frac{6.365}{d^2} + \frac{1}{2} \times \frac{6.365}{d^2} \left[\sqrt{4 + 4} \right] \\ &= \frac{6.365}{d^2} \left[1 + \frac{1}{2} \sqrt{4 + 4} \right] = \frac{15.365}{d^2} \text{ kN/mm}^2 = \frac{15\ 365}{d^2} \text{ N/mm}^2 \end{aligned}$$

According to maximum principal stress theory,

$$\sigma_{t1} = \sigma_{t(el)} \text{ or } \frac{15\ 365}{d^2} = 100$$

∴ $d^2 = 15\ 365/100 = 153.65$ or $d = 12.4 \text{ mm}$ **Ans.**

2. According to maximum shear stress theory

We know that maximum shear stress,

$$\begin{aligned} \tau_{max} &= \frac{1}{2} \left[\sqrt{(\sigma_1 - \sigma_2)^2 + 4 \tau^2} \right] = \frac{1}{2} \left[\sqrt{(\sigma_1)^2 + 4 \tau^2} \right] \quad \dots(\because \sigma_2 = 0) \\ &= \frac{1}{2} \left[\sqrt{\left(\frac{12.73}{d^2}\right)^2 + 4 \left(\frac{6.365}{d^2}\right)^2} \right] = \frac{1}{2} \times \frac{6.365}{d^2} \left[\sqrt{4 + 4} \right] \\ &= \frac{9}{d^2} \text{ kN/mm}^2 = \frac{9000}{d^2} \text{ N/mm}^2 \end{aligned}$$

According to maximum shear stress theory,

$$\tau_{max} = \frac{\sigma_{t(el)}}{2} \text{ or } \frac{9000}{d^2} = \frac{100}{2} = 50$$

∴ $d^2 = 9000 / 50 = 180$ or $d = 13.42 \text{ mm}$ **Ans.**

3. According to maximum principal strain theory

We know that maximum principal stress,

$$\sigma_{11} = \frac{\sigma_1}{2} + \frac{1}{2} \left[\sqrt{(\sigma_1)^2 + 4 \tau^2} \right] = \frac{15\,365}{d^2}$$

...(As calculated before)

and minimum principal stress,

$$\begin{aligned} \sigma_{12} &= \frac{\sigma_1}{2} - \frac{1}{2} \left[\sqrt{(\sigma_1)^2 + 4 \tau^2} \right] \\ &= \frac{12.73}{2 d^2} - \frac{1}{2} \left[\sqrt{\left(\frac{12.73}{d^2} \right)^2 + 4 \left(\frac{6.365}{d^2} \right)^2} \right] \\ &= \frac{6.365}{d^2} - \frac{1}{2} \times \frac{6.365}{d^2} \left[\sqrt{4 + 4} \right] \\ &= \frac{6.365}{d^2} \left[1 - \sqrt{2} \right] = \frac{-2.635}{d^2} \text{ kN/mm}^2 \\ &= \frac{-2635}{d^2} \text{ N/mm}^2 \end{aligned}$$



Front view of a jet engine. The rotors undergo high torsional and bending stresses.

We know that according to maximum principal strain theory,

$$\begin{aligned} \frac{\sigma_{11}}{E} - \frac{\sigma_{12}}{mE} &= \frac{\sigma_{t(el)}}{E} \text{ or } \sigma_{11} - \frac{\sigma_{12}}{m} = \sigma_{t(el)} \\ \therefore \frac{15\,365}{d^2} + \frac{2635 \times 0.3}{d^2} &= 100 \text{ or } \frac{16\,156}{d^2} = 100 \\ d^2 &= 16\,156 / 100 = 161.56 \text{ or } d = 12.7 \text{ mm Ans.} \end{aligned}$$

4. According to maximum strain energy theory

We know that according to maximum strain energy theory,

$$\begin{aligned} (\sigma_{11})^2 + (\sigma_{12})^2 - \frac{2 \sigma_{11} \times \sigma_{12}}{m} &= [\sigma_{t(el)}]^2 \\ \left[\frac{15\,365}{d^2} \right]^2 + \left[\frac{-2635}{d^2} \right]^2 - 2 \times \frac{15\,365}{d^2} \times \frac{-2635}{d^2} \times 0.3 &= (100)^2 \\ \frac{236 \times 10^6}{d^4} + \frac{6.94 \times 10^6}{d^4} + \frac{24.3 \times 10^6}{d^4} &= 10 \times 10^3 \\ \frac{23\,600}{d^4} + \frac{694}{d^4} + \frac{2430}{d^4} &= 1 \text{ or } \frac{26\,724}{d^4} = 1 \\ \therefore d^4 &= 26\,724 \text{ or } d = 12.78 \text{ mm Ans.} \end{aligned}$$

