

### 3) Linear Equation (first order first degree)

A linear of ordinary differential Equation (ODE) can always be put in to the standard form

$$\frac{dy}{dx} + \rho(x)y = Q(x)$$

$$\rho = (\text{integration factor}) \rightarrow \rho = e^{\int \rho(x)dx}$$

$$\text{equation}[y = e^{-\int \rho(x)dx} * (\int \rho * Q(x) + C)]$$

Example// Solve  $\frac{dy}{dx} = e^X - Y$

Solution/

$$\frac{dy}{dx} + Y = e^X \leftrightarrow \frac{dy}{dx} + \rho(x)y = Q(x)$$

$$\rho(x) = 1 \quad \& \quad Q(x) = e^X$$

$$\rho = e^{\int \rho(x)dx} = e^{\int 1dx} = e^X$$

$$y = e^{-\int \rho(x)dx} * \left( \int \rho * Q(x) + C \right) = e^{-X} * \left( \int e^X * e^X dx + c \right)$$

$$=$$

$$e^{-X} * \left( \int e^{2X} dx + c \right) = e^{-X} \left( \frac{1}{2} e^{2X} + c \right) = \frac{1}{2} e^X + e^{-X}c$$

Exercise: Solve  $\frac{dy}{dx} - 3y = x^2$

## # Bernoullis Equation

using when not linear equation

**The general form**

$$\frac{dy}{dx} + \rho(x)y = Q(x)y^n \quad (n \neq 1)$$

$$\text{Assume } u = y^{1-n} \rightarrow \frac{du}{dx} = (1-n)y^{-n} \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{1-n} y^n \frac{du}{dx} \text{ sub in (1)}$$

$$\frac{dy}{dx} + \rho(x)y = Q(x)y^n \div y^n$$

$$\rightarrow \left[ y^{-n} \frac{dy}{dx} + \rho(x)y^{1-n} = Q(x) \right] \dots \dots (1)$$

$$\rightarrow \left[ y^{-n} \frac{1}{1-n} y^n \frac{du}{dx} + \rho(x)u = Q(x) \right] * (1-n)$$

$$= \left[ \frac{du}{dx} + (1-n)\rho(x)u = (1-n)Q(x) \right]$$

**Example//** Solve  $\frac{dy}{dx} + \frac{3}{x}y = x^2y^2$

**Solution /**

$$\frac{dy}{dx} + \frac{3}{x}y = x^2y^2 \quad \div y^2 \quad (n = 2)$$

$$\frac{dy}{dx} y^{-2} + \frac{3}{xy} = x^2$$

Assume  $u = y^{1-n} \rightarrow u = y^{-1} \rightarrow \frac{du}{dx} = \frac{-1}{y^2} \frac{dy}{dx}$

$$\frac{dy}{dx} = -y^2 \frac{du}{dx} \quad \text{sub in equation}$$

$$\rightarrow -y^2 * y^{-2} \frac{du}{dx} + \frac{3}{xy} = x^2 \rightarrow \frac{du}{dx} - \frac{3u}{x} = -x^2 \quad \text{is linear}$$

$$\rho(x) = \frac{-3}{x} \quad \& \quad Q(x) = -x^2$$

$$\rho = e^{\int \frac{-3}{x} dx} = e^{-3 \ln x} = x^{-3}$$

$$\begin{aligned} y &= e^{-\int \rho(x) dx} * \left( \int \rho * Q(x) + C \right) \\ &= x^3 * \left( \int x^{-3} * (-x^2) dx + c \right) = \end{aligned}$$

$$x^3 * \left( \int \frac{-1}{x} dx + c \right) = x^3 (-\ln x + c) = -x^3 \ln x + x^3 c$$

**Exercise: Solve**  $\frac{dy}{dx} + \frac{y}{x} = xy^2$