# 3. Chapter Three: Equilibrium for a Rigid Body

The objectives of this chapter are:

- ✓ To develop the equations of equilibrium for a rigid body.
- ✓ To introduce the concept of the free-body diagram for a rigid body.
- ✓ To show how to solve rigid-body equilibrium problems using the equations of equilibrium.

## 3.1. Conditions for Rigid-Body Equilibrium

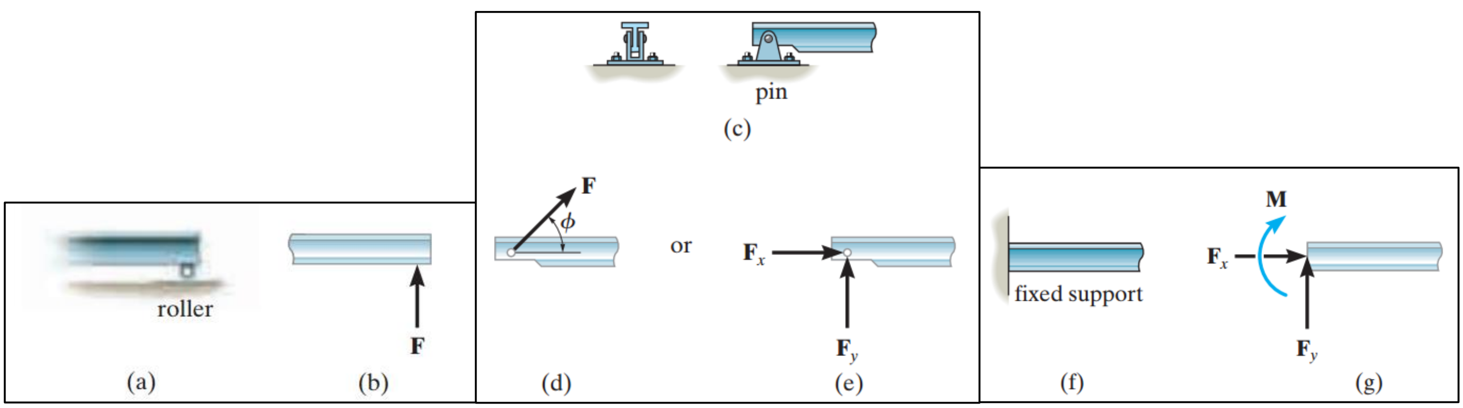
The body is said to be in ***equilibrium*** when resultant force and couple moment are both equal to zero. Mathematically, the equilibrium of a body is expressed as:

$$\sum Fx = 0$$

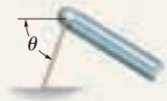
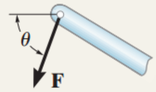
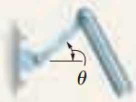
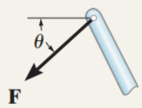
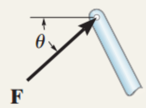
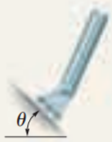
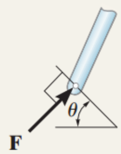
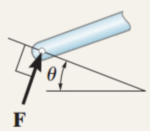
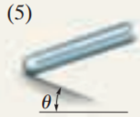
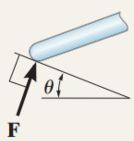
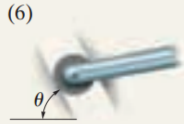
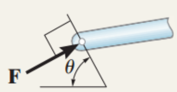
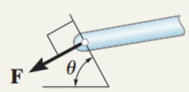
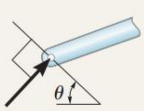
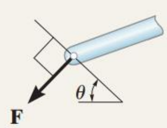
$$\sum Fy = 0$$

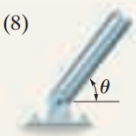
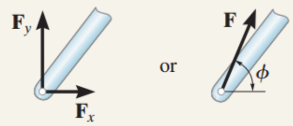
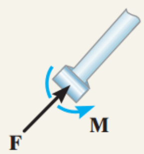
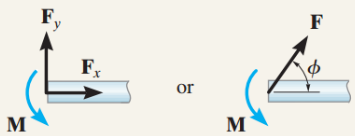
$$\sum M = 0$$

**<u>Main Support Reactions</u>**




Roller Pin (hinge) Fixed

| Types of Connection | Reaction | Number of Unknowns |
|---|---|---|
| (1)  cable |  | One unknown. The reaction is a tension force which acts away from the member in the direction of the cable. |
| (2)  weightless link |  or  | One unknown. The reaction is a force which acts along the axis of the link. |
| (3)  roller |  | One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact. |
| (4)  rocker |  | One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact. |
| (5)  smooth contacting surface |  | One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact. |
| (6)  roller or pin in confined smooth slot |  or  | One unknown. The reaction is a force which acts perpendicular to the slot. |
| (7)  member pin connected to collar on smooth rod |  or  | One unknown. The reaction is a force which acts perpendicular to the rod. |

| Types of Connection | Reaction | Number of Unknowns |
|---|---|---|
| (8)   smooth pin or hinge |   | Two unknowns. The reactions are two components of force, or the magnitude and direction $\phi$ of the resultant force. Note that $\phi$ and $\theta$ are not necessarily equal [usually not, unless the rod shown is a link as in (2)]. |
| (9)  member fixed connected to collar on smooth rod |   | Two unknowns. The reactions are the couple moment and the force which acts perpendicular to the rod. |
| (10)  fixed support |   | Three unknowns. The reactions are the couple moment and the two force components, or the couple moment and the magnitude and direction $\phi$ of the resultant force. |

## 3.2.Free-Body Diagrams

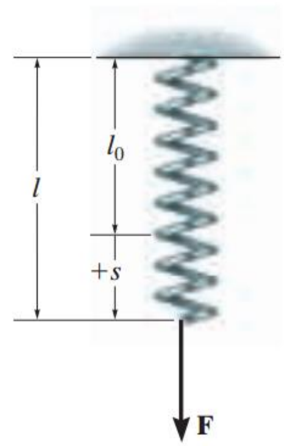
A ***free-body diagram*** is a sketch of the outlined shape of the body, which represents it as being isolated or "free" from its surroundings, i.e., a "free body." On this sketch it is necessary to show all the forces and couple moments that the surroundings exert on the body so that these effects can be accounted for when the equations of equilibrium are applied.