5. According to maximum distortion energy theory

According to maximum distortion energy theory,

$$\begin{aligned} (\sigma_{11})^2 + (\sigma_{12})^2 - 2 \sigma_{11} \times \sigma_{12} &= [\sigma_{t(el)}]^2 \\ \left[\frac{15\,365}{d^2} \right]^2 + \left[\frac{-2635}{d^2} \right]^2 - 2 \times \frac{15\,365}{d^2} \times \frac{-2635}{d^2} &= (100)^2 \\ \frac{236 \times 10^6}{d^4} + \frac{6.94 \times 10^6}{d^4} + \frac{80.97 \times 10^6}{d^4} &= 10 \times 10^3 \\ \frac{23\,600}{d^4} + \frac{694}{d^4} + \frac{8097}{d^4} &= 1 \text{ or } \frac{32\,391}{d^4} = 1 \\ \therefore d^4 &= 32\,391 \text{ or } d = 13.4 \text{ mm Ans.} \end{aligned}$$

Example 5.17. A cylindrical shaft made of steel of yield strength 700 MPa is subjected to static loads consisting of bending moment 10 kN-m and a torsional moment 30 kN-m. Determine the diameter of the shaft using two different theories of failure, and assuming a factor of safety of 2. Take $E = 210$ GPa and poisson's ratio = 0.25.

Solution. Given : $\sigma_{yt} = 700$ MPa = 700 N/mm²; $M = 10$ kN-m = 10×10^6 N-mm ; $T = 30$ kN-m = 30×10^6 N-mm ; $F.S. = 2$; $E = 210$ GPa = 210×10^3 N/mm²; $1/m = 0.25$

Let d = Diameter of the shaft in mm.

First of all, let us find the maximum and minimum principal stresses.

We know that section modulus of the shaft

$$Z = \frac{\pi}{32} \times d^3 = 0.0982 d^3 \text{ mm}^3$$

∴ Bending (tensile) stress due to the bending moment,

$$\sigma_1 = \frac{M}{Z} = \frac{10 \times 10^6}{0.0982 d^3} = \frac{101.8 \times 10^6}{d^3} \text{ N/mm}^2$$

and shear stress due to torsional moment,

$$\tau = \frac{16 T}{\pi d^3} = \frac{16 \times 30 \times 10^6}{\pi d^3} = \frac{152.8 \times 10^6}{d^3} \text{ N/mm}^2$$

We know that maximum principal stress,

$$\begin{aligned} \sigma_{t1} &= \frac{\sigma_1 + \sigma_2}{2} + \frac{1}{2} \left[\sqrt{(\sigma_1 - \sigma_2)^2 + 4 \tau^2} \right] \\ &= \frac{\sigma_1}{2} + \frac{1}{2} \left[\sqrt{(\sigma_1)^2 + 4 \tau^2} \right] \quad \dots (\because \sigma_2 = 0) \\ &= \frac{101.8 \times 10^6}{2d^3} + \frac{1}{2} \left[\sqrt{\left(\frac{101.8 \times 10^6}{d^3} \right)^2 + 4 \left(\frac{152.8 \times 10^6}{d^3} \right)^2} \right] \\ &= \frac{50.9 \times 10^6}{d^3} + \frac{1}{2} \times \frac{10^6}{d^3} \left[\sqrt{(101.8)^2 + 4 (152.8)^2} \right] \\ &= \frac{50.9 \times 10^6}{d^3} + \frac{161 \times 10^6}{d^3} = \frac{211.9 \times 10^6}{d^3} \text{ N/mm}^2 \end{aligned}$$

and minimum principal stress,

$$\begin{aligned} \sigma_{t2} &= \frac{\sigma_1 + \sigma_2}{2} - \frac{1}{2} \left[\sqrt{(\sigma_1 - \sigma_2)^2 + 4 \tau^2} \right] \\ &= \frac{\sigma_1}{2} - \frac{1}{2} \left[\sqrt{(\sigma_1)^2 + 4 \tau^2} \right] \quad \dots (\because \sigma_2 = 0) \\ &= \frac{50.9 \times 10^6}{d^3} - \frac{161 \times 10^6}{d^3} = \frac{-110.1 \times 10^6}{d^3} \text{ N/mm}^2 \end{aligned}$$

Let us now find out the diameter of shaft (d) by considering the maximum shear stress theory and maximum strain energy theory.

