2)Homogenous Equation (first order first degree)

If $f(tx.ty) = t^n\big(f(x.y)\big) is Homogenous$

$t > 0$ & $real\ number$

EX// $f(x.y) = x^2 + 3xy + 4y^2$ (test)

$f(tx.ty) = t^n\big(f(x.y)\big) \to f(tx.ty) = t^2x^2 + 3t^2xy + 4t^2y^2$

$=t^2(x^2 + 3xy + 4y^2) = t^2 * f(x.y)\ \ is Homogenous$

$*\frac{dy}{dx} = f\left(\frac{y}{x}\right)$ $to\ be Homogenous$

Assume $u = \frac{y}{x} \to y = ux \to \frac{dy}{dx} = u + \frac{du}{dx}$ Sub in *

$$\frac{dy}{dx} = f(u) \to u + x\frac{du}{dx} = f(u)$$

$$\left(\int \frac{dx}{x} + \int \frac{du}{u - f(u)} = 0\right)\ Homogenous$$

Example// Solve $(x^2 + y^2)dx + 2xydy = 0$

Solution/

<u>test</u>

$(x^2 + y^2)dx + 2xydy = 0 \ \to \frac{dy}{dx} = \frac{-(x^2+y^2)}{2xy} =$

$\frac{-(t^2x^2+t^2y^2)}{2txty} = \frac{-t^2(x^2+y^2)}{t^2 2xy}$ is $Homogenous$

Where u=$\frac{y}{x}$. $\frac{dy}{dx} = \frac{-(x^2+y^2)}{2xy} \div x^2$

$$\frac{dy}{dx} = \frac{-(1+(\frac{y}{x})^2}{2\frac{y}{x}} = f\left(\frac{y}{x}\right) = f(u)$$

$$f(u) = \frac{-(1+u^2)}{2u}$$

$$=\left(\int \frac{dx}{x} + \int \frac{du}{u-f(u)} = 0\right) = \left(\int \frac{dx}{x} + \int \frac{du}{u-\frac{-(1+u^2)}{2u}} = 0\right)$$

$$= \left(\int \frac{dx}{x} + \int \frac{2udu}{1+3u^2} = 0\right) \to ln|x| + \frac{1}{3} ln|1+3u^2| = c$$

$$\to ln|x| + \frac{1}{3} ln\left|1+3(\frac{y}{x})^2\right| = c$$

Example// Solve $\frac{dy}{dx} = \frac{x-y}{x+y}$

Solution //

test

$$\frac{dy}{dx} = \frac{x-y}{x+y} = \frac{tx-ty}{tx+ty} = \frac{t(x-y)}{t(x+y)} is\ Homogenous$$

Where u=$\frac{y}{x}$. $\frac{dy}{dx} = \frac{x-y}{x+y}$ $\div x$

$$\frac{dy}{dx}=\frac{1-\frac{y}{x}}{1+\frac{y}{x}}=f\left(\frac{y}{x}\right)=f(u)$$

$$f(u)=\frac{1-u}{1+u}$$

$$=\left(\int\frac{dx}{x}+\int\frac{du}{u-f(u)}=0\right)=\left(\int\frac{dx}{x}+\int\frac{du}{u-\frac{1-u}{1+u}}=0\right)$$

$$=\left(\int\frac{dx}{x}+\int\frac{(1+u)du}{u^2+2u-1}=0\right)$$

$$\rightarrow ln|x|+\frac{1}{2}ln|u^2+2u-1|=c$$

$$\rightarrow ln|x|+\frac{1}{2}ln\left|\frac{y^2}{x^2}+2\frac{y}{x}-1\right|=c$$

Example // Solve $\left(xe^{\frac{y}{x}}+y\right)dx-x\,dy=0$

Solution//

test

$$\rightarrow\frac{dy}{dx}=\frac{(xe^{\frac{y}{x}}+y)}{x}=\left(xe^{\frac{y}{x}}+y\right)dx-x\,dy=0$$

$$\frac{(txe^{\frac{ty}{tx}}+ty)}{tx}=\frac{t(xe^{\frac{y}{x}}+y)}{t(x)}\text{ is }Homogenous$$

Where u= $\frac{y}{x}$. $\frac{dy}{dx} = \frac{(xe^{\frac{y}{x}}+y)}{x} \quad \div x$

$$\frac{dy}{dx} = e^{\frac{y}{x}} + \frac{y}{x} = f\left(\frac{y}{x}\right) = f(u)$$

$$f(u) = e^{u} + u$$

$$=\left(\int \frac{dx}{x} + \int \frac{du}{u-f(u)} = 0\right) = \left(\int \frac{dx}{x} + \int \frac{du}{u-(e^{u}+u)} = 0\right)$$

$$= \left(\int \frac{dx}{x} + \int -e^{-u}\, du = 0\right) \to ln|x| + e^{-u} = c$$

$$\to ln|x| + e^{-\frac{y}{x}} = c$$