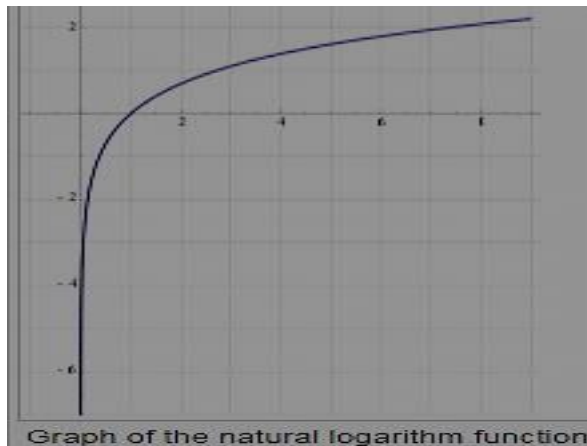


1.13 Derivative of natural logarithm, exponential and a^x function

1. Natural logarithm Function



Domain function = $x > 0$

Range function = $-\infty < y < +\infty$

Properties of Natural logarithm

$$\ln(x \cdot y) = \ln(x) + \ln(y)$$

$$\ln(x / y) = \ln(x) - \ln(y)$$

$$\ln(x^y) = y \cdot \ln(x)$$

$$\ln(1) = 0$$

Derivative of Natural logarithm

$$y = \ln u \rightarrow \frac{dy}{dx} = \frac{du}{u}$$

Example: $y = \ln(x^2 + 2x + 1)$ find $\bar{y}$

Solution //

$$\bar{y} = \frac{2x + 2}{x^2 + 2x + 1}$$

Example: $y = \ln x^{\sin x}$ find $\bar{y}$

Solution //

(From properties) $y = \sin x \ln x$

$$\bar{y} = \sin x * \frac{1}{x} + \ln x * \cos x$$

Example: $y = \ln \left(\frac{\sqrt{\cos x}}{x^2 \sin x} \right)$ find $\bar{y}$

Solution //

$$\bar{y} = \frac{x^2 \sin x * \left(\frac{1}{2} (\cos x)^{-\frac{1}{2}} * (-\sin x) - \sqrt{\cos x} * (x^2 * \cos x + \sin x * 2x) \right)}{\frac{\sqrt{\cos x}}{x^2 \sin x}}$$

Integration of Natural logarithm

$$\int \frac{du}{u} = \ln|u| + c$$

Example: Find $\int \frac{2x}{2x+2} dx$

Solution //

$$= \ln|2x + 2| + c$$

Example: Find $\int \frac{\cos x}{2+\sin x} dx$

Solution //

$$= \ln|2 + \sin x| + c$$

Example: Find $\int \frac{1}{\sqrt{x}(1+\sqrt{x})} dx$

Solution //

$$\begin{aligned} \int \frac{1}{\sqrt{x}(1+\sqrt{x})} dx & * \frac{2}{2} \\ &= 2 \ln|1 + \sqrt{x}| + c \end{aligned}$$

Example: Find $\int \tan x \, dx$

Solution //

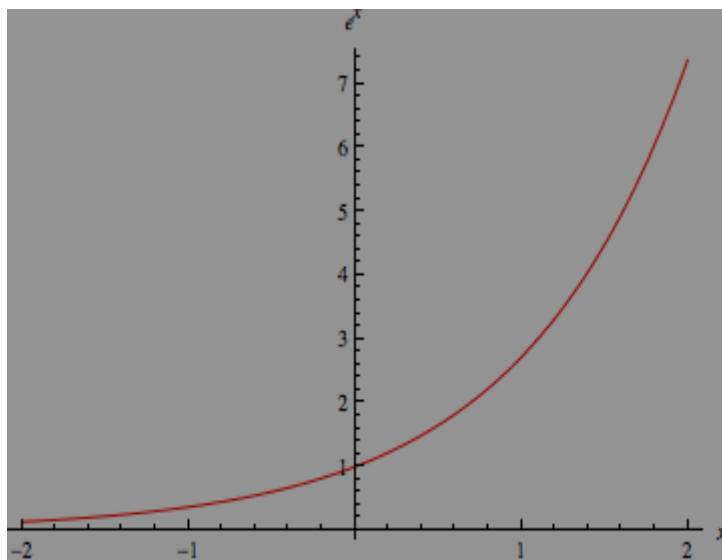
$$\int \frac{\sin x}{\cos x} dx = -\ln|\cos x| + c$$

Example: Find $\int 2x \tan(5x^2 - 1) \, dx$

Solution /

$$\int 2x \frac{\sin(5x^2 - 1)}{\cos(5x^2 - 1)} dx * \frac{-5}{-5} = \frac{-1}{5} \ln \cos(5x^2 - 1) + c$$

2. Exponential Function



Properties of e^x

1. $e^{\ln} \& e^{\ln} = 1$. Example: $e^{\ln \sin x} = \sin x$

2. $e^0 = 1 \& e^{-x} = \frac{1}{e^x}$

3. $(e^x)^r = e^{xr}$

4. $e^{x-y} = \frac{e^x}{e^y}$

5. $e^{x+y} = e^x \cdot e^y$

Derivative of e^x

$$y=e^u \rightarrow \frac{dy}{dx} = e^u du$$

Example: $y=e^{\sin^{-1} x}$ Find $\bar{y}$

Solution /

$$\bar{y} = e^{\sin^{-1} x} \frac{1}{\sqrt{1-x^2}}$$

Example: $y=\ln \frac{e^x}{1+e^x}$ Find $\bar{y}$

Solution //

$$\bar{y} = \frac{\frac{(1+e^x) * e^x * 1 - e^x * e^x * 1}{(1+e^x)^2}}{\frac{e^x}{1+e^x}} = \frac{1}{1+e^x}$$

Integration of e^x

$$\int e^u du = e^u + c$$

Example: Find $\int \frac{e^{\tan^{-1} x}}{1+x^2} dx$

Solution //

$$= e^{\tan^{-1} x}$$

Example: Find $\int x e^{x^2-3} dx$

Solution //

$$= \int x e^{x^2-3} dx * \frac{2}{2} = \frac{1}{2} e^{x^2-3} + c$$