1. According to maximum shear stress theory

We know that maximum shear stress,

$$\tau_{max} = \frac{\sigma_{t1} - \sigma_{t2}}{2} = \frac{1}{2} \left[\frac{211.9 \times 10^6}{d^3} + \frac{110.1 \times 10^6}{d^3} \right] = \frac{161 \times 10^6}{d^3}$$

We also know that according to maximum shear stress theory,

$$\tau_{max} = \frac{\sigma_{yt}}{2 F.S.} \quad \text{or} \quad \frac{161 \times 10^6}{d^3} = \frac{700}{2 \times 2} = 175$$

$$\therefore d^3 = 161 \times 10^6 / 175 = 920 \times 10^3 \text{ or } d = 97.2 \text{ mm Ans.}$$

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Note: The value of maximum shear stress (τ_{max}) may also be obtained by using the relation,

$$\begin{aligned}\tau_{max} &= \frac{1}{2} \left[\sqrt{(\sigma_1)^2 + 4\tau^2} \right] \\ &= \frac{1}{2} \left[\sqrt{\left(\frac{101.8 \times 10^6}{d^3} \right)^2 + 4 \left(\frac{152.8 \times 10^6}{d^3} \right)^2} \right] \\ &= \frac{1}{2} \times \frac{10^6}{d^3} \left[\sqrt{(101.8)^2 + 4(152.8)^2} \right] \\ &= \frac{1}{2} \times \frac{10^6}{d^3} \times 322 = \frac{161 \times 10^6}{d^3} \text{ N/mm}^2 \quad \dots(\text{Same as before})\end{aligned}$$

2. According to maximum strain energy theory

We know that according to maximum strain energy theory,

$$\begin{aligned}\frac{1}{2E} \left[(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\sigma_{t1} \times \sigma_{t2}}{m} \right] &= \frac{1}{2E} \left(\frac{\sigma_{yt}}{F.S.} \right)^2 \\ \text{or} \quad (\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\sigma_{t1} \times \sigma_{t2}}{m} &= \left(\frac{\sigma_{yt}}{F.S.} \right)^2 \\ \left[\frac{211.9 \times 10^6}{d^3} \right]^2 + \left[\frac{-110.1 \times 10^6}{d^3} \right]^2 - 2 \times \frac{211.9 \times 10^6}{d^3} \times \frac{-110.1 \times 10^6}{d^3} \times 0.25 &= \left(\frac{700}{2} \right)^2 \\ \text{or} \quad \frac{44\,902 \times 10^{12}}{d^6} + \frac{12\,122 \times 10^{12}}{d^6} + \frac{11\,665 \times 10^{12}}{d^6} &= 122\,500 \\ \frac{68\,689 \times 10^{12}}{d^6} &= 122\,500\end{aligned}$$

$$\therefore d^6 = 68\,689 \times 10^{12} / 122\,500 = 0.5607 \times 10^{12} \text{ or } d = 90.8 \text{ mm Ans.}$$

Example 5.18. A mild steel shaft of 50 mm diameter is subjected to a bending moment of 2000 N-m and a torque T . If the yield point of the steel in tension is 200 MPa, find the maximum value of this torque without causing yielding of the shaft according to 1. the maximum principal stress; 2. the maximum shear stress; and 3. the maximum distortion strain energy theory of yielding.

Solution. Given: $d = 50 \text{ mm}$; $M = 2000 \text{ N-m} = 2 \times 10^6 \text{ N-mm}$; $\sigma_{yt} = 200 \text{ MPa} = 200 \text{ N/mm}^2$
Let $T =$ Maximum torque without causing yielding of the shaft, in N-mm.

1. According to maximum principal stress theory

We know that section modulus of the shaft,

$$Z = \frac{\pi}{32} \times d^3 = \frac{\pi}{32} (50)^3 = 12\,273 \text{ mm}^3$$

\therefore Bending stress due to the bending moment,

$$\sigma_1 = \frac{M}{Z} = \frac{2 \times 10^6}{12\,273} = 163 \text{ N/mm}^2$$

and shear stress due to the torque,

$$\tau = \frac{16T}{\pi d^3} = \frac{16T}{\pi (50)^3} = 0.0407 \times 10^{-3} T \text{ N/mm}^2$$

$$\dots \left[\because T = \frac{\pi}{16} \times \tau \times d^3 \right]$$

We know that maximum principal stress,

$$\begin{aligned}\sigma_{t1} &= \frac{\sigma_1}{2} + \frac{1}{2} \left[\sqrt{(\sigma_1)^2 + 4\tau^2} \right] \\ &= \frac{163}{2} + \frac{1}{2} \left[\sqrt{(163)^2 + 4(0.0407 \times 10^{-3} T)^2} \right]\end{aligned}$$

