Ministry of Higher Education and Scientific Research
Almustaqbal University, College of Engineering
And Engineering Technologies
Computer Technology Engineering Department

Two week :

The Mean Values of Current and Voltage

Course Name : Fundamentals of Electricity
Stage : One
Academic Year : 2024

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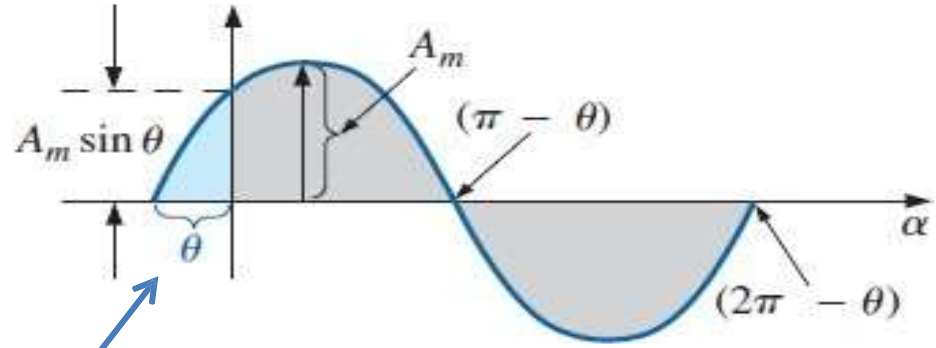
13.6 PHASE RELATIONS



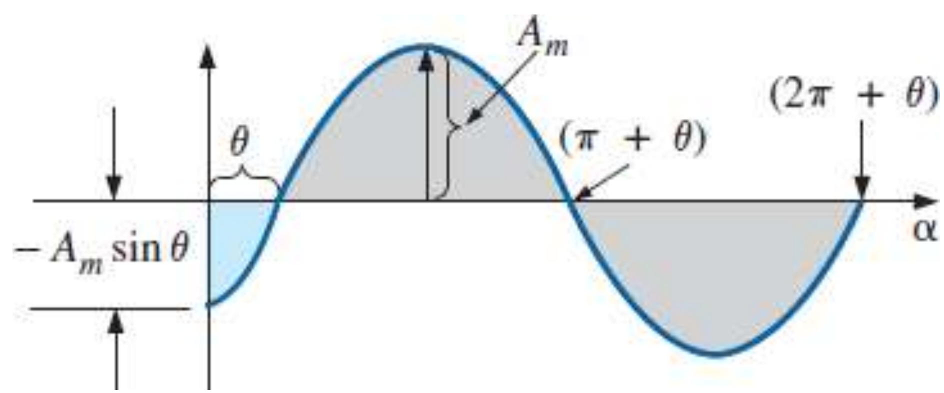
If the waveform is shifted to the right or left of 0°, the expression becomes

$$A_m \sin(\omega t \pm \theta)$$

If the waveform passes through the horizontal axis with a *positive going* (increasing with time) slope *before 0°, as shown in Fig. , the* expression is

$$A_m \sin(\omega t + \theta)$$

If the waveform passes through the horizontal axis with a positive-going slope *after* 0°, as shown in Fig. , the expression is



$$A_m \sin(\omega t - \theta)$$

If the waveform crosses the horizontal axis with a positive-going slope 90°

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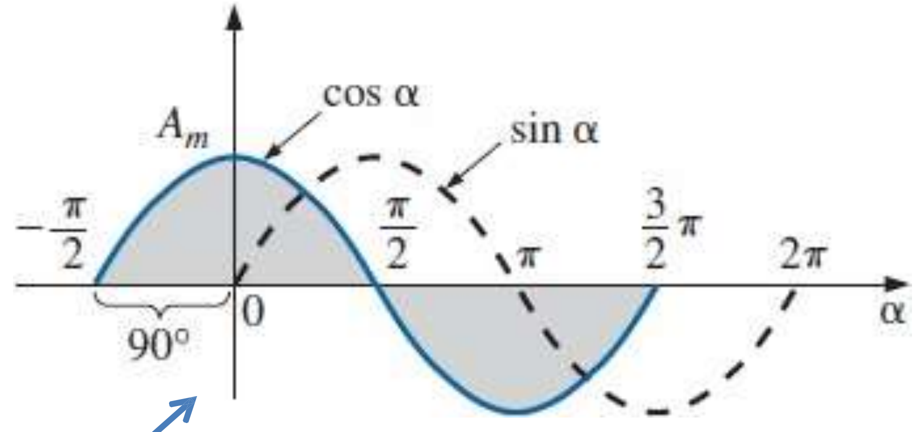
W

$$\sin(\omega t + 90°) = \sin\left(\omega t + \frac{\pi}{2}\right) = \cos \omega t$$

$$\sin \omega t = \cos(\omega t - 90°) = \cos\left(\omega t - \frac{\pi}{2}\right)$$



