Al-Mustaqbal University

College of Engineering and Technology

Department of Computer Techniques Engineering

Class: second Class

Subject: Instrumentation and Measurement

Lecturer: MSC.Zainab Kadum Jabber

Lecture Address: Bridges and Their Application

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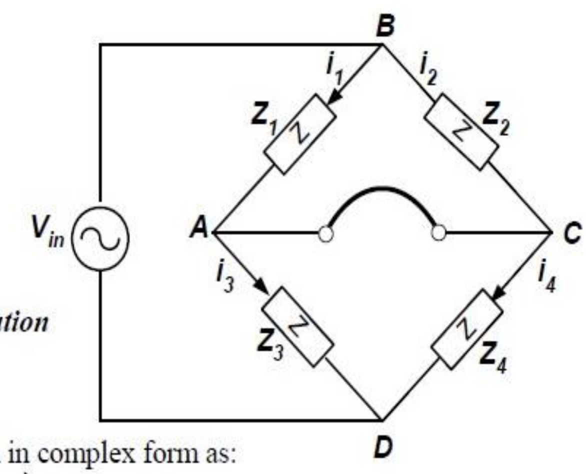
E-mail:zainab.kadum.jaber@uomus.edu.iq

2-*Ac Bridge and Their Application:*

The ac bridge is a natural outgrowth of the dc bridge and in its basic form consists of four bridge arms, a source of excitation, and a null ac detector.
For measurements at low frequencies, the power line may serve as the source of excitation; but at higher frequencies an oscillator generally supplies the excitation voltage.
The null ac detector in its cheapest effective form consists of a pair of headphones or may be oscilloscope.
The balance condition is reached when the detector response is zero or indicates null.
Then $V_{AC} = 0$ and $V_{Z1} = V_{Z2}$



$$V_{Z1} = Vin\frac{Z_1}{Z_1 + Z_3}$$

$$V_{Z2} = Vin\frac{Z_2}{Z_2 + Z_4} \quad \text{thus}$$

$\boxed{Z_1 Z_4 = Z_2 Z_3}$ *is the balance equation*

Or $\boxed{Y_1 Y_4 = Y_2 Y_3}$

The balance equation can be written in complex form as:

$$(Z_1\angle\theta_1)(Z_4\angle\theta_4) = (Z_2\angle\theta_2)(Z_3\angle\theta_3)$$

And $(Z_1 Z_4 \angle\theta_1 + \theta_4) = (Z_2 Z_3 \angle\theta_2 + \theta_3)$

So two conditions must be met simultaneously when balancing an ac bridge

1- $Z_1 Z_4 = Z_2 Z_3$
2- $\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_4$

Review on AC Impedance

Alternating Current (AC) Impedance, denoted by 'Z', represents the opposition to the flow of AC current in an electrical circuit.

It's a complex value comprised of real and imaginary parts, namely resistance 'R' and reactance 'X', respectively.

The AC impedance formula is represented as:

$Z = R + jX$

where:

- 'Z' is the total impedance of the circuit,
- 'R' is the resistive (real) component, and
- 'X' is the reactive (imaginary) component. Here, 'j' represents the imaginary unit in electrical engineering, equivalent to the square root of -1.

Furthermore, in AC circuits involving inductors and capacitors, the reactive component can be broken down into inductive reactance (X_L) and capacitive reactance (X_C).

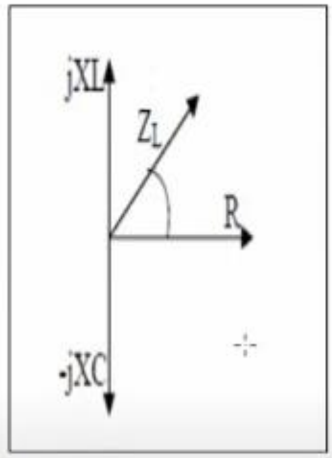
E-mail:zainab.kadum.jaber@uomus.edu.iq

a) **In series connection**

Impedance = resistance ± j reactance

$Z_L = R + jXL$ and $Z_L = R + j\omega L$

$Z_C = R - jXC$ and $Z_C = R - j\frac{1}{\omega C}$



Conversion from polar to rectangular

$Z\angle\theta$ in polar form R= Z Cosθ X= Z Sinθ become $Z = R \pm jX$

Conversion from rectangular to polar

$Z = R \pm jX$ in rectangular form $Z = \sqrt{R^2 + X^2}$ $\theta = \tan^{-1}\frac{X}{R}$ $\tan\theta = \frac{X}{R}$

