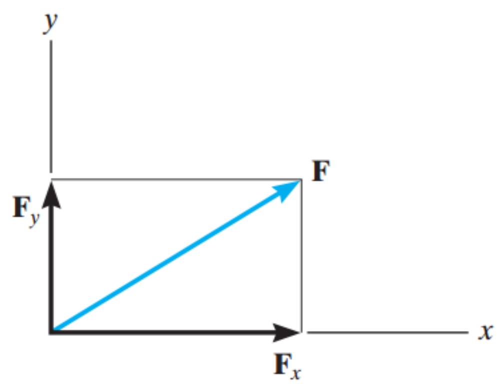
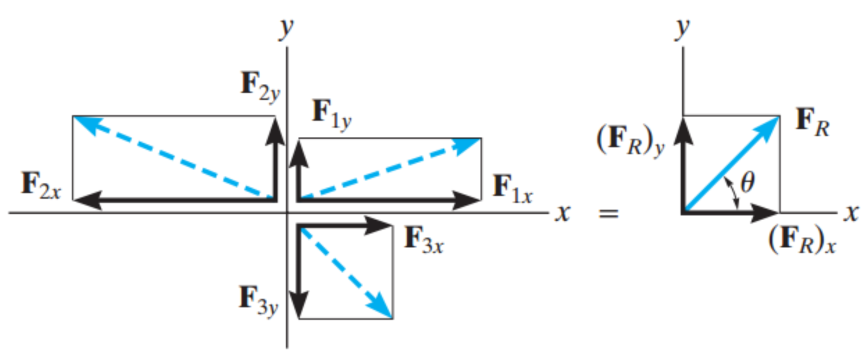
2.2. Rectangular Components : Two Dimensions

Vectors F_x and F_y are rectangular components of F.



The resultant force is determined from the algebraic sum of its components.



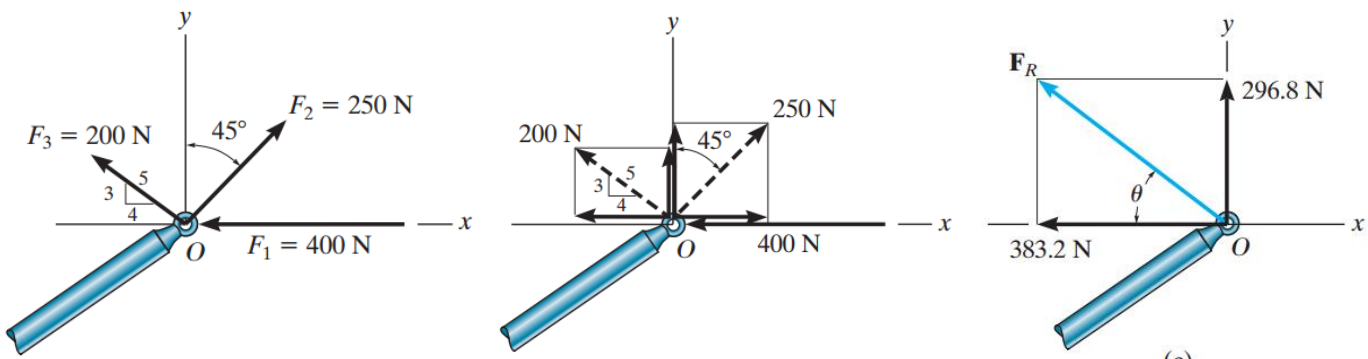
$$(F_R)_x = \Sigma F_x$$

$$(F_R)_y = \Sigma F_y$$

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

$$\theta = \tan^{-1}\left|\frac{(F_R)_y}{(F_R)_x}\right|$$

Example: The end of the boom O in Figure (a) below is subjected to three concurrent and coplanar forces. Determine the ***magnitude*** and ***direction*** of the resultant force.



