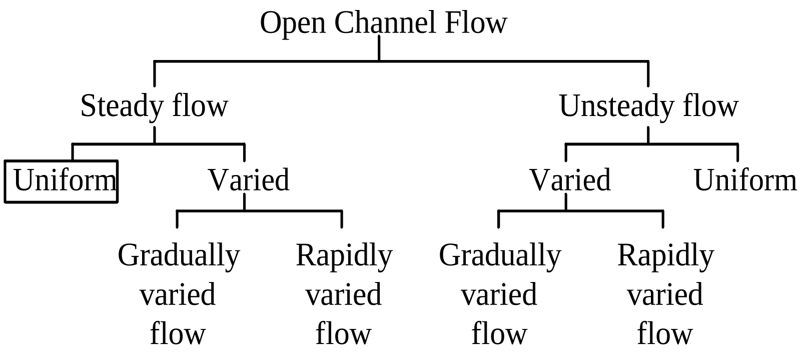
8 OPEN CHANNEL FLOW

8.1 Classification & Definition

- Open channel flows are flows in rivers, streams, artificial channels, irrigation ditches, partially filled pipe etc.

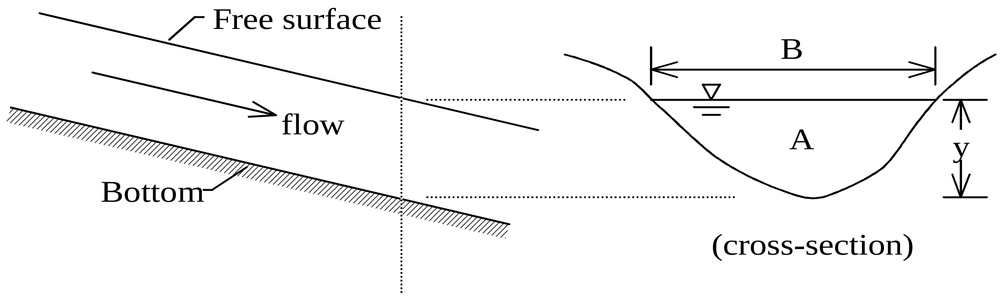
- Basically, it is a flow with **free surface**.
(Free surface is a surface with atmospheric pressure)



Classifications of Open Channel Flow

8.1.1 Open Channel Geometry

1. Depth of flow, **y**: **vertical** distance from the bottom to surface.



2 Top width, **B**:
– the width of the channel at the free surface

3 Flow area, **A**:
– cross-sectional area of the flow

4 Wetted perimeter, **P**:
– the length of the channel cross-section in contact with the fluid

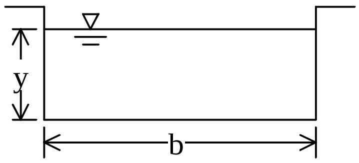
5 Hydraulic radius (hydraulic mean depth), **R**:

$$R = \frac{\text{Flow area}}{\text{Wetted perimeter}} = \frac{A}{P}$$

6 Average depth (hydraulic average depth), $\mathbf{y_{ave}}$:

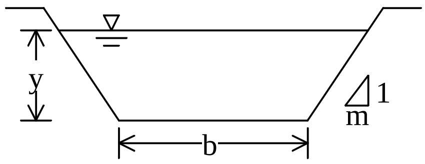
$$y_{ave} = \frac{\text{Flow area}}{\text{Top width}} = \frac{A}{B}$$

8.1.2 Rectangular channel



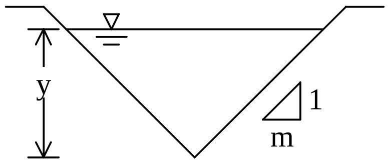
- $B = b$
- $A = b*y$
- $P = b+2*y$
- $R = \dfrac{b*y}{b+2*y}$
- $y_{ave} = y$

8.1.3 Trapezoidal channel



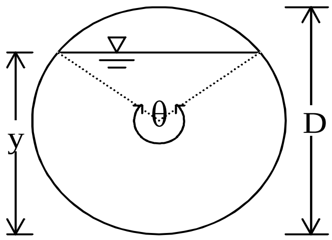
- $B = b + 2*m*y$
- $A = y*(b+m*y)$
- $P = b+2*y*\sqrt{1+m^2}$
- $R = \dfrac{y*(b+m*y)}{b+2*y*\sqrt{1+m^2}}$
- $y_{ave} = \dfrac{y*(b+m*y)}{b+2*m*y}$