So two conditions must be met simultaneously when balancing an ac bridge

Magnitude balance: $Z_1 Z_4 = Z_2 Z_3$

Phase balance: $\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3$

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Example (1):

The impedance of the basic a.c bridge are given as follows:

$Z_1 = 100\angle 80^\circ$ (inductive impedance) $\quad Z_2 = 250\Omega \quad Z_3 = 400\angle 30^\circ$ (inductive impedance)

$Z_4 = unknown$

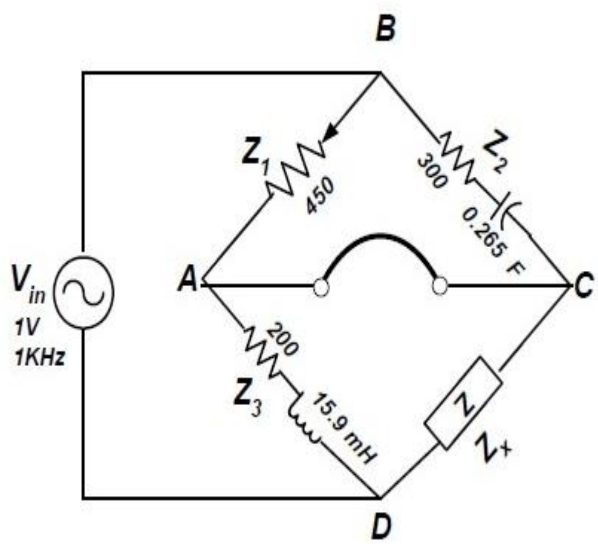
Sol:

$$\boxed{Z_4 = \frac{Z_2 Z_3}{Z_1}} \qquad Z_4 = \frac{250\times 400}{100} = 1k\Omega \qquad \boxed{\theta_4 = \theta_2 + \theta_3 - \theta_1} \qquad \theta_4 = 0 + 30 - 80 = -50^\circ$$

$Z_4 = 1000\angle -50^\circ$ (capacitive impedance)

Example (2):

For the following bridge find Zx?



The balance equation $Z_1 Z_4 = Z_2 Z_3$

$Z_1 = R = 450\Omega$

$Z_2 = R + \frac{1}{j\omega C} = R - \frac{j}{\omega C}$

$Z_2 = 300 - j600$

$Z_3 = R + j\omega L$

$Z_3 = 200 + j100$

$Z_4 = Z_x = unknown$

$$Z_4 = \frac{Z_2 Z_3}{Z_1} \qquad Z_4 = \frac{(300 - j600)(200 + j100)}{450} = 266.6 - j200$$

$$R = 266.6\Omega \qquad C = \frac{1}{2\pi F\times 200} = 0.79\mu F$$

Comparison Bridge:

There are two types of Comparison Bridge, Namely

1. Capacitance Comparison Bridge
2. Inductance Comparison Bridge

1. Capacitance Comparison Bridge:

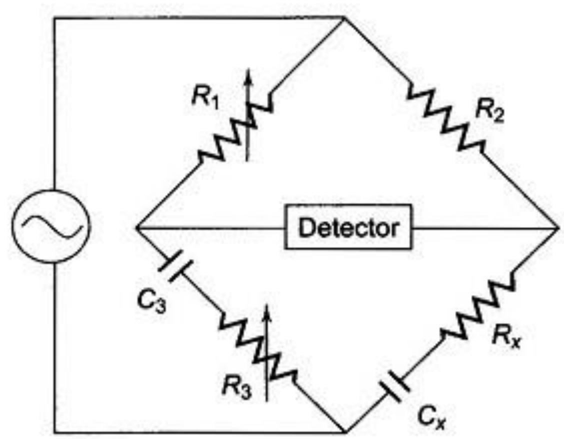


Fig. 11.18 Capacitance Comparison Bridge

Figure 11.18 shows the circuit of a capacitance comparison bridge. The ratio arms R_1, R_2 are resistive.
The known standard capacitor C_3 is in series with R_3.
R_3 may also include an added variable resistance needed to balance the bridge.