To construct a free-body diagram for a rigid body or any group of bodies considered as a single system, the following steps should be performed:

- ✓ **Draw Outlined Shape.**
- ✓ **Show All Forces and Couple Moments.**
- ✓ **Identify Each Loading and Give Dimensions.**

**Note:** the weight **W** of the body locates at the **center of gravity**.

**Springs:**



(a)

If a linearly elastic spring (or cord) of undeformed length $l_0$ is used to support a particle, the length of the spring will change in direct proportion to the force **F** acting on it, Fig. a.

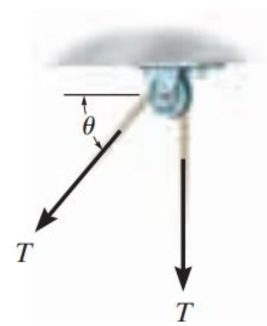
A characteristic that defines the "elasticity" of a spring is the spring constant or stiffness **k**. The magnitude of force exerted on a linearly elastic spring which has a stiffness k and is deformed (elongated or compressed) a distance $s = l - l_0$, measured from its unloaded position, is:

$$F = ks$$

For example, if the spring in **Fig. a** has an unstretched length of 0.8 m and a stiffness **k = 500 N/m** and it is stretched to a length of 1 m, so that $s = l - l_0 = 1\ m - 0.8\ m = 0.2\ m$, then a force:

**F = ks =500 N /m (0.2 m) = 100 N**.

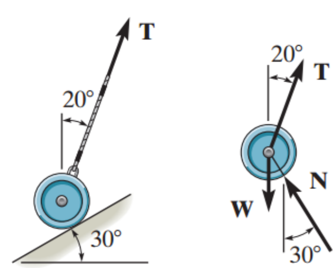
**Cables and Pulleys**



Cable is in tension
(b)

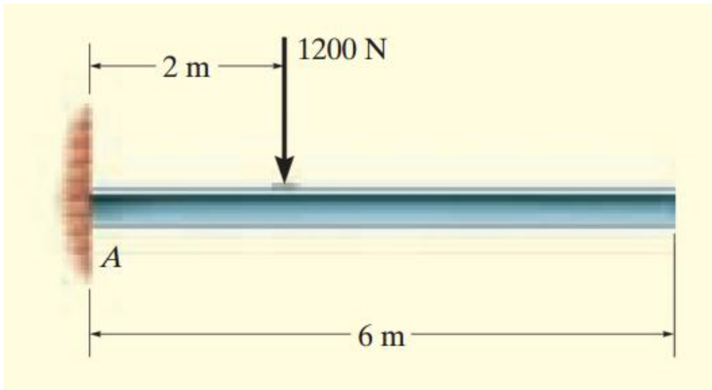
A cable can support only a tension or "pulling" force, and this force always acts in the direction of the cable. Hence, for any angle θ, shown in Fig. b, the cable is subjected to a constant tension **T** throughout its length.

**Smooth Contact**:




If an object rests on a smooth surface, then the surface will exert a force on the object that is normal to the surface at the point of contact.

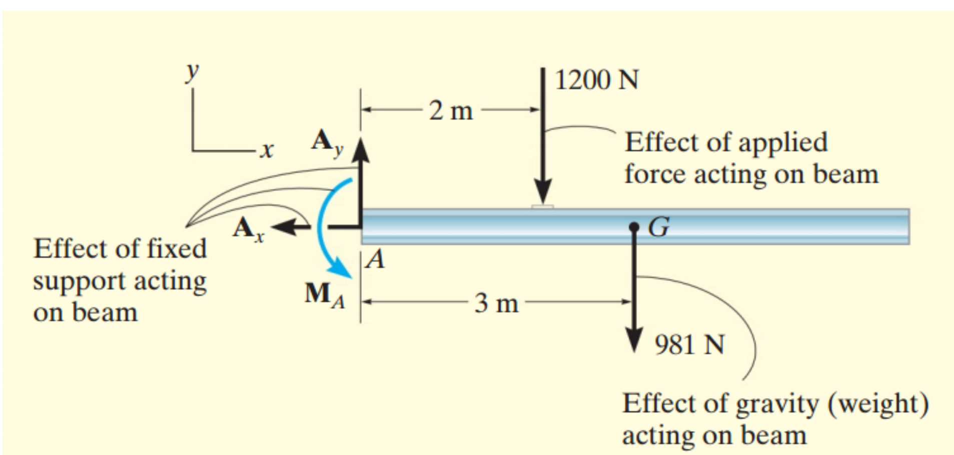
**Example:** Draw the free-body diagram of the uniform beam shown in Fig. below. The beam has a mass of 100 kg.



(a)

## Solution:

Since the support at A is fixed, the wall exerts three reactions on the beam, denoted as Ax, Ay, and MA. The magnitudes of these reactions are unknown, and their sense has been assumed. The weight of the beam, W = 100(9.81) N = 981 N, acts through the beam's center of gravity G, which is 3 m from A since the beam is uniform.



(b)