(a) (b) (c)

Solution:

Each force is resolved into its x and y components, Figure (b), Summing the x-components and y-components:

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = -400\text{ N} + 250 \sin 45^\circ\text{ N} - 200\left(\tfrac{4}{5}\right)\text{ N}$$
$$= -383.2\text{ N} = 383.2\text{ N} \leftarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 250 \cos 45^\circ\text{ N} + 200\left(\tfrac{3}{5}\right)\text{ N}$$
$$= 296.8\text{ N}\uparrow$$

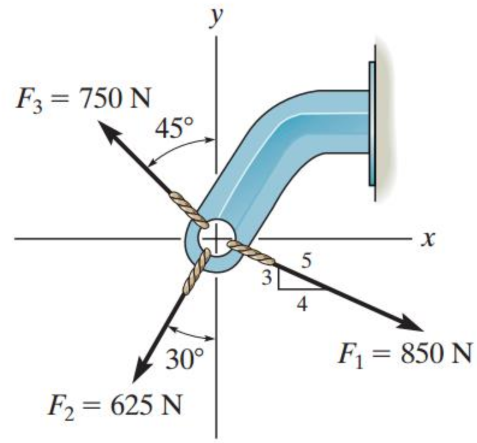
The resultant force, shown in Figure c, has a magnitude of:

$$F_R = \sqrt{(-383.2\text{ N})^2 + (296.8\text{ N})^2}$$
$$= 485\text{ N}$$

The direction angle θ is:

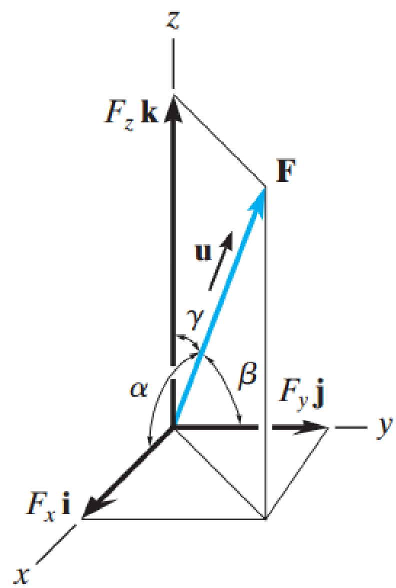
$$\theta = \tan^{-1}\left(\frac{296.8}{383.2}\right) = 37.8^\circ$$

HW: Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



2.3.Rectangular Components: Three Dimensions

The magnitude of F is determined from the positive square root of the sum of the squares of its components.



$$F = \sqrt{{F_x}^2 + {F_y}^2 + {F_z}^2}$$

To determine α, β, and γ:

$$\cos\alpha = \frac{F_x}{F}, \qquad \cos\beta = \frac{F_y}{F}, \qquad \cos\gamma = \frac{F_z}{F}$$

If only *two* of the coordinate angles are known, the third angle can be found using this equation:

$$\cos\alpha + \cos\beta + \cos\gamma = 1$$

Example: Two forces act on the hook shown in Fig. below. Specify the magnitude of F_2 and its coordinate direction angles so that the resultant force F_R acts along the positive y-axis and has a magnitude of 800 N.

Solution:

$\sum Fx = 0$ (Because F_R acts along the y-axis)

$0 = 300 \cos 45^0 + F_{2x}$

$F_{2x} = -212.1$ N

$\sum Fy = 800\ N$ (Because F_R acts along the y-axis)

$800 = 300 \cos 60^0 + F_{2y}$

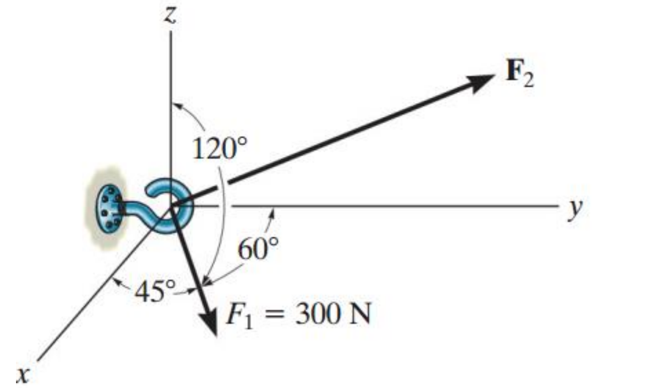
$F_{2y} = 650$ N

$\sum Fz = 0$ (Because F_R acts along the y-axis)

