boundary between inelastic and elastic columns, Equations 4.8 and 4.9 give the same value of F_{cr}. This occurs when KL/r is approximately

$$4.71\sqrt{\frac{E}{F_y}}$$

To summarize,

$$\text{When } \frac{KL}{r} \le 4.71\sqrt{\frac{E}{F_y}}, \quad F_{cr} = (0.658^{F_y/F_e})F_y \tag{4.10}$$

$$\text{When } \frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}}, \quad F_{cr} = 0.877F_e \tag{4.11}$$

The AISC Specification provides for separating inelastic and elastic behavior based on either the value of KL/r (as in equations 4.10 and 4.11) or the value of the ratio F_y/F_e. The limiting value of F_y/F_e can be derived as follows. From AISC Equation E3-4,

$$\frac{KL}{r} = \sqrt{\frac{\pi^2 E}{F_e}}$$

For $$\frac{KL}{r} \le 4.71\sqrt{\frac{E}{F_y}},$$

$$\sqrt{\frac{\pi^2 E}{F_e}} \le 4.71\sqrt{\frac{E}{F_y}}$$

$$\frac{F_y}{F_e} \le 2.25$$

The complete AISC Specification for compressive strength is as follows:

$$\text{When } \frac{KL}{r} \le 4.71\sqrt{\frac{E}{F_y}} \quad \text{or} \quad \frac{F_y}{F_e} \le 2.25,$$

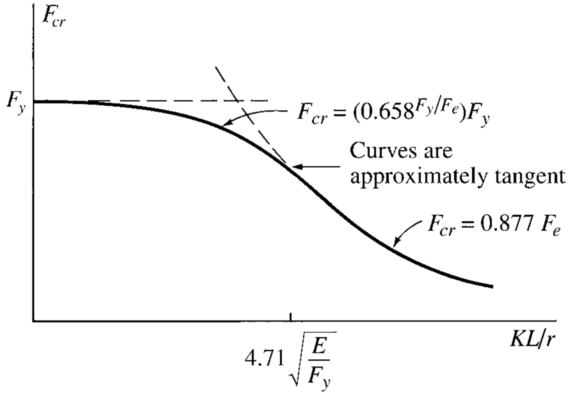
$$F_{cr} = (0.658^{F_y/F_e})F_y \qquad \text{(AISC Equation E3-2)}$$

$$\text{When } \frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}} \quad \text{or} \quad \frac{F_y}{F_e} > 2.25,$$

$$F_{cr} = 0.877F_e \qquad \text{(AISC Equation E3-3)}$$

In this book, we will usually use the limit on KL/r, as expressed in Equations 4.10 and 4.11. These requirements are represented graphically in Figure 4.8.

FIGURE 4.8



AISC Equations E3-2 and E3-3 are a condensed version of five equations that cover five ranges of KL/r (Galambos, 1988). These equations are based on experimental and theoretical studies that account for the effects of residual stresses and an initial out-of-straightness of $L/1500$, where L is the member length. A complete derivation of these equations is given by Tide (2001).

Although AISC does not require an upper limit on the slenderness ratio KL/r, an upper limit of 200 is recommended (see user note in AISC E2). This is a practical upper limit, because compression members that are any more slender will have little strength and will not be economical.

EXAMPLE 4.2

A W14 × 74 of A992 steel has a length of 20 feet and pinned ends. Compute the design compressive strength for LRFD and the allowable compressive strength for ASD.

SOLUTION Slenderness ratio:

$$\text{Maximum } \frac{KL}{r} = \frac{KL}{r_y} = \frac{1.0(20 \times 12)}{2.48} = 96.77 < 200 \quad \text{(OK)}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29,000}{50}} = 113$$

Since 96.77 < 113, use AISC Equation E3-2.

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29,000)}{(96.77)^2} = 30.56 \text{ ksi}$$

$$F_{cr} = 0.658^{(F_y/F_e)} F_y = 0.658^{(50/30.56)}(50) = 25.21 \text{ ksi}$$