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Chapter (3)

Measures of Location

When raw data is classified in to a frequency distribution table and presented graphically, the major features of the sample become apparent. However, to make quantitative decisions, further condensation in to a number of statistical parameters is needed.

Measures of location are statistical parameters, giving an estimate of the data centre, being typical of all measurement.

Mode : Is the measurement that occurs with the greatest frequency.

e. g. for sample :

14 , 19 , 16 , 21 , 19 , 24 , 18 , 19

Mode = 19

For sample : 6 , 7 , 7 , 3 , 8 , 3 , 9 , 5

Mode = 3 , 7

(bimodal)

For grouped data, the mode corresponds to the top of the frequency curve.

$$mode = L_m + \frac{\Delta L}{\Delta L + \Delta H} C_m$$

Where:

L_m is lower boundary of modal class

$\Delta L = f_m - f_{lower\ class}$

$\Delta H = f_m - f_{higher\ class}$

C_m = width of modal class

If there are two or more classes having the same highest frequency the formula to be used is:-

Mode = 3 (median) – 2 (mean)

e. g. for electric bulbs sample :

	Class limit	Class bound.	Class mark	Freq.	
1.	663-675	662.5-675.5	669	4	4
2.	676-688	675.5-688.5	682	10	14
3.	689-701	688.5-701.5	695	15	29
4.	702-714	701.5-714.5	708	11	40
5.	715-727	714.5-727.5	721	6	46
6.	728-740	727.5-740.5	734	4	50
				$\sum f = 50$	

$$mode = 688.5 + \frac{15 - 10}{(15 - 10) + (15 - 11)} (13) = 695.7$$

Median : Is the middle measurement of an ordered array (odd). Or the arithmetic mean of the two middle values (even).

e. g. for sample : 3 , 4 , 4 , 5 , 6 , 8 , 8 , 10 , 11

median = 6

for sample : 5 , 5 , 7 , 9 , 11 , 12 , 15 , 18

median = 10

* For grouped data, the median line halves the area under the frequency curve.

$$median = L_m + \frac{\frac{N}{2} - f_{CL}}{f_m} C_m$$

Where :

L_m is lower boundary of median class

N is sample size

F_{CL} is cumulative frequency of lower class

f_m is frequency of median class

C_m is width of median class

e. g. for electric bulbs sample :

3rd class is median class, since $f_c = 29 > \frac{N}{2}$

$$\text{median} = 688.5 + \frac{\frac{50}{2} - 14}{15} (13) = 689.0$$

Arithmetic Mean: is the sum of measurements divided by sample size.

$$\bar{x} = \frac{\sum x_i}{N}$$

For grouped data :

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

e. g. for electric bulbs sample:

Xi = class mark, $\sum$ fi = N

Class Mark (xi)	fi	fi xi
669	4	2676
682	10	6820
695	15	10425
708	11	7788
721	6	4326
734	4	2936
		$\sum$ fi xi = 34971

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

$$= \frac{34971}{50} = 699.42$$

Other Mean Measures :

** Geometric Mean* $G = (\pi x_i)^{\frac{1}{N}}$, $\log G = \frac{\sum f_i \log X_i}{N}$

** Harmonic Mean* $H = \frac{N}{\sum \frac{1}{x_i}}$, $H = \frac{N}{\sum \frac{f_i}{x_i}}$

** Root Mean Square*

$$RMS = \sqrt{\frac{\sum x_i^2}{N}} \quad , \quad RMs = \sqrt{\frac{\sum f_i x_i^2}{N}}$$

For a sample of positive measurements,

$H \le G \le \bar{x} \le RMS$

Other Mean Measures

1- Geometric Mean

Log G = $\frac{\sum \text{fi log xi}}{N}$

xi	fi	fi log xi
669	4	11.3017
682	10	28.3378
695	15	42.6297
708	11	31.3503
721	6	17.1476
734	4	11.4627
		∑ fi log xi = 142.2298