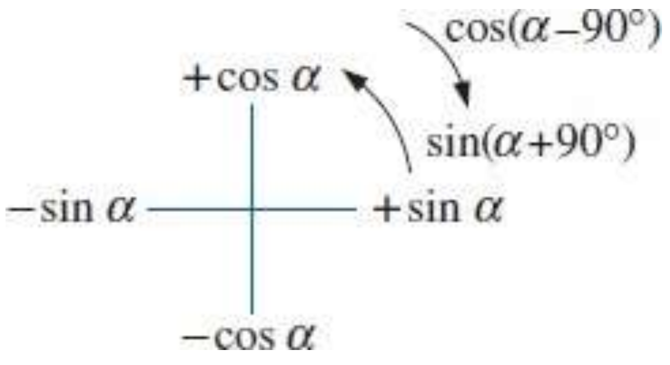
In Fig. , the **cosine curve** is said to *lead the sine* curve by 90°, and the sine curve is said to *lag the cosine curve by 90°.*

The terms **leading and lagging** are used to indicate the relationship between two sinusoidal waveforms of the *same frequency plotted on the* same set of axes.

The geometric relationship between various forms of the sine and cosine functions can be derived from Fig.

For instance, starting at the +sin α position, we find that cos α is an additional 90° in the counterclockwise direction. Therefore, cos α = sin(α + 90°).

For -sin α we must travel 180° in the counterclockwise (or clockwise) direction so that -sin α = sin (α ± 180°), and so on, as listed below:



$$\cos\alpha = \sin(\alpha + 90^\circ)$$
$$\sin\alpha = \cos(\alpha - 90^\circ)$$
$$-\sin\alpha = \sin(\alpha \pm 180^\circ)$$
$$-\cos\alpha = \sin(\alpha + 270^\circ) = \sin(\alpha - 90^\circ)$$
etc.

In addition, note that

$$\sin(-\alpha) = -\sin\alpha$$
$$\cos(-\alpha) = \cos\alpha$$

Since $-\sin\omega t = \sin(\omega t \pm 180^\circ)$

the expression can also be written

$$e = E_m \sin(\omega t \pm 180^\circ)$$

$$e = -E_m \sin\omega t = E_m \sin(\omega t + 180^\circ)$$
$$= E_m \sin(\omega t - 180^\circ)$$

EXAMPLE 13.12 What is the phase relationship between the sinusoidal waveforms of each of the following sets?

a. $v = 10 \sin(\omega t + 30°)$
   $i = 5 \sin(\omega t + 70°)$

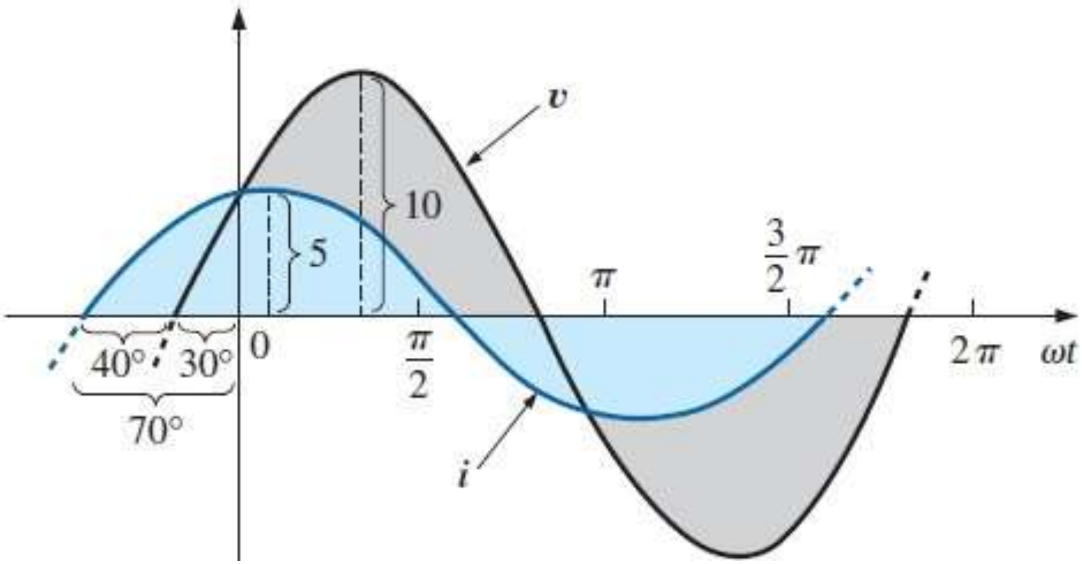
b. $i = 15 \sin(\omega t + 60°)$
   $v = 10 \sin(\omega t - 20°)$

