Al-Mustaqbal University

College of Engineering & Technology

Department of

Electrical Engineering Techniques

AC ELECTRICAL CIRCUITS

Asst. Lec.

ALI IMAD

8-Representing AC Voltages and Currents by Complex Numbers:

The sinusoidal voltage $e(t)$ =200 sin(wt + 40°) of Figure 14(a) and (b) can be represented by its phasor equivalent, $E = 200V\llcorner 40$, as in (c).

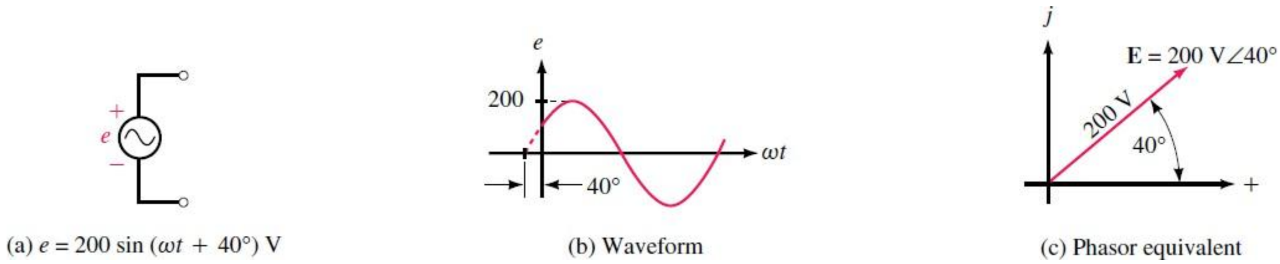


Figure 14

To convert between forms:

$$C = a + jb \qquad (rectangular\ form)$$

$$C = c\angle\theta \qquad (polar\ form)$$

$$a = c\ cos\theta, \qquad b = c\ sin\theta, \qquad c = \sqrt{a^2 + b^2},\ and \qquad \theta = tan^{-1}\frac{b}{a}$$

$$j = \sqrt{-1}, \qquad j^2 = -1, \qquad j^3 = -j, \qquad j^4 = 1,\ and \qquad j = \frac{-1}{j}$$

The conjugate of a complex number (denoted by an asterisk *) is a complex number with the same real part but the opposite imaginary part. Thus, the conjugate of $C = c\angle\theta = a + jb$ is $C^* = c\angle - \theta = a - jb$. For example, if $C = 3 + j4 = 5\angle 53.13^{\circ}$, then $C^* = 5\angle - 53.13^{\circ} = 3 - j4$.

EXAMPLE For Figure 15, $v_1 = \sqrt{2}(16) sinwt\ V, v_2 = \sqrt{2}(24)\sin(wt + 90^\circ)\,V,\quad v_3 = \sqrt{2}(15)\sin(wt - 90^\circ)\,V$. Determine source voltage *e*.

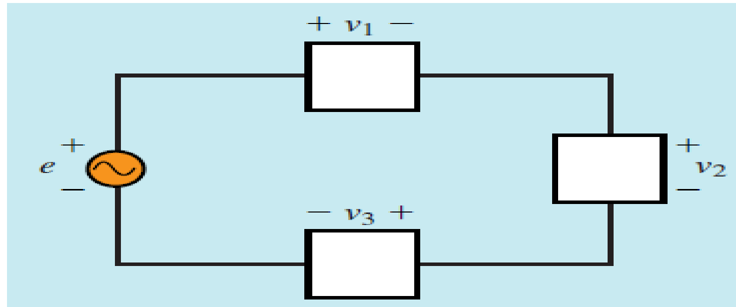


Figure 15

solution:

Applying KVL:

$$E = V_1 + V_2 + V_3 = 16\angle 0^\circ + 24\angle 90^\circ + 15\angle{-90}^\circ$$

$$= 16 + j0 + 24j - 15j = 16 + 9j = 18.36\angle 29.36^\circ$$

$$= \sqrt{2}(18.36)\sin(wt + 29.36^\circ)\,V$$

H.W: For figure 16, $i_1 = \sqrt{2}(23) sinwt\ mA,\ i_2 = \sqrt{2}(0.29)\sin(wt + 63^\circ)\,A$, and $i_2 = \sqrt{2}(127) \times 10^{-3}\sin(wt - 72^\circ)\,A$. Determine current i_T.