Log G = $\frac{142.2298}{50}$ = 2.84459

G = 699.2

2- Harmonic Mean

$$H = \frac{N}{\sum \frac{fi}{xi}}$$

xi	fi	$\frac{fi}{xi}$
669	4	0.005979
682	10	0.0146627
695	15	0.021582
708	11	0.155367
721	6	0.00832177
734	4	0.005449
		$\sum \frac{fi}{xi} = 0.071532$

$$H = \frac{N}{\sum \frac{fi}{xi}} = \frac{50}{0.071532} = 699$$

3- Root Mean Square

$$R.\ M.\ S = \sqrt{\frac{\sum fi\ xi^2}{N}}$$

xi	fi	xi^2	$fi\ xi^2$
669	4	447561	1790244
682	10	465124	4651240
695	15	483025	7245375
708	11	501264	5513904
721	6	519841	3119046
734	4	538756	2155024
			$\sum fi\ xi^2$=24474833

R. M. S = $\sqrt{\frac{24474833}{50}}$

R. M. S = 699.6

$H \leq G \leq \bar{x} \leq RMS$

699 ≤ 699.2 ≤ 699.4 ≤ 699.6

Properties of the Arithmetic Mean :

1. The sum of deviations of the data from their arithmetic mean is zero.

$\sum(x_i - \bar{x}) = 0$ (prove)

2. For several samples, the combined mean is given by:

$$\bar{x} = \frac{N_1\overline{x_1} + N_2\overline{x_2} + \ldots}{N_1 + N_2 + \ldots}$$

3. If the deviations (d_i) from any value (A) are available, then :

$\bar{x} = A + \frac{\sum d_i}{N}$ $\quad$ *where* $d_i = x_i - A$ (prove)

Or $\bar{x} = A + \frac{\sum f_i d_i}{N}$ (grouped data)

– Properties of the Arithmetic Mean :-

A – Properties of $\bar{x}$

1. the sum of deviations of the data from their arithmetic mean is zero

$\sum f_i (x_i - \bar{x}) = 0$ Prove that for group data

الحل/

التوزيع

$\sum f_i (x_i - \bar{x}) = 0 \quad \ldots (1)$

where :-

x_i = متغير , $\bar{x}$ = ثابت , f_i = متغير

$\sum f_i x_i - \bar{x} \sum f_i = 0 \quad \ldots (2)$

(* الثوابت تخرج خارج الـ Summation وهذا من خواص الجمع

$\sum a x_i = a \sum x_i$)

$\because \bar{x} = \frac{\sum f_i x_i}{\sum f_i}$ من تعريف Arithmetic

وبالتعويض بالمعادلة (2)

$\therefore \sum f_i x_i - \frac{\sum f_i x_i}{\sum f_i} * \sum f_i = 0$

$\therefore \sum f_i x_i - \sum f_i x_i = 0$

2- For several Samples, the combind mean is given by:-

$$\bar{X} = \frac{N_1\bar{X}_1 + N_2\bar{X}_2 + N_3\bar{X}_3 \ldots}{N_1 + N_2 + N_3 \ldots}$$

اشتقاق الخاصيه الذاتيه

where data (1) → $\bar{X}_1$, data (2) → $\bar{X}_2$

Sample	N_1	$\bar{X}_1$
Sample	N_2	$\bar{X}_2$
Sample	N_3	$\bar{X}_3$

} combined $\bar{X}$

if $N_1 = N_2 = N_3$

$$\therefore \bar{X} = \frac{N_1\bar{X}_1 + N_1\bar{X}_2 + N_1\bar{X}_3}{N_1 + N_1 + N_1}$$

$$\therefore \bar{X} = \frac{N_1(\bar{X}_1 + \bar{X}_2 + \bar{X}_3)}{2N_1}$$

$$\boxed{\bar{X} = \frac{\bar{X}_1 + \bar{X}_2}{2}}$$

3- if the deviations (d_i) from any value (A) are available, then:

if $d_i = x_i - A$ ---① A constant

Prove That

$$\bar{X} = A + \frac{\sum f_i d_i}{N}$$ for group data

sol/ $d_i = x_i - A$

from equ①

$x_i = d_i + A$

حسب قانون Arithmetic

$$\bar{X} = \frac{\sum f_i x_i}{\sum f_i}$$

$$\therefore \bar{X} = \frac{\sum f_i (d_i + A)}{\sum f_i}$$

$$\bar{X} = \frac{\sum f_i d_i + A \sum f_i}{\sum f_i}$$

$$\bar{X} = \frac{\sum f_i d_i}{\sum f_i} + \frac{A \sum f_i}{\sum f_i}$$

$$\therefore \bar{X} = A + \frac{\sum f_i d_i}{\sum f}$$

$\because \sum f_i = N$

$$\boxed{\bar{X} = A + \frac{\sum f_i d_i}{N}}$$