c. $i = 2 \cos(\omega t + 10°)$
   $v = 3 \sin(\omega t - 10°)$

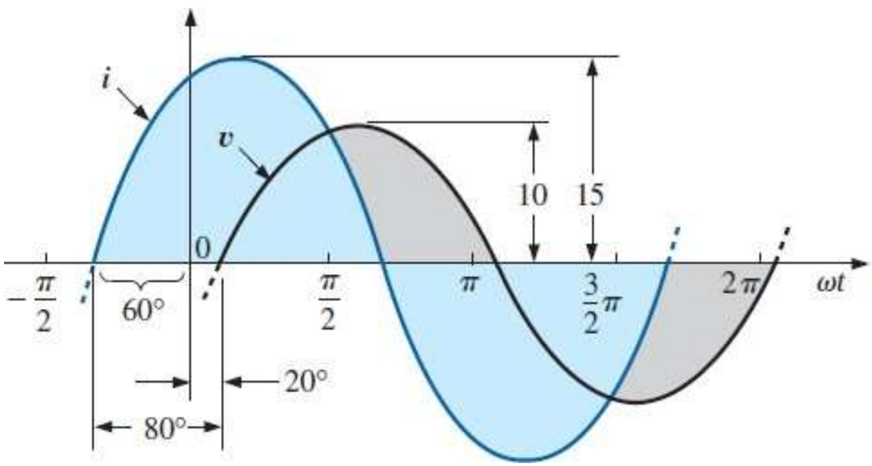
d. $i = -\sin(\omega t + 30°)$
   $v = 2 \sin(\omega t + 10°)$

e. $i = -2 \cos(\omega t - 60°)$
   $v = 3 \sin(\omega t - 150°)$

a. ***i* leads *v* by 40°, or *v* lags *i* by 40°.**

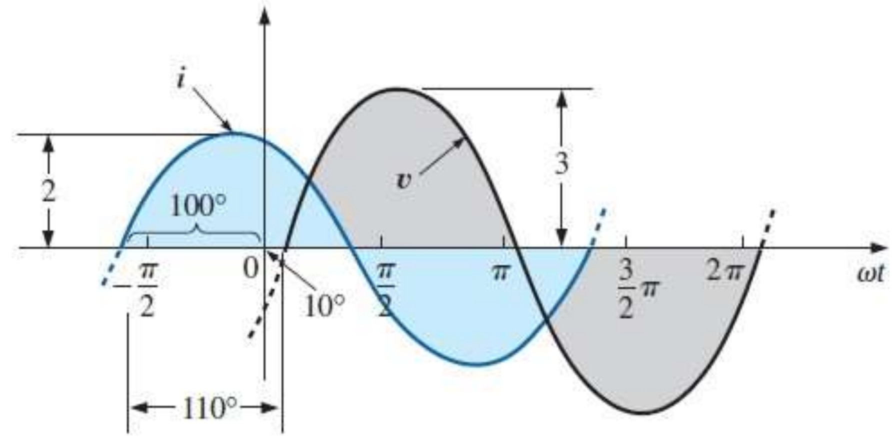


b. ***i* leads *v* by 80º, or *v* lags *i* by 80º.**



c. $i = 2\cos(\omega t + 10°) = 2\sin(\omega t + 10° + 90°)$
$= 2\sin(\omega t + 100°)$