8.1.4 Triangular channel



- B $= 2*m*y$
- A $= m*y^2$
- P $= 2*y*\sqrt{1+m^2}$
- R $= \dfrac{m*y}{2*\sqrt{1+m^2}}$
- y_{ave} $= \dfrac{y}{2}$

8.1.5 Circular channel



(θ in radian)

- B $= 2*\sqrt{y*(D-y)}$
- A $= \dfrac{D^2*(\theta-\sin\theta)}{8}$
- P $= \dfrac{\theta*D}{2}$
- R $= \dfrac{D}{4}\left(1-\dfrac{\sin\theta}{\theta}\right)$
- y_{ave} $= \dfrac{D*(\theta-\sin\theta)}{8*\sin\theta/2}$

8.2 Steady Uniform Flow

- For a steady uniform flow
  - **depth** is **constant** along the flow
  - **velocity** is **constant** over the cross-section
  - **time independent**

8.2.1 Manning Equations

- In 1890, Manning, an Irish engineer derived a better and more accurate relationship, **Manning equation**, based on many field measurement.

$$V = \frac{1}{n} * R^{2/3} * S^{1/2} \qquad (8.1)$$

- **n - Manning's coefficient, s/m$^{1/3}$**
  (can be found in most of the hydraulic handbooks)

- To incorporate the continuity equation, Manning equation becomes

$$Q = \frac{A}{n} * R^{2/3} * S^{1/2} \qquad (8.2)$$

- As the flow according to Manning equations is for normal steady uniform flow,
  - the flow is **Normal Flow**
  - the depth is **Normal Depth**

Worked examples:

1. Water flows in a rectangular, concrete, open channel that is 12 m wide at a depth of 2.5m. The channel slope is 0.0028. Find the water velocity and the flow rate. (n = 0.013)

Answer

By Manning equation,

$$V = \frac{1}{n} * R^{2/3} * S^{1/2}$$

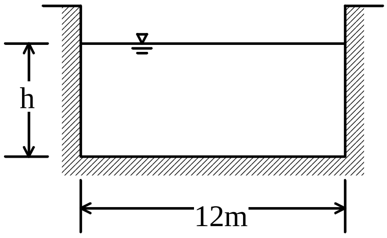
with n = 0.013
S = 0.0028

A = 12* 2.5 m^2 = 30 m^2
P = 12 + 2*2.5 m = 17 m
∴ R = A/P
= 30 / 17 m = 1.765 m

hence V $= \frac{1}{0.013} * (1.765)^{2/3} * (0.0028)^{1/2}$
= 5.945 m/s

Discharge, Q = A*V
= 30*5.945 m^3/s
= 178.3 m^3/s

2. Water flows in a rectangular, concrete, open channel that is 12 m wide. The channel slope is 0.0028. If the velocity of the flow is 6 m/s, find the depth of the flow. (n = 0.013)



Answer

By Manning equation,

$$V = \frac{1}{n} * R^{2/3} * S^{1/2}$$

with V = 6 m/s
n = 0.013
S = 0.0028

$A = 12 * h \ m^2$

$P = 12 + 2*h$ m

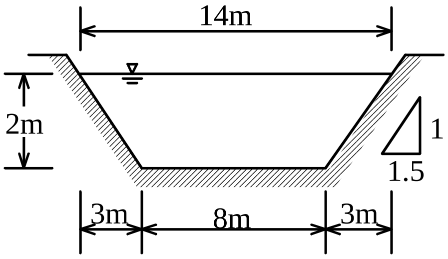
$\therefore \quad R = \frac{A}{P} = \frac{12*h}{12+2*h} = \frac{6*h}{6+h}$

$\frac{6*h}{6+h} = 1.790$

$h = 2.551$ m

Depth of the flow = 2.551m

3. A trapezoidal channel with side slopes of 2/3, a depth of 2 m, a bottom width of 8 m and a channel slope of 0.0009 has a discharge of 56 m^3/s. Find the Manning's n.