E-mail:zainab.kadum.jaber@uomus.edu.iq

C_x is the unknown capacitor and R_x is the small leakage resistance of the capacitor.

In this case an unknown capacitor is compared with a standard capacitor and the value of the former, along with its leakage resistance, is obtained. Hence.

$$Z_1 = R_1$$
$$Z_2 = R_2$$
$$Z_3 = R_3 \text{ in series with } C_3 = R_3 - j/\omega C_3$$
$$Z_x = R_x \text{ in series with } C_x = R_x - j/\omega C_x$$

The condition for balance of the bridge is

$$Z_1 Z_x = Z_2 Z_3$$

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$$R_1\left(R_x-\frac{j}{\omega C_x}\right)=R_2\left(R_s-\frac{j}{\omega C_s}\right)$$

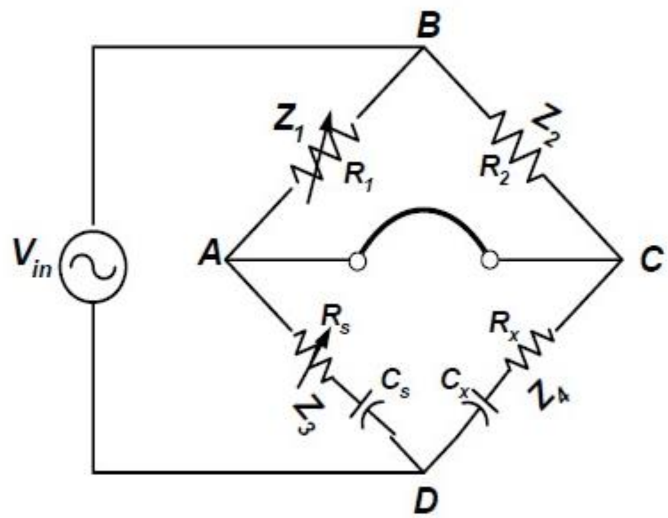
$$R_1R_x-\frac{jR_1}{\omega C_x}=R_2R_s-\frac{jR_2}{\omega C_s}$$

By equating the real term with the real and imaginary term with imaginary we get:

$$R_1R_x=R_2R_s \qquad R_x=\frac{R_2R_s}{R_1}$$

$$\frac{-jR_1}{\omega C_x}=\frac{-jR_2}{\omega C_s} \qquad C_x=\frac{R_1C_s}{R_2}$$

We can ***note*** that the bridge is ***independent*** on ***frequency*** of applied source.



$$\therefore R_x=\frac{R_2\,R_3}{R_1}$$

$$C_x=\frac{C_3\,R_1}{R_2}$$

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2.Inductance Comparison Bridge:

Figure 11.20 gives a schematic diagram of an inductance comparison bridges. In this, values of the unknown inductance Lx and its internal resistance Rx are obtained by comparison with the standard inductor and resistance, i.e. L3 and R3.

