

Al-Mustaqbal University

Department of Medical Instrumentation Techniques Engineering



Mathematics

LECTURE 1

Introduction of Integration

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In this course, we'll discuss the definition of integral calculus, indefinite and definite integration, key integration rules and some essential integration methods with some examples.

What is Integral Calculus?

You are probably already familiar with differentiation, which is the process used to calculate the instantaneous rate of change of a function. What is the difference between integration and differentiation? Well, you can think about integration as the reverse operation of differentiation. Together, differentiation and integration make up the essential operations of calculus and are related by the fundamental theorems of calculus.

That is:

Integration is the opposite of differentiation, meaning finding the function if its derivative is known.

$$\text{If } \frac{d}{dx} (F(x)) = f(x) \text{ then } \int f(x)dx = F(x).$$

Where \int is called the *integral sign*. This symbol indicates that we're calculating the anti-derivative function of $f(x)$.

The function $f(x)$ is called the *integrand*, and it's the function we're taking the integral of.

The letters dx in calculus represent the differential dx . The differential dx indicates that we're integrating $f(x)$ with respect to the variable x .

$F(x)$ is the anti-derivative function that gives back $f(x)$ when differentiated.

Types of integration

1. Indefinite integral
2. Definite integral



1. Indefinite integral

$$\int f(x)dx = F(x) + C.$$

The capital letter C represents a constant value called the constant of integration.

Standard Integration Rules

1- $\int a \, dx = ax + C$ for some constant a (Constant Rule)

2- $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$ (Power Rule)

3- $\int kf(x)dx = k \int f(x)dx$ for some constant (Constant Multiplier Rule)

4- $\int [f(x) - g(x)]dx = \int f(x)dx - \int g(x)dx$ (Difference Rule)

5- $\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$ (Sum Rule)

Examples

[1] $\int 6 \, dx = 6x + C$



$$[2] \int -3 \, dx = -3x + C$$

$$[3] \int \frac{1}{4} \, dy = \frac{1}{4}y + C$$

$$[5] \int x^4 \, dx = \frac{x^{4+1}}{4+1} + C = \frac{x^5}{5} + C$$

$$[6] \int x^{-3} \, dx = \frac{x^{-2}}{-2} + C = -\frac{1}{2x^2} + C$$

$$[7] \int 4x^3 \, dx = \frac{4x^4}{4} + C = x^4 + C$$

$$[8] \int -6x^{-5} \, dx = \frac{-6x^{-4}}{-4} + C = \frac{3}{2x^4} + C$$

$$[9] \int \sqrt{x} \, dx = \int x^{\frac{1}{2}} \, dx$$

$$= \frac{2}{3}x^{\frac{3}{2}} + C$$

$$[10] \int (3x^2 + 5) \, dx = \frac{3x^3}{3} + 5x + C = x^3 + 5x + C$$

$$[11] \int (6x^2 - 5x + 3) \, dx = \frac{6x^3}{3} - \frac{5x^2}{2} + 3x + C$$
$$= 2x^3 - \frac{5}{2}x^2 + 3x + C$$

$$[12] \int (x^4 - 2x^3 + x^2) \, dx = \frac{x^5}{5} - \frac{2x^4}{4} + \frac{x^3}{3} + C = \frac{x^5}{5} - \frac{2x^4}{4} + \frac{x^3}{3} + C$$