***i* leads *v* by 110º, or *v* lags *i* by 110º.**



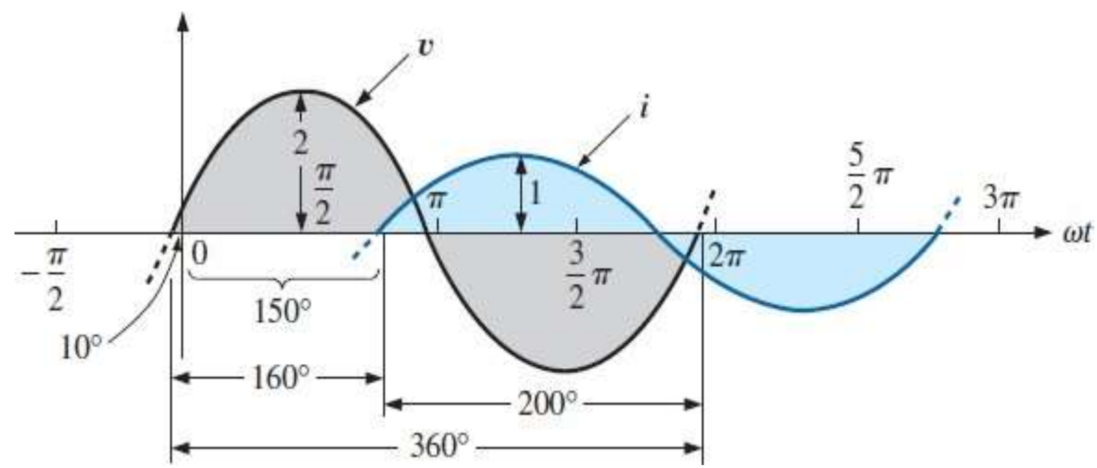
d. $-\sin(\omega t + 30°) = \sin(\omega t + 30° - 180°)$ (Note)
$= \sin(\omega t - 150°)$

***v* leads *i* by 160º, or *i* lags *v* by 160º.**

Or using

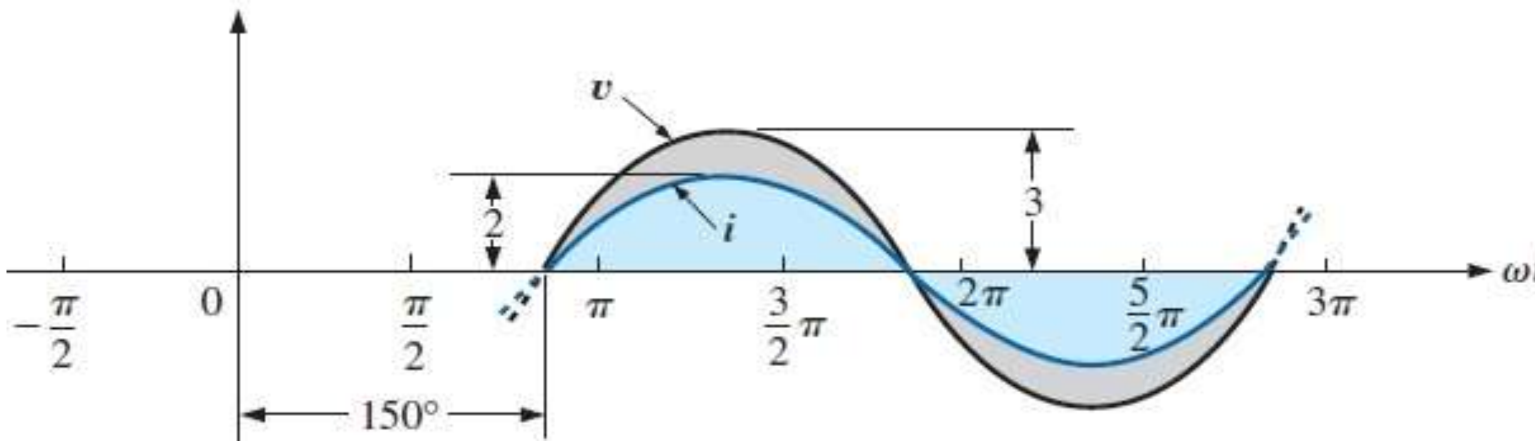
$-\sin(\omega t + 30°) = \sin(\omega t + 30° + 180°)$ (Note)
$= \sin(\omega t + 210°)$

***i* leads *v* by 200º, or *v* lags *i* by 200º.**



By choice

e. $i = -2\cos(\omega t - 60^\circ) = 2\cos(\omega t - 60^\circ - 180^\circ)$
$= 2\cos(\omega t - 240^\circ)$

