Inverse Trigonometric Function and It's Derivatives, Logarithm and Exponential Functions and Their Derivatives

مقلوب الدوال المثلثية ومشتقاتها، الدوال اللوغارتمية والاسية ومشتقاتها

Inverse Trigonometric Functions
الدوال المثلثية العكسية

Inverse trigonometric Functions are defined as the inverse functions of the basic trigonometric Funs, which are sine, Cosine, tangent, cotangent, secant & cosecant.

(1) Evaluate $\sin^{-1}$ Fun

The Range of $\sin^{-1}$ Fun is between $-\frac{\pi}{2}$ & $+\frac{\pi}{2}$

$\therefore$ Range $\Rightarrow [-\frac{\pi}{2}, \frac{\pi}{2}]$

So, it is only exist in this range.

Ex)

- $\sin^{-1} 0 = 0$
- $\sin^{-1} 1 = \frac{\pi}{2}$
- $\sin^{-1}(-1) = -\frac{\pi}{2}$
- $\sin^{-1} \frac{1}{2} = \frac{\pi}{6}$
- $\sin^{-1} \frac{\sqrt{3}}{2} = \pi/3 = 60$
- $\sin^{-1}(-\frac{1}{2}) = -\frac{\pi}{6}$
- $\sin^{-1}(-\frac{1}{\sqrt{2}}) = -\pi/4$
- ~~sin~~

(2) Evaluate $\cos^{-1}$ Fun :-

The Range of $\cosine^{-1}$ Fun is between Zero & π

$\therefore$ Range $\Rightarrow [0, \pi]$, so $\cos^{-1}$ is only exist in this range

Ex)

- $\cos^{-1} \frac{1}{2} = \frac{\pi}{3}$
- $\cos^{-1}(-\frac{\sqrt{3}}{2}) = 150 = \frac{5\pi}{6}$

- $\cos^{-1}(-\frac{1}{\sqrt{2}}) = 135 = \frac{3\pi}{4}$
- $\cos^{-1}(0) = \frac{\pi}{2}$
- $\cos^{-1}(1) = 0$
- $\cos(-1) = \pi$

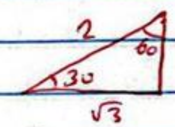
③ **Evaluate $\tan^{-1}$ Fun**

The Range of $\tan^{-1}$ Fun is similar to $\sin^{-1}$, which is between $-\pi/2$ to $\pi/2$

$\therefore$ Range $\longrightarrow [-\frac{\pi}{2}, \frac{\pi}{2}]$

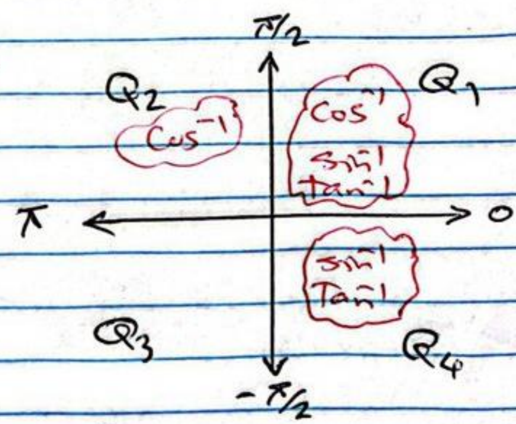
Ex/

- $\tan^{-1}(0) = 0$
- $\tan^{-1}(1) = \pi/4$
- $\tan^{-1}(-1) = -\pi/4$
- $\tan^{-1}(\sqrt{3}) = \pi/3 = 60$
- $\tan^{-1}(-\frac{1}{\sqrt{3}}) = -\pi/6 = -30$



$\tan 60 = \frac{\sqrt{3}}{1} = \sqrt{3}$

Summary



Arc sine	Range
$\cos^{-1}x$	$\longrightarrow [0, \pi]$
$\sin^{-1}x$, $\tan^{-1}x$	$\longrightarrow [-\pi/2, \pi/2]$