H.W

$$[1] \int 3x^2 + 5x + 7 \, dx$$

$$[2] \int \left(\frac{1}{x^2} + x \right) \, dx$$

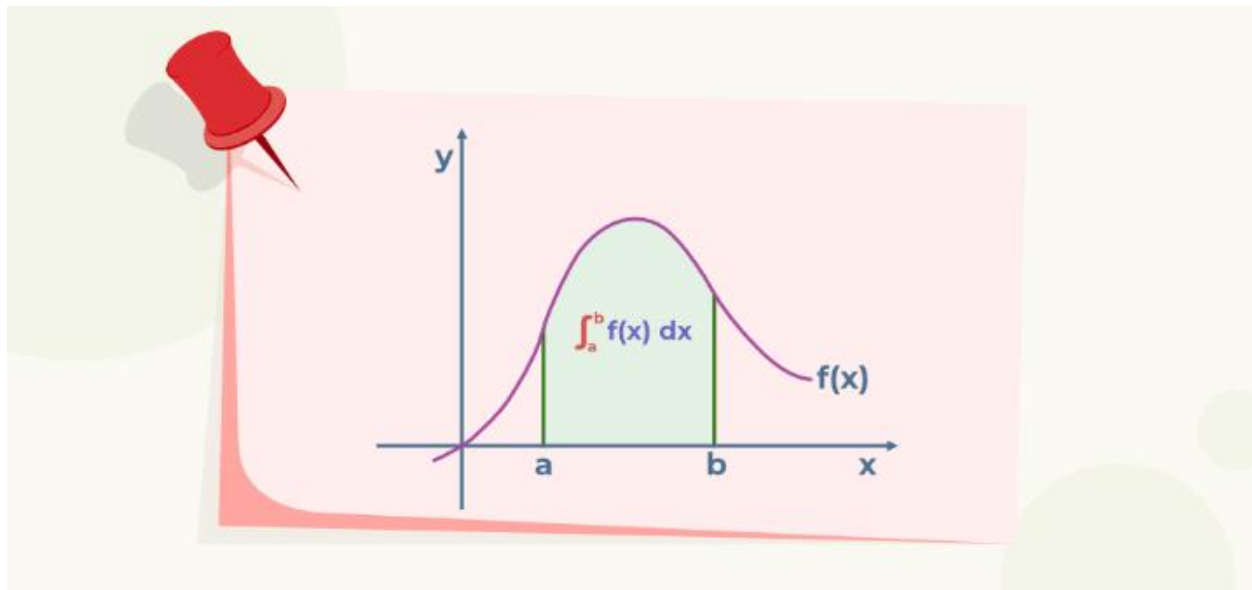
$$[3] \int \sqrt[3]{x^2} \, dx$$

$$[5] \int x^{-\frac{3}{4}} \, dx$$

$$[6] \int 8\sqrt{x} \, dx$$

2- Definite integral

These types of integrals have different outputs. The definite integral outputs a unique number that represents the area enclosed by a function's curve and the x-axis over some interval $[a,b]$. The indefinite integral outputs a function's antiderivative function, accompanied by the constant of integration C .



Definite Integral



$$\int_a^b f(x) dx = F(x) \Big|_a^b = F(b) - F(a)$$

The letters **a** and **b** are called integral bounds or limits. The letter **a** represents the lower bound, while **b** represents the upper bound.

Examples

$$[1] \int_1^3 x^3 dx = \left[\frac{x^4}{4} \right]_1^3 = \frac{(3)^4}{4} - \frac{(1)^4}{4} = \frac{81}{4} - \frac{1}{4} = \frac{80}{4} = 20$$

$$[2] \int_{-1}^0 (3x^2 + 2x + 1) dx = \left[\frac{3x^3}{3} + \frac{2x^2}{2} + x \right]_{-1}^0 = [x^3 + x^2 + x]_{-1}^0 \\ = (0) - (-1 + 1 - 1) = -(-1) = 1$$

$$[3] \int_1^2 (x^{-2} + 2x + 1) dx = \left[\frac{x^{-1}}{-1} + \frac{2x^2}{2} + x \right]_1^2 = \left[\frac{-1}{x} + x^2 + x \right]_1^2 \\ = \left[\frac{-1}{2} + 4 + 2 \right] - [-1 + 1 + 1] \\ = \frac{-1}{2} + 6 - 1 = \frac{-1}{2} + 5 = \frac{-1 + 10}{2} = \frac{9}{2}$$



$$\begin{aligned} [4] \int_1^3 (x^4 + 4x) dx &= \left[\frac{x^5}{5} + \frac{4x^2}{2} \right]_1^3 = \left[\frac{x^5}{5} + 2x^2 \right]_1^3 \\ &= \left[\frac{(3)^5}{5} + 2(3)^2 \right] - \left[\frac{(1)^5}{5} + 2(1) \right] \\ &= \left[\frac{243}{5} + 18 \right] - \left[\frac{1}{5} + 2 \right] \\ &= \left[\frac{243 + 90}{5} \right] - \left[\frac{1 + 10}{5} \right] = \frac{333}{5} - \frac{11}{5} = \frac{322}{5} \end{aligned}$$

$$\begin{aligned} [5] \int_1^3 \frac{2x^3 - 4x^2 + 5}{x^2} dx &= \int_1^3 \left(\frac{2x^3}{x^2} - \frac{4x^2}{x^2} + \frac{5}{x^2} \right) dx \\ &= \int_1^3 (2x - 4 + 5x^{-2}) dx = \left[\frac{2x^2}{2} - 4x + \frac{5x^{-1}}{-1} \right]_1^3 \\ &= \left[x^2 - 4x - \frac{5}{x} \right]_1^3 \\ &= \left[9 - 12 - \frac{5}{3} \right] - \left[1 - 4 - 5 \right] = \left[-3 - \frac{5}{3} \right] - [-8] \\ &= \frac{-9 - 5}{3} + 8 = \frac{-14}{3} + 8 = \frac{-14 + 24}{3} = \frac{10}{3} \end{aligned}$$



$$[6] \int_1^4 \sqrt{x} + 3x \, dx$$
$$= \int_1^4 (x)^{\frac{1}{2}} + 3x \, dx$$

$$= \left[\frac{x^{3/2}}{3/2} + \frac{3x^2}{2} \right]_1^4 = \left[\frac{4^{3/2}}{3/2} + \frac{3(4^2)}{2} \right] - \left[\frac{1^{3/2}}{3/2} + \frac{3(1^2)}{2} \right]$$

$$= \left[\frac{2}{3} (8) + 24 \right] - \left[\frac{2}{3} (1) - \frac{3}{2} \right]$$

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$$= 27.2$$

Good Luck ..