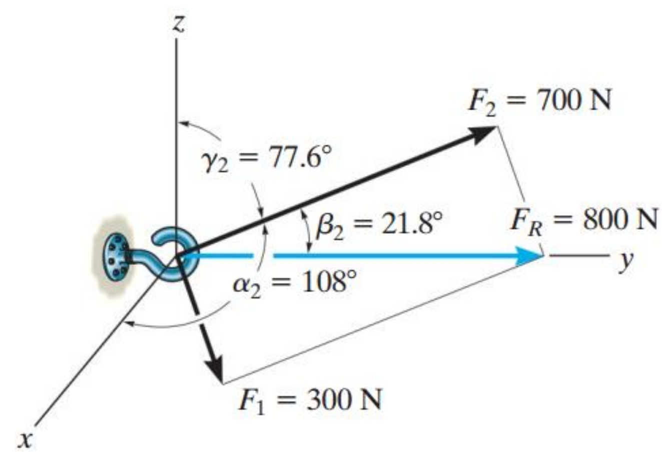
$0 = 300 \cos 120^0 + F_{2z}$

$0 = -150 + F_{2z}$

$F_{2z} = -150$ N



(a)



(b)

$$F_2 = \sqrt{{F_{2x}}^2 + {F_{2y}}^2 + {F_{2z}}^2}$$

$$F_2 = \sqrt{(-212.1)^2 + (650)^2 + (150)^2}$$

$$F_2 = 700\ N$$

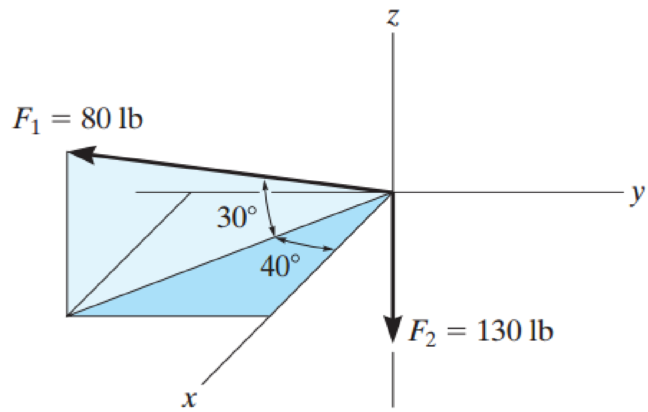
$\cos \alpha_2 = \frac{F_{2x}}{F} = \frac{-212.1}{700};$ $\alpha_2 = 108^0$

$\cos \beta_2 = \frac{F_{2y}}{F} = \frac{650}{700};$ $\beta_2 = 21.8^0$

$\cos \gamma_2 = \frac{F_{2z}}{F} = \frac{150}{700};$ $\gamma_2 = 77.6^0$

HW: Determine the magnitude and coordinate direction angles of the resultant force and sketch this vector on the coordinate system.

Solution:



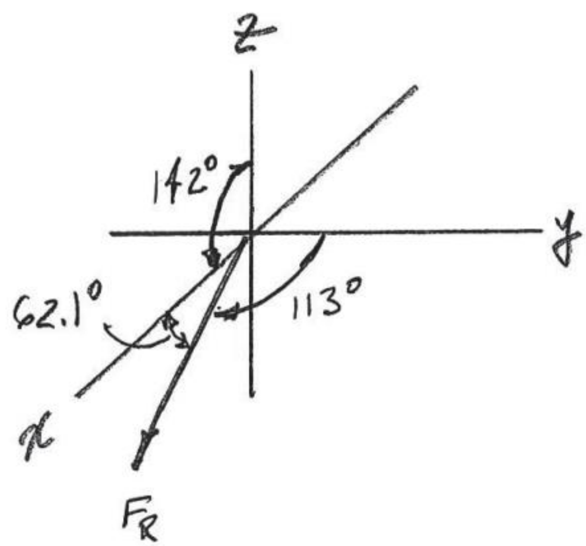
$F_{1x} = 80\ \cos 30\ \cos 40 = 53.1\ ^{Ib} = F_x$

$F_{1y} = 80\ \cos 30\ \sin 40 = \ (-)\ 44.5\ ^{Ib} = \ F_y$

$F_{1z} = 80\ \sin 30 = \ 40\ ^{Ib}$

$F_z = 40 - 130 = -90\ ^{Ib}$

$F_R = \sqrt{53.1\ ^2 + 44.5^2 + (-90)^2} = \ 113.6^{Ib}$



$\cos\alpha = \frac{F_x}{F_R} = \frac{53.1}{113.6};$ $\alpha = 62.1^0$

$\cos\beta = \frac{F_y}{F_R} \ = \frac{-44.5}{113.6};$ $\beta = 113^0$

$\cos\gamma = \frac{F_z}{F_R} \ = \frac{-90}{113.6};$ $\gamma = 142^0$