Answer

A = (14+8)*2/2 m^2
= 22 m^2

P = 8 + 2* $\sqrt{2^2 + 3^2}$ m
= 15.211 m

A/P = 22 / 15.211 m
= 1.446 m

By Manning equation,

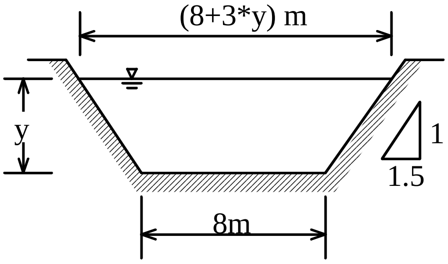
$$Q = \frac{A}{n} * R^{2/3} * S^{1/2}$$

Q = 56 m^3/s, S = 0.0009

$$56 = \frac{22}{n} * (1.446)^{2/3} * (0.0009)^{1/2}$$

n = 0.01507

4. Determine the depth in a trapezoidal channel with side slopes of 1 to 1.5, a bottom width of 8 m and a channel slope of 0.0009. The discharge is 56 m^3/s and n = 0.017.



Answer

A = (8+8+3*y)*y/2 m^2
= (8+1.5*y)*y m^2

P = 8 + 2*y* $\sqrt{1^2 + 1.5^2}$ m
= 8+3.6056*y m

R = A/P
= (8+1.5*y)*y / 8+3.6056*y

By Manning equation,

$$Q = \frac{A}{n} * R^{2/3} * S^{1/2}$$

Q = 56 m^3/s, S = 0.0009

$$\therefore \quad 56 = \frac{(8+1.5*y)*y}{0.017} * \left[\frac{(8+1.5*y)*y}{8+3.6056*y}\right]^{2/3} * (0.0009)^{1/2}$$

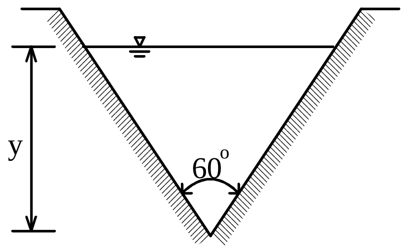
or

$$\frac{[(8+1.5*y)*y]^{5/3}}{[8+3.6056*y]^{2/3}} = 31.7333$$

$$\frac{[(1+0.1875*y)*y]^{5/3}}{[1+0.4507*y]^{2/3}} - 3.9667 = 0$$

By trial & error, y = 2.137 m.
The depth of the trapezoidal channel is 2.137m.

5. Water flows in the triangular steel channel shown in the figure below. Find the depth of flow if the channel slope is 0.0015 and the discharge is 0.22 m^3/s. (n=0.014)



Answer

$A = 2y\tan 30^\circ * y/2\ m^2$

$\quad = y^2 * \tan 30^\circ\ m^2$

$P = 2y/\cos 30^\circ\ m$

$R = A/P = y^2 * \tan 30^\circ / 2y/\cos 30^\circ\ m$

$\quad = y\sin 30^\circ/2\ m$

By Manning equation,

$$Q = \frac{A}{n} * R^{2/3} * S^{1/2}$$

$Q = 0.22\ m^3/s, \quad S = 0.0015$

$$0.22 = \frac{y^2 \tan 30^\circ}{0.014} * \left(\frac{y\sin 30^\circ}{2}\right)^{2/3} * \sqrt{0.0015}$$

$= y^{8/3} * 0.6338$

or $\quad y = \left(\frac{0.22}{0.6338}\right)^{3/8}\ m$

$= 0.672\ m$

Depth of the channel is 0.672 m.

8.2.2 Optimum Hydraulic Cross-sections (REFERENCE ONLY)

- From Manning equation,

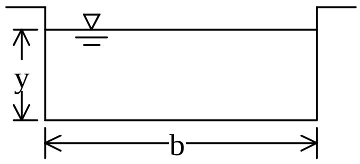
$$Q = \frac{1}{n} * \frac{A^{5/3} * \sqrt{S}}{P^{2/3}}$$

Hence, Q will be **maximum** when P is a minimum.

- For a given cross-sectional area, A of an open channel, the discharge, Q is maximum when the wetted perimeter, P is minimum. Hence if the wetted perimeter, P for a given flow area is minimised, the area, A will give the least expensive channel to be construct.

- This corresponding cross-section is the **optimum hydraulic section** or the **best hydraulic section**.

8.2.2.1 Rectangular section



width = b

depth = y

area, A = by

$$P = b + 2*y$$

$$= \frac{A}{y} + 2y$$

Hence $\frac{dP}{dy} = -\frac{A}{y^2} + 2 = 0$

i.e. $y = \sqrt{\frac{A}{2}}$ or $b = 2y$

Therefore, the optimum rectangular section is