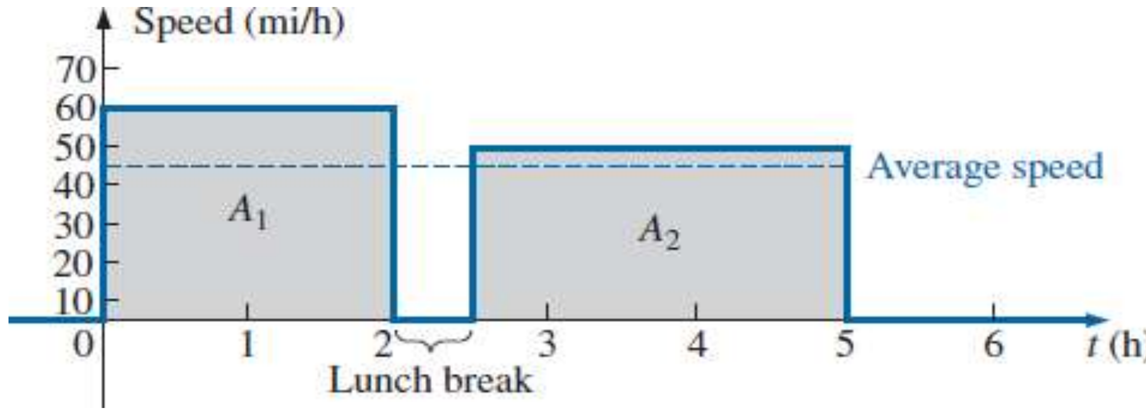


However, $\cos\alpha = \sin(\alpha + 90^\circ)$

so that $2\cos(\omega t - 240^\circ) = 2\sin(\omega t - 240^\circ + 90^\circ)$
$= 2\sin(\omega t - 150^\circ)$

v **and** ***i*** **are in phase.**

13.7 AVERAGE VALUE



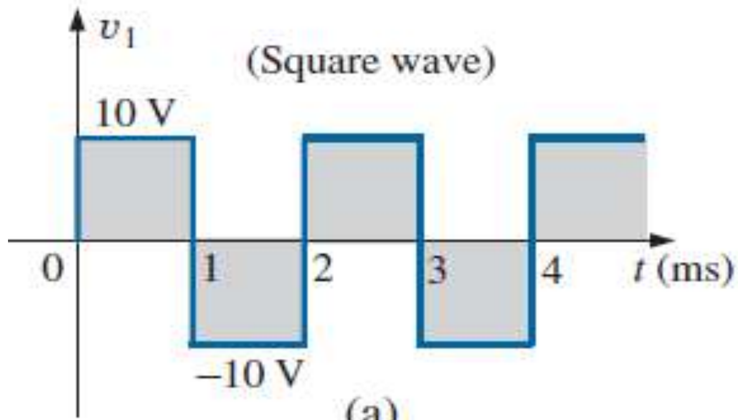
$$\text{Average speed} = \frac{\text{area under curve}}{\text{length of curve}}$$

$$\text{Average speed} = \frac{A_1 + A_2}{5\text{ h}} = \frac{(60\text{ mi/h})(2\text{ h}) + (50\text{ mi/h})(2.5\text{ h})}{5\text{ h}}$$

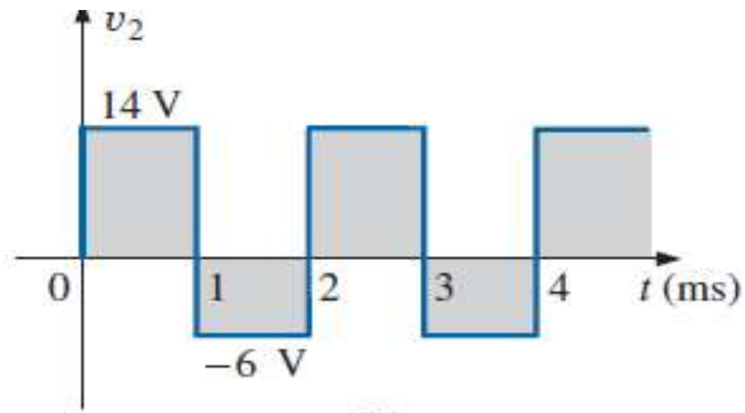
$$= \frac{225}{5}\text{ mi/h} = \mathbf{45\ mi/h}$$

$$G\text{ (average value)} = \frac{\text{algebraic sum of areas}}{\text{length of curve}}$$

EXAMPLE 13.14 Determine the average value of the waveforms in Fig.