The Derivative of Inverse Trigonometric Functions :-

① $\frac{d}{dx}(\sin^{-1}u) = \frac{u'}{\sqrt{1-u^2}}$

② $\frac{d}{dx}(\cos^{-1}u) = \frac{-u'}{\sqrt{1-u^2}}$

③ $\frac{d}{dx}(\tan^{-1}u) = \frac{u'}{1+u^2}$

④ $\frac{d}{dx}(\sec^{-1}u) = \frac{u'}{|u|\sqrt{u^2-1}}$

⑤ $\frac{d}{dx}(\csc^{-1}u) = \frac{-u}{|u|\sqrt{u^2-1}}$

⑥ $\frac{d}{dx}(\cot^{-1}u) = \frac{-u'}{1+u^2}$

Examples

1- $\frac{d}{dx}(\sin^{-1}x^3) \longrightarrow \frac{d}{dx}(\sin^{-1}x^3) = \boxed{\frac{3x^2}{\sqrt{1-x^6}}}$

2- $\frac{d}{dx}(\cos^{-1}(5x-4)) = \boxed{\frac{-5}{\sqrt{1-(5x-4)^2}}}$

3- $\frac{d}{dx}(\tan^{-1}\sqrt{x}) = \boxed{\frac{\frac{1}{2\sqrt{x}}}{1+x}}$

4- $\frac{d}{dx}(\sec^{-1}x^3) = \frac{3\cancel{x^2}}{x^{\cancel{3}\,1}\sqrt{x^6-1}} = \boxed{\frac{3}{x\sqrt{x^6-1}}}$

Logarithmic & Exponential Functions :-

In mathematics, the logarithmic is the inverse function to exponential. i.e the logarithm of a number x to the base b is the exponent to which b must be raised to produce x.

- Ex 1)

$$\log_b x = y \quad = \quad b^y = x$$

base — logarithmic form fun — exponential form fun

- Ex 2)

$$\log_{10} 1000 = 3 \quad = \quad 10^3 = 1000$$

* Common logarithm :-
It is the logarithm with base 10.

Evaluate logarithmic Function :-

Ex)

1- $\log_2 4 = 2$

2- $\log_2 8 = 3$

3- $\log_3 9 = 2$

4- $\log_3 27 = 3$

5- $\log_4 16 = 2$

6- $\log_2 32 = 5$

7- $\log_3 1 = 0$

8- $\log_{10} 10 = 1$

ملاحظة: في حالة لم يتم ذكر اساس (base) اللوغاريتم فهنا يعتبر 10

9- $\log_{10} 100 = 2$

10- $\log 10000 = 4$

11- $\log(0.1) = -1$

12- $\log(0.01) = -2$

13- $\log(-5) = DNE$ "does not exist"

14- $\log(0) = DNE$

15- $\log_4 (16) = 2$

16- $\log_4 \left(\frac{1}{16}\right) = -2$

Note)

$$\log_a b = \frac{\log b}{\log a}$$

Ex)

- $\log_4 16 = 2 = \frac{\log 16}{\log 4} = 2$
- $\log_2 8 = 3 = \frac{\log 8}{\log 2} = 3$

Natural logarithm :-

$$\log_e x = \ln x \quad ; \quad \ln 1 = 0 \,,\ \ln e = 1$$

properities of logarithms :-

1. $\log(xy) = \log(x) + \log(y)$
2. $\log\left(\frac{x}{y}\right) = \log(x) - \log(y)$
3. $\log x^n = n \log(x)$

Ex)

1. Re-write the Following log expression into a single log : $\log(x) + \log(y) - \log(z)$

Sol.)