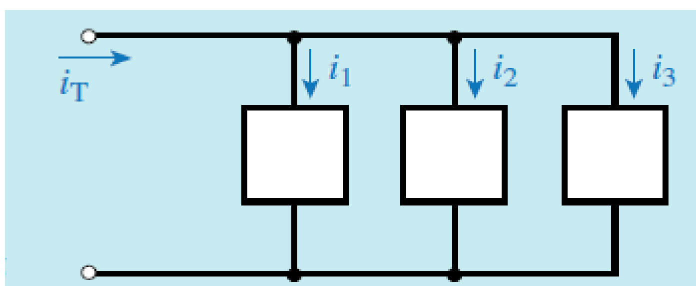


Figure 16

Ans. $i_T = \sqrt{2}(238) \times 10^{-3}\sin(wt + 35.4^\circ)\,A$

9.Ohm's Law for AC Circuits:

-Resistors

when a resistor is subjected to a sinusoidal voltage as shown in Figure 17, the resulting current is also sinusoidal and in phase with the voltage.

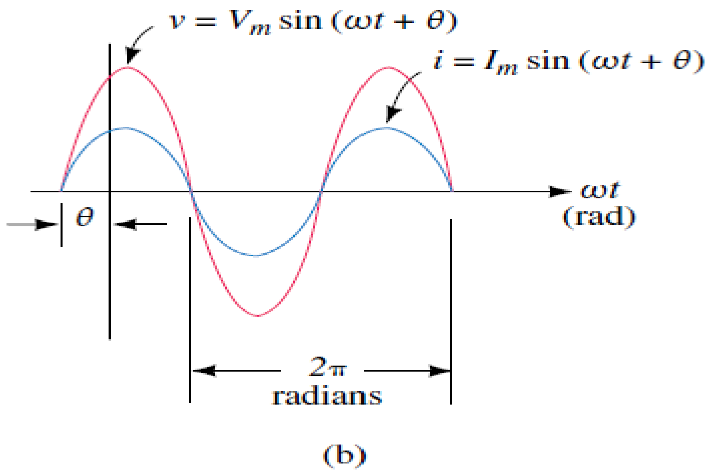


Figure 17

The sinusoidal voltage $v=V_m\sin(wt+\theta)$ may be written in phasor form as $\mathbf{V}=V\angle\theta^{\circ}$.Whereas the sinusoidal expression gives the instantaneous value of voltage for awaveform having an amplitude of V_m (volts peak), the phasor form has amagnitude which is the effective (or rms) value.

The voltage and current phasors may be shown on a phasor diagram as
in Figure 18.

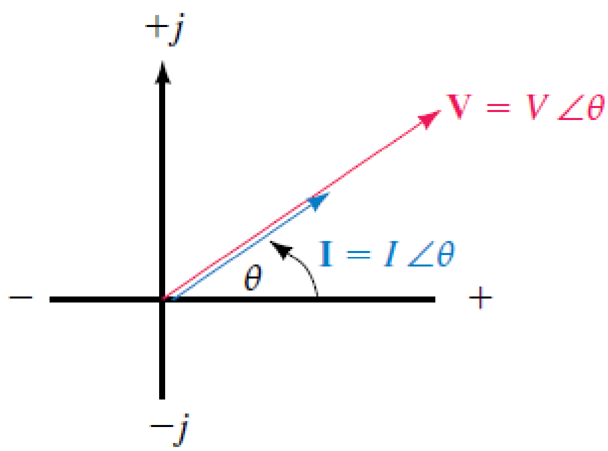


Figure 18

Example:

Refer to the resistor shown in Figure 19:

a. Find the sinusoidal current *i* using phasors.

b. Sketch the sinusoidal waveforms for *v* and *i*.

c. Sketch the phasor diagram of **V** and **I.**

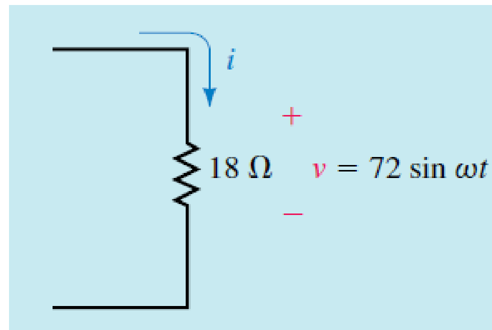


Figure 19

Solution:

a. The phasor form of the voltage is determined as follows:

$$v = 72 sinwt \quad \leftrightarrow 50.9V\angle 0^{\circ}$$

From Ohm's law, the current phasor is determined to be

$$I = \frac{V}{Z_R} = \frac{50.9V\angle 0^{\circ}}{18\Omega\angle 0^{\circ}} = 2.83A\angle 0^{\circ}$$

which results in the sinusoidal current waveform having an amplitude of $i = 4\ sinwt$

b &c. The voltage- current waveforms, and phasor are shown in Figure 20.

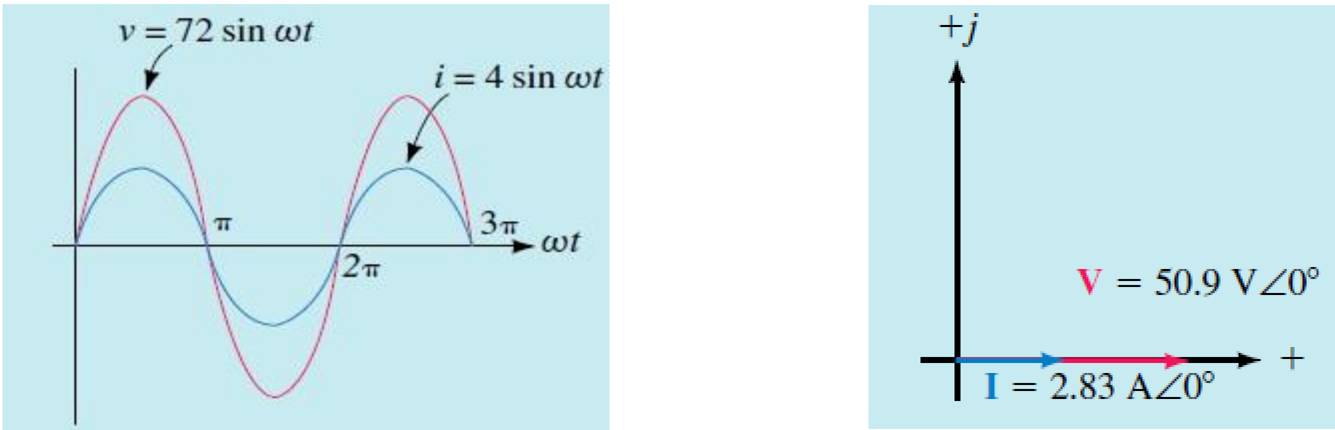


Figure 20

H.W:

Refer to the resistor of Figure 21:

a. Use phasor algebra to find the sinusoidal voltage, *v*.

b. Sketch the sinusoidal waveforms for *v* and *i*.