(a)



(b)

a. By inspection, the area above the axis equals the area below over one cycle, resulting in an average value of zero volts.

$$G = \frac{(10\text{ V})(1\text{ ms}) - (10\text{ V})(1\text{ ms})}{2\text{ ms}} = \frac{0}{2\text{ ms}} = \mathbf{0\text{ V}}$$

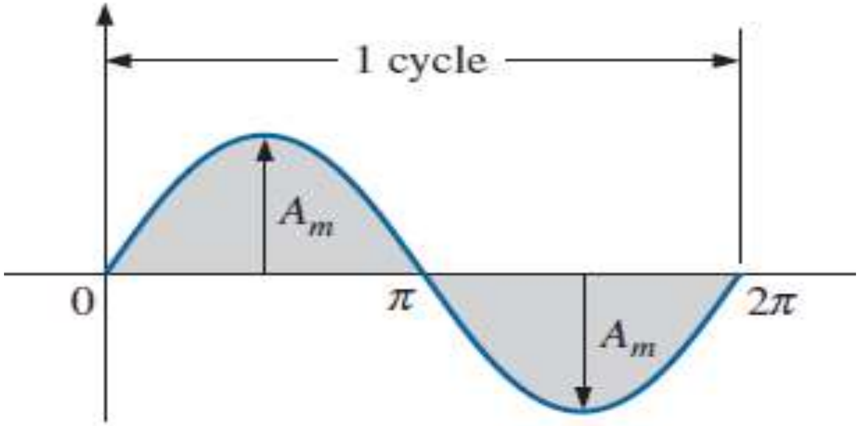
b.

$$G = \frac{(14\text{ V})(1\text{ ms}) - (6\text{ V})(1\text{ ms})}{2\text{ ms}} = \frac{14\text{ V} - 6\text{ V}}{2} = \frac{8\text{ V}}{2} = \mathbf{4\text{ V}}$$

EXAMPLE 13.16 Determine the average value of the sinusoidal waveform in Fig.

The average value of a pure sinusoidal waveform over one full cycle is zero.

$$G = \frac{+2A_m - 2A_m}{2\pi} = 0\ \mathrm{V}$$



13.8 EFFECTIVE or Root Mea

$$P_{ac} = (i_{ac})^2R = (I_m \sin \omega t)^2R = (I_m^2 \sin^2 \omega t)R$$

However,

$$\sin^2 \omega t = \frac{1}{2}(1 - \cos 2\omega t) \qquad \text{(trigonometric identity)}$$

Therefore, $P_{ac} = I_m^2\left[\frac{1}{2}(1 - \cos 2\omega t)\right]R$

$$P_{ac} = \frac{I_m^2R}{2} - \frac{I_m^2R}{2}\cos 2\omega t$$

The average power delivered by the ac source is just the first term, since the average value of a cosine wave is zero

$$P_{av(ac)} = P_{dc}$$

$$\frac{I_m^2R}{2} = I_{dc}^2R$$

$$I_{dc} = \frac{I_m}{\sqrt{2}} = 0.707I_m$$

Calculus format: $$I_{\text{rms}} = \sqrt{\frac{\int_0^T i^2(t)\,dt}{T}}$$

which means: $$I_{\text{rms}} = \sqrt{\frac{\text{area } (i^2(t))}{T}}$$

$$I_{\text{rms}} = \frac{1}{\sqrt{2}} I_m = 0.707 I_m$$
$$E_{\text{rms}} = \frac{1}{\sqrt{2}} E_m = 0.707 E_m$$