$\log(x) + \log(y) - \log(z) = \boxed{\log\left(\frac{xy}{z}\right)}$

2. $\log x - \log y + \log z - \log R = \boxed{\log\left(\frac{xz}{yR}\right)}$

3. $2\log x + 3\log y - 4\log z = \log x^2 + \log y^3 - \log z^4$
$= \boxed{\log \frac{x^2 y^3}{z^4}}$

Ex) Expand the log into multiple $\log\left(\frac{x^2y^5}{z^6}\right)$

Sol.)

$$\log\left(\frac{x^2y^5}{z^6}\right) = \log x^2 + \log y^5 - \log z^6$$

$$= \boxed{2\log x + 5\log y - 6\log z}$$

Derivative of Logarithmic & Exponential Fns:

1- $\frac{d}{dx}(\log_a u) = \frac{1}{u \ln a} \cdot \frac{du}{dx}$

2- $\frac{d}{dx}(\ln u) = \frac{1}{u} \cdot \frac{du}{dx}$

3- $\frac{d}{dx}(a^u) = a^u \ln a \cdot \frac{du}{dx}$; a — constant

4- $\frac{d}{dx}(e^u) = e^u \cdot \frac{du}{dx}$

Ex)

1- $\frac{d}{dx}(\log_2 x) = \boxed{\frac{1}{x \ln 2}}$

2- $\frac{d}{dx}(\log_4 x^2) = \boxed{\frac{2}{x \ln 4}}$

3- $\frac{d}{dx}(\log_7(5-2x)) = \frac{1}{(5-2x)\ln 7} * -2 = \boxed{\frac{-2}{(5-2x)\ln 7}}$

4- $\frac{d}{dx}(\log_5(\tan x)) = \frac{1}{\tan x \cdot \ln 5} * \sec^2 x = \boxed{\frac{\sec^2 x}{\tan x \cdot \ln 5}}$

5- $\frac{d}{dx}(\ln x^2) = \frac{1}{x^2} * 2x = \boxed{\frac{2}{x}}$

6- $\frac{d}{dx}(\ln(x^2+5)) = \frac{1}{x^2+5} * 2x = \boxed{\frac{2x}{x^2+5}}$

7- $\frac{d}{dx}(\ln(\sin x)) = \frac{1}{\sin x} * \cos x = \frac{\cos x}{\sin x} = \boxed{\cot x}$

8- $\frac{d}{dx}(\sqrt[3]{\ln x}) = \frac{d}{dx}(\ln x)^{1/3} = \frac{1}{3}(\ln x)^{1/3-1} * \frac{1}{x}$

$= \boxed{\frac{1}{3x\,(\ln x)^{2/3}}}$

9- $\frac{d}{dx}(\ln(\ln x)) = \frac{1}{\ln x} * \frac{1}{x} = \boxed{\frac{1}{x\,\ln x}}$

10- $\frac{d}{dx}(e^x) = e^x * 1 = \boxed{e^x}$

11- $\frac{d}{dx}(e^{5x+3}) = e^{5x+3} * 5 = \boxed{5\,e^{5x+3}}$

12- $\frac{d}{dx}(e^{x^3}) = e^{x^3} * 3x^2 = \boxed{3x^2\,e^{x^3}}$

13- $\frac{d}{dx}(3^{x^2}) = 3^{x^2} * 2x * \ln 3 = \boxed{2x\,3^{x^2}\ln 3}$

14- $\frac{d}{dx}(7^{2x+5}) = 7^{2x+5} * (2) * \ln 7 = \boxed{2 * 7^{2x+5} * \ln 7}$

15- $\frac{d}{dx}[9^{x^3}] = 9^{x^3} * 3x^2 * \ln 9 = \boxed{3x^2 \cdot 9^{x^3} \cdot \ln 9}$

16- $\frac{d}{dx}(4^{\tan x}) = \boxed{4^{\tan x} * \sec^2 x * \ln 4}$

17- $\frac{d}{dx}(x^4 e^{4x}) = x^4 * e^{4x} * 4 + e^{4x} * 4x^3$

$= \boxed{4x^3 e^{4x}(x+1)}$

18- $\frac{d}{dx}\left[\frac{e^x + e^{-x}}{e^x - e^{-x}}\right] = \frac{(e^x - e^{-x})(e^x - e^{-x}) - (e^x + e^{-x})(e^x + e^{-x})}{(e^x - e^{-x})^2}$

$= \frac{(e^x - e^{-x})^2 - (e^x + e^{-x})^2}{(e^x - e^{-x})^2} = \boxed{1 - \frac{(e^x + e^{-x})^2}{(e^x - e^{-x})^2}}$

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