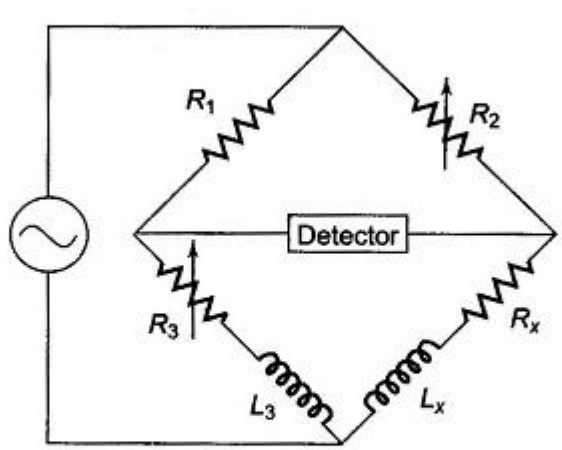

Fig. 11.20 Inductance Comparison Bridge

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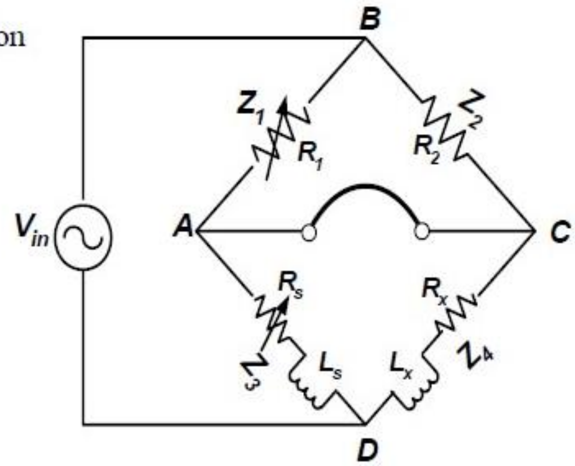
The unknown inductance is determined by comparing it with a known standard inductor. At balance we get

$$R_x = \frac{R_2 R_s}{R_1}$$ represent resistive balance equation

$$L_x = \frac{R_2 L_s}{R_1}$$ inductive balance equation



b) ***Maxwell bridge:***

This bridge measure ***unknown inductance*** in terms of ***a known capacitance***, at balance:

$$Z_1 Z_4 = Z_2 Z_3 \qquad Z_1 = \frac{1}{Y_1} \quad \text{thus}$$

$$\boxed{Z_4 = Z_2 Z_3 Y_1} \quad \text{where}$$

$$Z_2 = R_2 \qquad Z_3 = R_3 \quad Y_1 = \frac{1}{R_1} + j\omega C_1$$

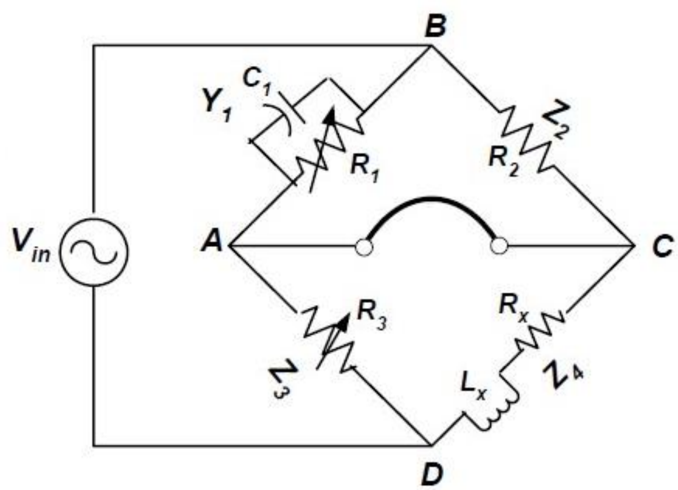
$$Z_4 = R_x + j\omega L_x$$

So

$$R_x + j\omega L_x = R_2 R_3 \left(\frac{1}{R_1} + j\omega C_1 \right)$$

$$R_x = \frac{R_2 R_3}{R_1}$$

$$L_x = R_2 R_3 C_1$$



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c) Hay Bridge:

Hay bridge convening for ***measuring high Q coils***

$Z_1 = R_1 - \frac{j}{\omega C_1} \qquad Z_2 = R_2 \qquad Z_3 = R_3$

$Z_4 = R_x + j\omega L_x$

At balance $\qquad Z_1 Z_4 = Z_2 Z_3$

$\left(R_1 - \frac{j}{\omega C_1}\right)(R_x + j\omega L_x) = R_2 R_3$

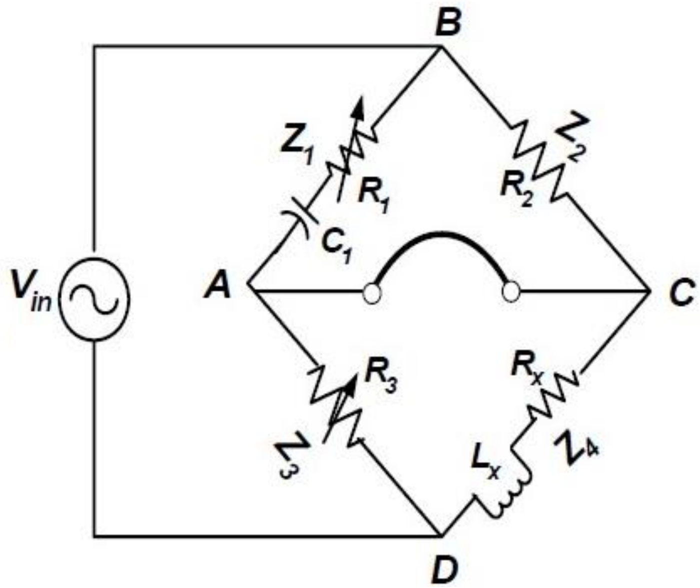
$R_1 R_x + \frac{L_x}{C_1} - \frac{jR_x}{\omega C_1} + j\omega R_1 L_x = R_2 R_3$

Separating the real and imaginary terms

$R_1 R_x + \frac{L_x}{C_1} = R_2 R_3$ ….. (1)

$\frac{R_x}{\omega C_1} = \omega R_1 L_x$ …… … (2)

Solving equ.(1) and (2) yields



$$R_x = \frac{\omega^2 C_1^2 R_1 R_2 R_3}{1 + \omega^2 C_1^2 R_1^2}$$

$$L_x = \frac{R_2 R_3 C_1}{1 + \omega^2 C_1^2 R_1^2}$$

$$\theta 1 = -\theta 4 \quad \text{because} \quad \theta 2 = \theta 3 = zero$$

E-mail:zainab.kadum.jaber@uomus.edu.iq

d) *Schering Bridge:*

Schering bridge used extensively for capacitive measurement, (C3) is standard high mica capacitor for general measurement work, or (C3) may be an air capacitor for insulation measurements. The balance condition require that $\theta 1+\theta 4=\theta 2+\theta 3$ but $\theta 1+\theta 4=-90$
Thus $\theta 2+\theta 3$ must equal (-90) to get balance

At balance $\boxed{Z_4 = Z_2 Z_3 Y_1}$

$$Y_1 = \frac{1}{R_1} + j\omega C_1 \qquad Z_2 = R_2 \qquad Z_3 = \frac{-j}{\omega C_3}$$

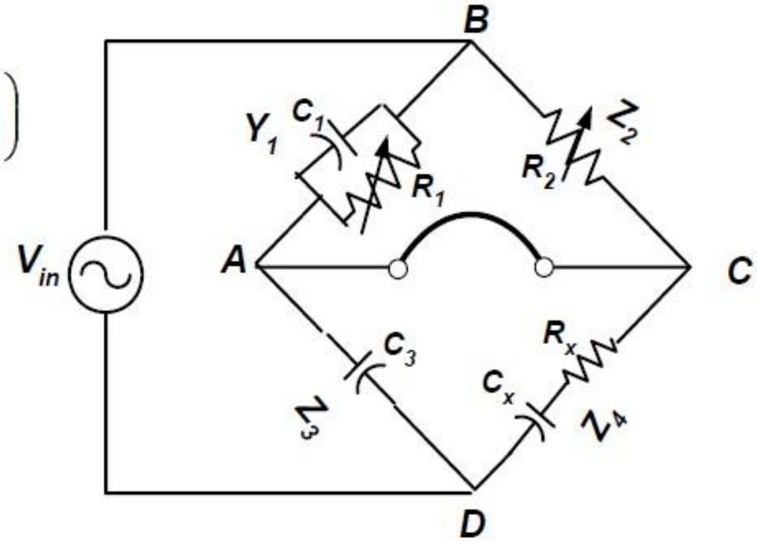
$$Z_4 = R_x - \frac{j}{\omega C_x}$$

$$R_x - \frac{j}{\omega C_x} = R_2\left(\frac{-j}{\omega C_3}\right)\left(\frac{1}{R_1} + j\omega C_1\right)$$