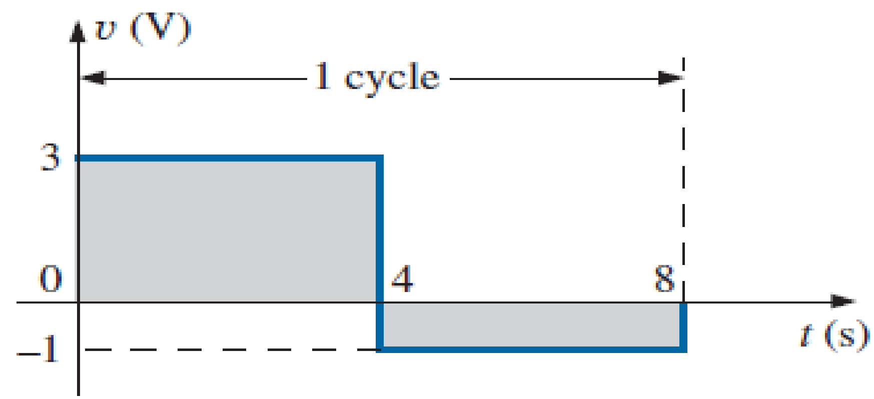
Similarly,

$$I_m = \sqrt{2} I_{\text{rms}} = 1.414 I_{\text{rms}}$$
$$E_m = \sqrt{2} E_{\text{rms}} = 1.414 E_{\text{rms}}$$

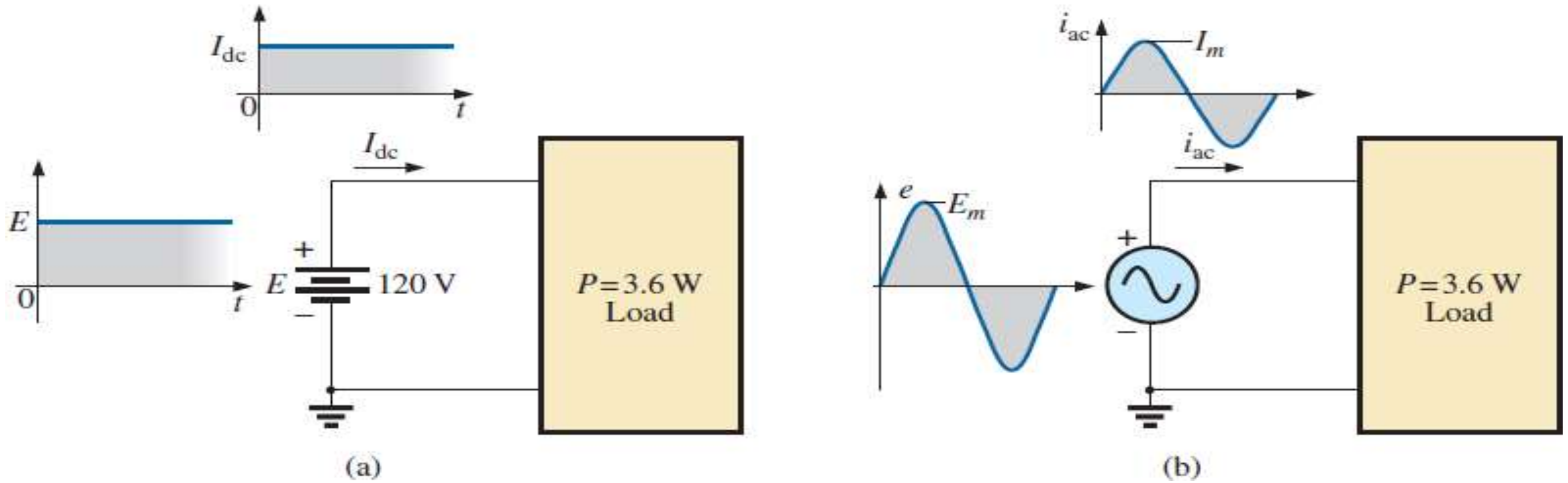
EXAMPLE 13.22 Find the rms value of the waveform in Fig.

v^2

$$V_{\text{rms}} = \sqrt{\frac{(9)(4) + (1)(4)}{8}} = \sqrt{\frac{40}{8}} = \mathbf{2.24\ V}$$



EXAMPLE 13.21 The 120 V dc source in Fig. (a) delivers 3.6 W to the load. Determine the peak value of the applied voltage (Em) and the current (Im) if the ac source [Fig. (b)] is to deliver the same power to the load.



