

3. a^x function ($a=\text{constant}$)& $a > 0$

$$Y = a^x$$

$$\text{Domain function} = -\infty < x < +\infty$$

$$\text{Range function} = y > 0$$

Properties of a^x

$$1. a^x = e^{\ln a^x} = e^{x \ln a}$$

$$2. x^x = e^{\ln x^x} = e^{x \ln x}$$

Derivative of a^x

$$y = a^u \rightarrow \frac{dy}{dx} = a^u \ln a \frac{du}{dx}$$

Integration of a^x

$$\int a^u du = \frac{a^u}{\ln a} + c$$

Example: $y = 3^{\tan^{-1} x^2}$ Find $\bar{y}$

Solution //

$$\bar{y} = 3^{\tan^{-1} x^2} \frac{2x}{1 + (x^2)^2} * \ln 3$$

Example: $y = \ln x \cdot 3^{\sin x}$ Find $\bar{y}$

Solution //

$$\bar{y} = \ln x * (3^{\sin x} * \cos x * \ln 3) + 3^{\sin x} * \frac{1}{x}$$

Example: $y = \pi^x + x^\pi$ Find $\bar{y}$

Solution //

$$\bar{y} = \pi^x * (1) * \ln \pi + \pi x^{\pi-1}$$

Example: Find $\bar{y}$. $\pi^y = (\sin x)^x + \ln x + \pi^{\cos x} + x^{2\pi}$

Solution //

$$(\sin x)^x = e^{\ln (\sin x)^x} = e^{x \ln \sin x}$$

$$\pi^y = e^{x \ln \sin x} + \ln x + \pi^{\cos x} + x^{2\pi}$$

$$\pi^y * \bar{y} * \ln \pi$$

$$= e^{x \ln \sin x} \left(x \cdot \frac{\cos x}{\sin x} + \ln \sin x * 1 \right) + \frac{1}{x} + \pi^{\cos x} * (-\sin x) * \ln \pi + 2\pi x^{2\pi-1}$$

$$\bar{y} = \frac{e^{x \ln \sin x} \left(x \cdot \frac{\cos x}{\sin x} + \ln \sin x * 1 \right) + \frac{1}{x} + \pi^{\cos x} * (-\sin x) * \ln \pi + 2\pi x^{2\pi-1}}{\pi^y * \ln \pi}$$

Example: Find $\int 2^{\cos x} \sin x dx$

Solution //

$$= \frac{2^{\cos x} * (-1)}{\ln 2} + c$$

Example: Find $\int \frac{2^{\sin^{-1} x}}{\sqrt{1+x^2}} dx$

Solution//

$$= \frac{2^{\sin^{-1} x}}{\ln 2} + c$$

Example: Find $\int \frac{2^x}{1+4^x} dx$

Solution //

$$\begin{aligned} &= \int \frac{2^x}{1 + (2^x)^2} dx \\ &= \frac{\tan^{-1} 2^x}{\ln 2} + c \end{aligned}$$

4. Function of logarithm ($\log \frac{u}{a}$)

$$Y = \log \frac{u}{a} = \frac{\ln u}{\ln a}$$

Relation between logarithm & natural logarithm

Properties of logarithm function

$$1. \log_a \left(\frac{x}{y} \right) = \log_a x - \log_a y$$

$$2. \log_a (x \cdot y) = \log_a x + \log_a y$$

$$3. \log_a 1 = 0. \log_a a = 1. \log_a a^x = x. \log_2 10 = \frac{\ln 10}{\ln 2} = \frac{2.3}{0.6}$$

Derivative

$$y = \log_a u \rightarrow \frac{dy}{dx} = \frac{1}{\ln a} \frac{du}{u}$$

Example: $y = \log_3 \sin x$. Find $\bar{y}$

Solution //

$$\bar{y} = \frac{1}{\ln 3} \frac{\cos x}{\sin x}$$