Minimum principal stress,

$$= 81.5 + \sqrt{6642.5 + 1.65 \times 10^{-9} T^2} \text{ N/mm}^2$$

$$\begin{aligned} \sigma_{r2} &= \frac{\sigma_1}{2} - \frac{1}{2} \left[\sqrt{(\sigma_1)^2 + 4 \tau^2} \right] \\ &= \frac{163}{2} - \frac{1}{2} \left[\sqrt{(163)^2 + 4 (0.0407 \times 10^{-3} T)^2} \right] \\ &= 81.5 - \sqrt{6642.5 + 1.65 \times 10^{-9} T^2} \text{ N/mm}^2 \end{aligned}$$

and maximum shear stress,

$$\begin{aligned} \tau_{max} &= \frac{1}{2} \left[\sqrt{(\sigma_1)^2 + 4 \tau^2} \right] = \frac{1}{2} \left[\sqrt{(163)^2 + 4 (0.0407 \times 10^{-3} T)^2} \right] \\ &= \sqrt{6642.5 + 1.65 \times 10^{-9} T^2} \text{ N/mm}^2 \end{aligned}$$

We know that according to maximum principal stress theory,

$$\begin{aligned} \sigma_{f1} &= \sigma_{yt} && \dots(\text{Taking } F.S. = 1) \\ \therefore 81.5 + \sqrt{6642.5 + 1.65 \times 10^{-9} T^2} &= 200 \\ 6642.5 + 1.65 \times 10^{-9} T^2 &= (200 - 81.5)^2 = 14\,042 \\ T^2 &= \frac{14\,042 - 6642.5}{1.65 \times 10^{-9}} = 4485 \times 10^9 \\ \text{or } T &= 2118 \times 10^3 \text{ N-mm} = 2118 \text{ N-m } \mathbf{Ans.} \end{aligned}$$

2. According to maximum shear stress theory

We know that according to maximum shear stress theory,

$$\begin{aligned} \tau_{max} &= \tau_{yt} = \frac{\sigma_{yt}}{2} \\ \therefore \sqrt{6642.5 + 1.65 \times 10^{-9} T^2} &= \frac{200}{2} = 100 \\ 6642.5 + 1.65 \times 10^{-9} T^2 &= (100)^2 = 10\,000 \\ T^2 &= \frac{10\,000 - 6642.5}{1.65 \times 10^{-9}} = 2035 \times 10^9 \\ \therefore T &= 1426 \times 10^3 \text{ N-mm} = 1426 \text{ N-m } \mathbf{Ans.} \end{aligned}$$

3. According to maximum distortion strain energy theory

We know that according to maximum distortion strain energy theory

$$\begin{aligned} (\sigma_{r1})^2 + (\sigma_{r2})^2 - \sigma_{r1} \times \sigma_{r2} &= (\sigma_{yt})^2 \\ \left[81.5 + \sqrt{6642.5 + 1.65 \times 10^{-9} T^2} \right]^2 + \left[81.5 - \sqrt{6642.5 + 1.65 \times 10^{-9} T^2} \right]^2 \\ - \left[81.5 + \sqrt{6642.5 + 1.65 \times 10^{-9} T^2} \right] \left[81.5 - \sqrt{6642.5 + 1.65 \times 10^{-9} T^2} \right] &= (200)^2 \\ 2 \left[(81.5)^2 + 6642.5 + 1.65 \times 10^{-9} T^2 \right] - \left[(81.5)^2 - 6642.5 + 1.65 \times 10^{-9} T^2 \right] &= (200)^2 \\ (81.5)^2 + 3 \times 6642.5 + 3 \times 1.65 \times 10^{-9} T^2 &= (200)^2 \\ 26\,570 + 4.95 \times 10^{-9} T^2 &= 40\,000 \\ T^2 &= \frac{40\,000 - 26\,570}{4.95 \times 10^{-9}} = 2713 \times 10^9 \\ \therefore T &= 1647 \times 10^3 \text{ N-mm} = 1647 \text{ N-m } \mathbf{Ans.} \end{aligned}$$