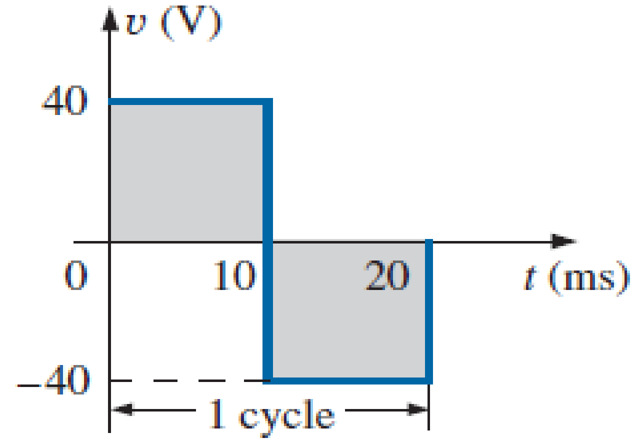
$$P_{dc} = V_{dc} I_{dc}$$

$$I_{dc} = \frac{P_{dc}}{V_{dc}} = \frac{3.6 \text{ W}}{120 \text{ V}} = 30 \text{ mA}$$

$$I_m = \sqrt{2} I_{dc} = (1.414)(30 \text{ mA}) = \mathbf{42.42 \text{ mA}}$$

$$E_m = \sqrt{2} E_{dc} = (1.414)(120 \text{ V}) = \mathbf{169.68 \text{ V}}$$

EXAMPLE 13.24 Determine the average and rms values of the square wave in Fig.

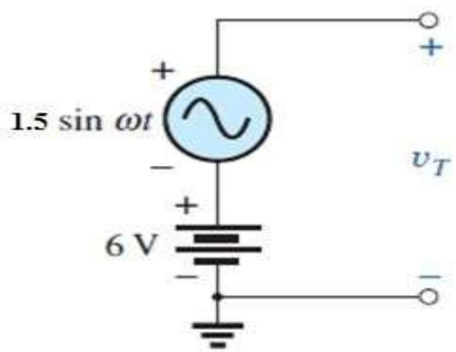


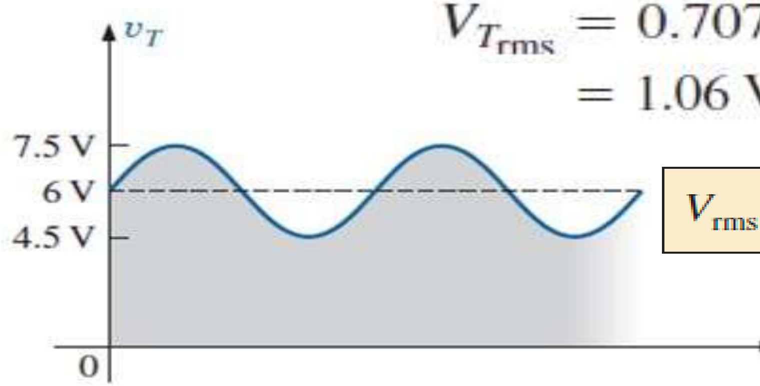
By inspection, the average value is zero.

v^2

$$V_{rms} = \sqrt{\frac{(1600)(10 \times 10^{-3}) + (1600)(10 \times 10^{-3})}{20 \times 10^{-3}}}$$

$$= \sqrt{\frac{(32{,}000 \times 10^{-3})}{20 \times 10^{-3}}} = \sqrt{1600} = \mathbf{40\ V}$$





$$V_{T_{rms}} = 0.7071(1.5\ V) + 6\ V$$
$$= 1.06\ V + 6\ V = 7.06\ V$$

$$V_{rms} = \sqrt{V_{dc}^2 + V_{ac(rms)}^2}$$

Generation and display of a waveform having a dc and an ac component.

However, the rms value is actually determined by

$$V_{\text{rms}} = \sqrt{V_{\text{dc}}^2 + V_{\text{ac(rms)}}^2}$$

which for the waveform in Fig.

$$V_{\text{rms}} = \sqrt{(6\text{ V})^2 + (1.06\text{ V})^2} = \sqrt{37.124}\text{ V} \cong \mathbf{6.1\text{ V}}$$

PROBLEMS

SECTION 13.2 Sinusoidal ac Voltage Characteristics and Definitions: 1, 4

SECTION 13.4 The Sinusoidal Waveform: 10, 11, 12 , 15, 16

SECTION 13.5 General Format for the Sinusoidal Voltage or Current: 17, 18, 20, 23

SECTION 13.6 Phase Relations: 25, 27, 30, 34

SECTION 13.7 Average Value: 37, 39, 40

SECTION 13.8 Effective (rms) Values: 42, 43, 44, 47

THANK YOU