



جامعة المستقبل
AL MUSTAQBAL UNIVERSITY

كلية العلوم قسم الأمن السيبراني

Lecture: (5)

Code efficiency and redundancy

Subject: Coding Techniques

First Stage

Lecturer: Asst. Lecturer. Suha Alhussieny

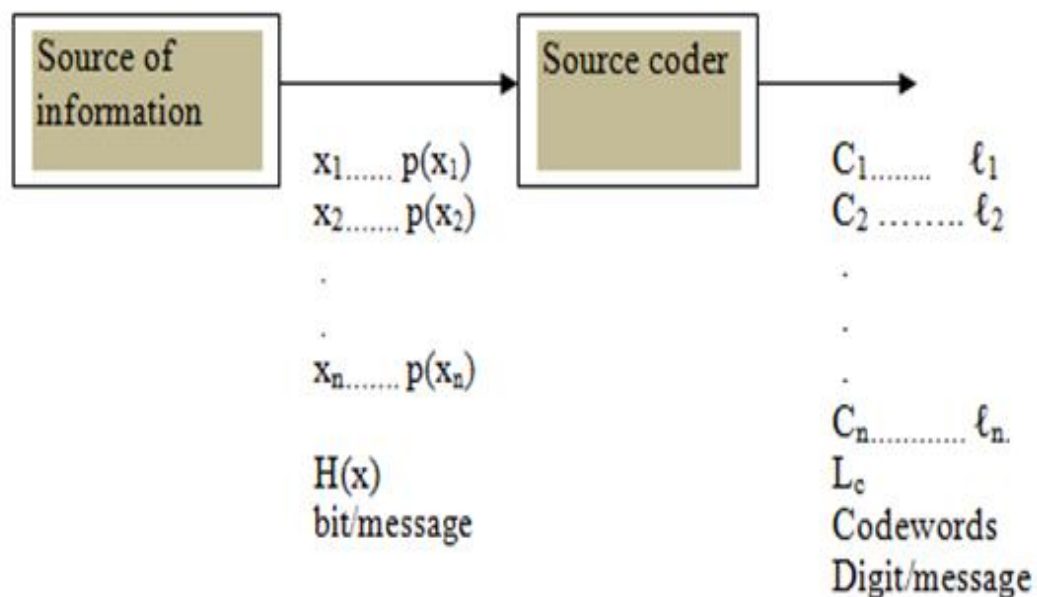


Source codes:

The source coder will transform the messages into a finite sequence of digits, called the codeword of the message. If binary digits (bits) are used in this codeword, then we obtain what is called " Binary Source Coding".

The aim of source coding is to produce a code which, on average, requires the transmission of the maximum amount of information for the fewest binary digits. This can be quantified by calculating the **efficiency** η of the code.

A code is a mapping from the discrete set of symbols $\{0, \dots, M - 1\}$ to finite binary sequences.





However before calculating efficiency we need to establish the length of the code. The length of a code is the average length of its code words and is obtained by:

$$L = \sum_{i=1}^n P_i l_i$$

For the purposes of efficiency. The average code length is minimized, where l_i is the number of digits in the i^{th} symbol and n is the number of symbols the code contains.

For fixed length code

$$L = l_i = \lceil \log_2 M \rceil \text{ where } M \text{ is the number of symbols.}$$

OR

1- $L_c = \log_2 n$ bit/message if $n = 2^r$ ($n = 2, 4, 8, 16, \dots$ and r is an integer) which gives $\eta = 100\%$

2- $L_c = \text{Int}[\log_2 n] + 1$ bits/message if $n \neq 2^r$ which gives less efficiency



Source Code Efficiency:

L = average length of the code

$$L = \sum_{i=1}^n P_i l_i \text{ bits/symbol.}$$

$$\xi_{code} = \frac{H(x)}{L} * 100\% \text{ where } \xi_{code} = \text{code Efficiency}$$

Redundancy of the Code:

$$R_{code} = \frac{L - H(x)}{L} * 100\% = \left(1 - \frac{H(x)}{L} \right) * 100\%$$
$$= (1 - \xi_{code}) * 100\% \text{ where } R_{code} = \text{Code Redundancy}$$

Example 1:

Let $x = \{ x_1, x_2, \dots, x_{16} \}$ where $P_i = 1/16$ for all i , find $\xi_{source code}$

Sol:

$$H(x) = \log_2 M = \log_2 16 = 4 \text{ bits/symbol (because } P_1 = P_2 = \dots = P_{16} = 1/M)$$

$$L = \lceil \log_2 M \rceil$$

$$L = \lceil \log_2 16 \rceil = 4 \text{ bits/symbol.}$$

$$\text{Code Redundancy} = L - H.$$



$$R = 4 - 4 = 0.$$

$$\therefore \xi_{source\ code} = \frac{H(x)}{L} * 100\% = \frac{4}{4} * 100\% = 100\%$$

Example 2:

Let $x = \{ x_1, x_2, \dots, x_{12} \}$ where $P_i = 1/12$ for all i , find $\xi_{source\ code}$

Sol:

$$H(x) = \log_2 M = \log_2 12 = 3.585 \text{ bit/symbol (because } P_1 = P_2 = \dots = P_{12} = 1/M \text{)}$$

$$L_c = \text{Int}[\log_2 n] + 1 \quad \text{bits/message}$$

$$L = \text{Int} [\log_2 12] + 1 = 4 \text{ bits/symble}$$

$$\text{Code Redundancy} = L - H.$$

$$R = 4 - 3.585 = 0.415.$$

$$\therefore \xi_{source\ code} = \frac{H(x)}{L} * 100\% = \frac{3.585}{4} * 100\% = 89 \%$$



Example 3:

For ten equi-probable messages coded in a fixed length code, find the efficiency.

Sol:

$$p(x_i) = \frac{1}{10} \text{ and } L_C = \text{Int}[\log_2 10] + 1 = 4 \text{ bits}$$

$$\eta = \frac{H(X)}{L_C} \times 100\% = \frac{\log_2 10}{4} \times 100\% = 83.048\%$$

Example 4:

For eight equi-probable messages coded in a fixed length code, find the efficiency

Sol:

$$p(x_i) = \frac{1}{8} \text{ and } L_C = \log_2 8 = 3 \text{ bits and } \eta = \frac{3}{3} \times 100\% = 100\%$$