$$R_x - \frac{j}{\omega C_x} = \frac{R_2 C_1}{C_2} - \frac{jR_2}{\omega C_3 R_1}$$

$$\boxed{R_x = R_2 \frac{C_1}{C_3}} \quad \ldots\ldots (1)$$

$$\boxed{C_x = C_3 \frac{R_1}{R_2}} \quad \ldots\ldots (2)$$



The power factor (pf):

$$pf = Cos\,\theta_c = \frac{R_x}{Z_x}$$

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The dissipation factor (D):

$$D = Cot\theta_c = \frac{R_x}{XC_x} = \frac{1}{Q} = \omega R_x C_x \quad \ldots\ldots (3)$$

Substitute equs. (1) & (2) into (3), we get

$$D = \omega R_1 C_1$$

e) Wien Bridge:

This bridge is used to measured ***unknown frequency***

$$Z_1 = R_1 - \frac{j}{\omega C_1} \qquad Z_2 = R_2 \qquad Y_3 = \frac{1}{R_3} + j\omega C_3 \qquad Z_4 = R_4$$

$$Z_1 Z_4 = \frac{Z_2}{Y_3}$$

$$Z_2 = Z_1 Z_4 Y_3$$

$$R_2 = \left(R_1 - \frac{j}{\omega C_1}\right) R_4 \left(\frac{1}{R_3} + j\omega C_3\right)$$

$$R_2 = \frac{R_1 R_4}{R_3} + \frac{R_4 C_3}{C_1}$$

Dividing by R_4 we get

$$\frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{C_3}{C_1} \quad \ldots\ldots. (1)$$

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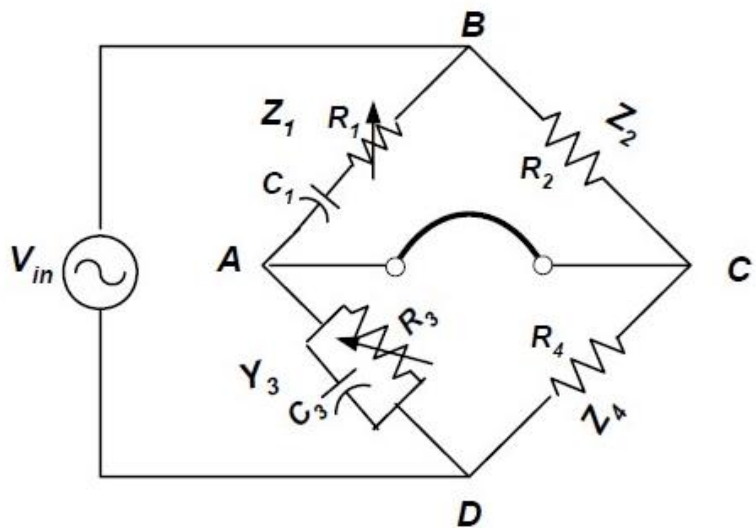
Equating the imaginary terms, yield

$$\omega C_3 R_1 R_4 = \frac{R_4}{\omega C_1 R_3} \quad \text{Since } \omega = 2\pi F$$

Thus $\boxed{F = \frac{1}{2\pi\sqrt{C_1 C_3 R_1 R_3}}}$ if $R_1 = R_3$ and $C_1 = C_3$ then $\frac{R_2}{R_4} = 2$ in equ.(1)

And $F = \frac{1}{2\pi RC}$ ***this is the general equation for Wien bridge***



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