c. Sketch a phasor diagram showing **V** and **I.**

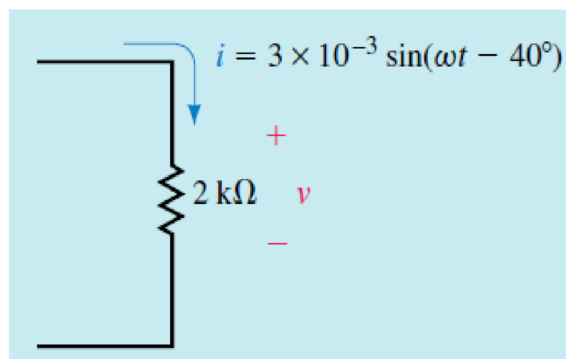


Figure 21

- **Inductors**

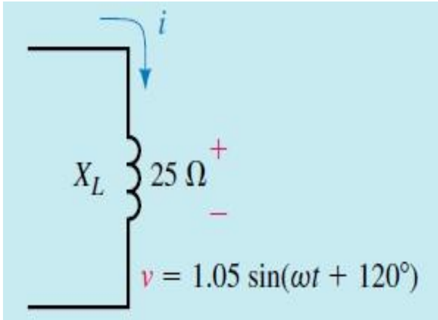
When an inductor is subjected to a sinusoidal current, a sinusoidal voltage is induced across the inductor such that the voltage across the inductor leads the current waveform by exactly 90°. If we know the reactance of an inductor, then from Ohm's law the current in the inductor may be expressed in phasor form as:

$$I = \frac{V}{Z_L} = \frac{V\angle\theta^\circ}{X_L\angle 90^\circ} = \frac{V}{X_L}\angle(\theta^\circ - 90^\circ)$$

In vector form, the reactance of the inductor is given as: $Z_L = X_L \angle 90^\circ$

$$where\ \ X_L = wL = 2\pi f L$$

Example: Consider the inductor shown in Figure 22:



a. Determine the sinusoidal expssion for the current *i* using phasors.

b. Sketch the sinusoidal waveforms for *v* and *i.*

c. Sketch the phasor diagram showing **V** and **I.**

Solution:

a. The phasor form of the voltage is determined as follows:

$$v = 1.05\sin(wt + 120^\circ)\ \leftrightarrow 0.742V\angle 120^\circ$$

From Ohm's law, the current phasor is determined to be

$$I = \frac{V}{Z_R} = \frac{0.742V\angle 120^\circ}{25\Omega\angle 90^\circ} = 29.7mA\angle 30^\circ$$

The amplitude of the sinusoidal current is

$$I_m = (\sqrt{2})(29.7) = 42\ mA$$

The current *i* is now written as

$$i = 0.042\sin(wt + 30^\circ)$$

b. Figure 23 shows the sinusoidal waveforms of the voltage and current.

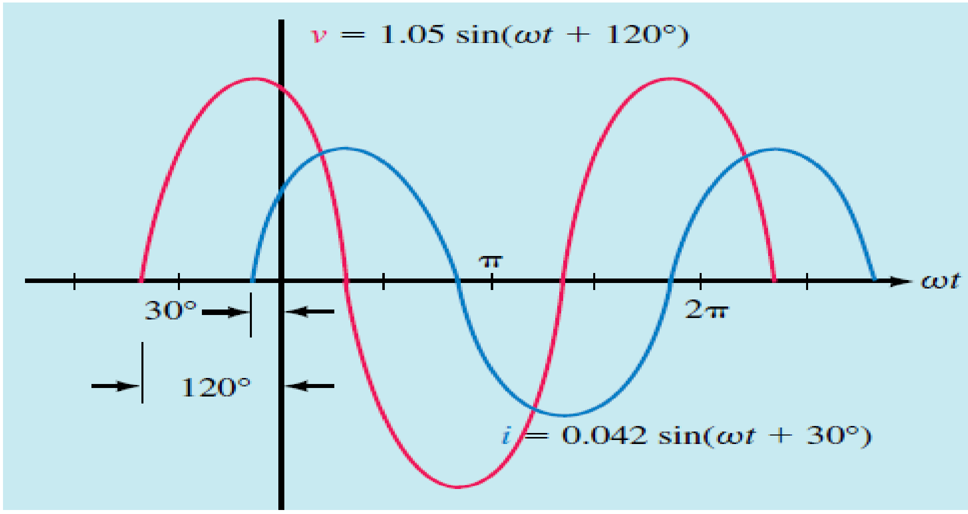


Figure 23

c. The voltage and current phasors are shown in Figure 23.

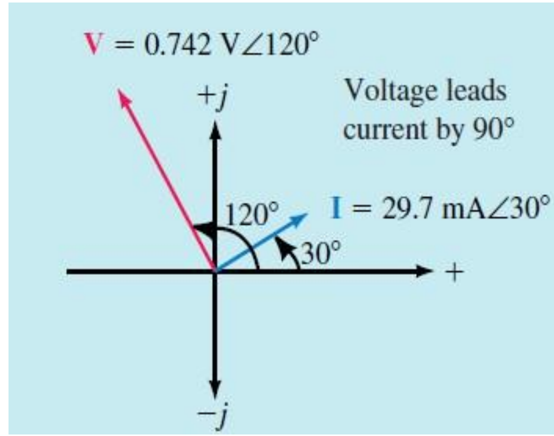


Figure23

- **Capacitors:**

When a capacitor is subjected to a sinusoidal voltage, a sinusoidal current results. The current through the capacitor leads the voltage by exactly 90°. If we know the reactance of a capacitor, then from Ohm's law the current in the capacitor expressed in phasor form is:

$$I = \frac{V}{Z_C} = \frac{V\angle\theta^\circ}{X_C\angle -90^\circ} = \frac{V}{X_C}\angle(\theta^\circ + 90^\circ)$$

In vector form, the reactance of the capacitor is given as

$$Z_C = X_C \angle -90^\circ \; where \;\; X_C = \frac{1}{wC} = \frac{1}{2\pi f C}$$

H.W: Consider the capacitor of Figure 24.

a. Find the voltage *v* across the capacitor.

b. Sketch the sinusoidal waveforms for *v* and *i*.

c. Sketch the phasor diagram showing **V** and **I.**

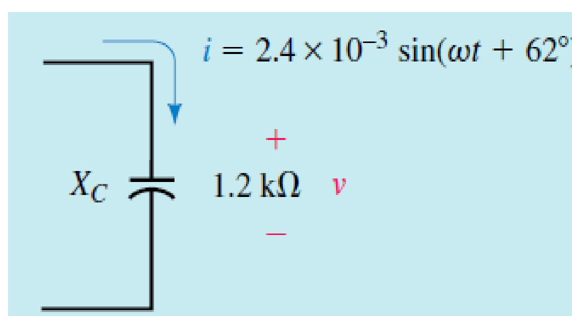


Figure 24

Practice problems:

Q1:

A voltage source, $\mathbf{E} = 10\ \text{V}\angle 30°$, is applied to an inductive impedance of $50\ \Omega$.

a. Solve for the phasor current, **I.**

b. Sketch the phasor diagram for **E** and **I.**

c. Write the sinusoidal expressions for *e* and *i.*

d. Sketch the sinusoidal expressions for *e* and *i.*

Answers:

a. $\mathbf{I} = 0.2\ \text{A}\angle -60°$

c. $e = 14.1 \sin(\omega t + 30°)$

$i = 0.283 \sin(\omega t - 60°)$

Q2:

A voltage source, $\mathbf{E} = 10\ \text{V}\angle 30°$, is applied to a capacitive impedance of $20\ \Omega$.

a. Solve for the phasor current, **I.**

b. Sketch the phasor diagram for **E** and **I.**

c. Write the sinusoidal expressions for *e* and *i.*

d. Sketch the sinusoidal expressions for *e* and *i.*

Answers:

a. $\mathbf{I} = 0.5\ \text{A}\angle 120°$

c. $e = 14.1 \sin(\omega t + 30°)$

$i = 0.707 \sin(\omega t + 120°)$